VERSION 1 CAPS

GRADE 11
PHYSICAL SCIENCES

WRITTEN BY VOLUNTEERS

EVERYTHING SCIENCE
BY
SIYAVULA
TECHNOLOGY-POWERED LEARNING

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

METROPOLITAN

Periodic Table of the Elements

No EN **Element** AMU

1 IA	2 IIA	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB	8 VII	9 VII	10 VII	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 0
1 2,1 **H** 1,01																	2 **He** 4,00
3 1,0 **Li** 6,94	4 1,5 **Be** 9,01											5 2,0 **B** 10,8	6 2,5 **C** 12,0	7 3,0 **N** 14,0	8 3,5 **O** 16,0	9 4,0 **F** 19,0	10 **Ne** 20,2
11 0,9 **Na** 23,0	12 1,2 **Mg** 24,3											13 1,5 **Al** 27,0	14 1,8 **Si** 28,1	15 2,1 **P** 31,0	16 2,5 **S** 32,1	17 3,0 **Cl** 35,45	18 **Ar** 39,9
19 0,8 **K** 39,1	20 1,0 **Ca** 40,1	21 1,3 **Sc** 45,0	22 1,5 **Ti** 47,9	23 1,6 **V** 50,9	24 1,6 **Cr** 52,0	25 1,5 **Mn** 54,9	26 1,8 **Fe** 55,8	27 1,8 **Co** 58,9	28 1,8 **Ni** 58,7	29 1,9 **Cu** 63,5	30 1,6 **Zn** 65,4	31 1,6 **Ga** 69,7	32 1,8 **Ge** 72,6	33 2,0 **As** 74,9	34 2,4 **Se** 79,0	35 2,8 **Br** 79,9	36 **Kr** 83,8
37 0,8 **Rb** 85,5	38 1,0 **Sr** 87,6	39 1,2 **Y** 88,9	40 1,4 **Zr** 91,2	41 1,6 **Nb** 92,9	42 1,8 **Mo** 95,9	43 1,9 **Tc** (98)	44 2,2 **Ru** 101,1	45 2,2 **Rh** 102,9	46 2,2 **Pd** 106,4	47 1,9 **Ag** 107,9	48 1,7 **Cd** 112,4	49 1,7 **In** 114,8	50 1,8 **Sn** 118,7	51 1,9 **Sb** 121,8	52 2,1 **Te** 127,6	53 2,5 **I** 126,9	54 **Xe** 131,3
55 0,7 **Cs** 132,9	56 0,9 **Ba** 137,3	57-71 **La-Lu** Lanthanides	72 1,6 **Hf** 178,5	73 1,5 **Ta** 180,9	74 2,4 **W** 183,8	75 1,9 **Re** 186,2	76 2,2 **Os** 190,2	77 2,2 **Ir** 192,2	78 2,2 **Pt** 195,1	79 2,5 **Au** 197,0	80 2,0 **Hg** 200,6	81 1,6 **Tl** 204,4	82 1,8 **Pb** 207,2	83 1,9 **Bi** 209,0	84 2,0 **Po** (209)	85 2,5 **At** (210)	86 **Rn** (222)
87 0,7 **Fr** (223)	88 0,9 **Ra** 226,0	89-103 **Ac-Lr** Actinides	104 **Rf** (261)	105 **Db** (262)	106 **Sg** (263)	107 **Bh** (262)	108 **Hs** (265)	109 **Mt** (266)	110 **Ds** (269)	111 **Rg** (272)	112 **Cn** (277)	113 **Uut** (284)	114 **Uuq** (289)	115 **Uup** (288)	116 **Uuh** (293)	117 **Uus** (282)	118 **Uuo** (282)
			57 1,1 **La** 138,9	58 1,1 **Ce** 140,1	59 1,1 **Pr** 140,9	60 1,1 **Nd** 144,2	61 **Pm** (145)	62 1,1 **Sm** 150,4	63 **Eu** 152,0	64 1,2 **Gd** 157,3	65 **Tb** 158,9	66 1,2 **Dy** 162,5	67 1,2 **Ho** 164,9	68 1,2 **Er** 167,3	69 1,3 **Tm** 168,9	70 **Yb** 173,0	71 1,3 **Lu** 175,0
			89 1,1 **Ac** 227,0	90 1,3 **Th** 232,0	91 1,5 **Pa** 231,0	92 1,4 **U** 238,0	93 1,3 **Np** 237,0	94 1,3 **Pu** (244)	95 1,3 **Am** (243)	96 1,3 **Cm** (247)	97 1,3 **Bk** (247)	98 1,3 **Cf** (251)	99 1,3 **Es** (252)	100 1,3 **Fm** (257)	101 1,3 **Md** (258)	102 1,3 **No** (258)	103 **Lr** (260)

- Transition Metal
- Metal
- Metalloid
- Non-metal
- Noble Gas
- Lanthanide
- Actinide

EVERYTHING SCIENCE

GRADE 11 PHYSICAL SCIENCES

VERSION 1 CAPS

WRITTEN BY VOLUNTEERS

AUTHORS AND CONTRIBUTORS

Siyavula Education

Siyavula Education is a social enterprise launched in 2012 with capital and support from the PSG Group Limited and the Shuttleworth Foundation. The Everything Maths and Science series is one of the titles developed and openly released by Siyavula. For more information about the writing and distribution of these or other openly licensed titles:

www.siyavula.com
info@siyavula.com
021 469 4771

Siyavula Authors

Dr. Mark Horner; Heather Williams

Siyavula and DBE team

Jayanthi SK Maharaj (Veena); Marongwa Masemula; Ewald Zietsman; Bridget Nash; Prof. Gilberto Isquierdo; Karen Kornet; Dr. Kevin Reddy; Enoch Ndwamato Makhado; Clive Mhaka

Siyavula and Free High School Science Text contributors

Dr. Mark Horner; Dr. Samuel Halliday; Dr. Sarah Blyth; Dr. Rory Adams; Dr. Spencer Wheaton

Iesrafeel Abbas; Sarah Abel; Dr. Rory Adams; Andrea Africa; Wiehan Agenbag; Matthew Amundsen; Ben Anhalt; Prashant Arora; Amos Baloyi; Bongani Baloyi; Raymond Barbour; Caro-Joy Barendse; Richard Baxter; Tara Beckerling; Tim van Beek; Mariaan Bester; Jennifer de Beyer; Dr. Sarah Blyth; Sebastian Bodenstein; Martin Bongers; Thinus Booysen; Gareth Boxall; Stephan Brandt; Hannes Breytenbach; Alexander Briell; Wilbur Britz; Graeme Broster; Craig Brown; Michail Brynard; Deanne de Bude; Richard Burge; Bianca B˚hmer; Jan Buys; George Calder-Potts; Eleanor Cameron; Mark Carolissen; Shane Carollisson; Richard Case; Sithembile Cele; Alice Chang; Richard Cheng; Fanny Cherblanc; Dr. Christine Chung; Brett Cocks; RochÈ Compaan; Willem Conradie; Stefaan Conradie; Rocco Coppejans; Tim Craib; Andrew Craig; Tim Crombie; Dan Crytser; Jock Currie; Dr. Anne Dabrowski; Laura Daniels; Gareth Davies; Sandra Dickson; Sean Dobbs; Buhle Donga; William Donkin; Esmi Dreyer; Matthew Duddy; Christel Durie; Fernando Durrell; Dr. Dan Dwyer; Frans van Eeden; Alexander Ellis; Tom Ellis; Andrew Fisher; Giovanni Franzoni; Olivia Gillett; Ingrid von Glehn; Tamara von Glehn; Lindsay Glesener; Kevin Godby; Dr. Vanessa Godfrey; Terence Goldberg; Dr. Johan Gonzalez; Saaligha Gool; Hemant Gopal; Dr. Stephanie Gould; Umeshree Govender; Heather Gray; Lynn Greeff; Jaco Greyling; Martli Greyvenstein; Carine Grobbelaar; Suzanne GrovÈ; Dr. Tom Gutierrez; Brooke Haag; Kate Hadley; Alex Hall; Dr. Sam Halliday; Asheena Hanuman; Dr. Melanie Dymond Harper; Ebrahim Harris; Dr. Nicholas Harrison; Neil Hart; Nicholas Hatcher; Jason Hayden; Laura Hayward; Dr. William P. Heal; Pierre van Heerden; Dr. Fritha Hennessy; Dr. Colleen Henning; Shaun Hewitson; Millie Hilgart; Grant Hillebrand; Nick Hobbs; Chris Holdsworth; Dr. Benne Holwerda; Dr. Mark Horner; Robert Hovden; Mfandaidza Hove; Jennifer Hsieh; George Hugo; Laura Huss; Prof. Ed Jacobs

Hester Jacobs; Stefan Jacobs; Rowan Jelley; Grant Jelley; Clare Johnson; Francois Jooste; Luke Jordan; Tana Joseph; Corli Joubert; Dr. Fabian Jutz; Brian Kamanzi; Herman Kamper; Dr. Lutz Kampmann; Simon Katende; Natalia Kavalenia; Rabia Khan; Nothando Khumalo; Paul Kim; Lizl King; Melissa Kistner; Dr. Jennifer Klay; Andrea Koch; Grove Koch; Bishop Komolafe; Dr. Timo Kriel; Lara Kruger; Sihle Kubheka; Andrew Kubik; Luca Lategan; Dr. Jannie Leach; Nkoana Lebaka; Dr. Marco van Leeuwen; Dr. Tom Leinster; Ingrid Lezar; Henry Liu; Christopher Loetscher; Linda Loots; Michael Loseby; Bets Lourens; Chris Louw; Amandla Mabona; Malothe Mabutho; Stuart Macdonald; Dr. Anton Machacek; Tshepo Madisha; Batsirai Magunje; Dr. Komal Maheshwari; Michael Malahe; Masoabi Malunga; Kosma von Maltitz; Masilo Mapaila; Bryony Martin; Nicole Masureik; Jacques Masuret ; John Mathew; Dr. Will Matthews; Chiedza Matuso; JoEllen McBride; Nikolai Meures; Margaretha Meyer; Riana Meyer; Filippo Miatto; Jenny Miller; Rossouw Minnaar; Abdul Mirza; Colin Mkhize; Mapholo Modise; Carla Moerdyk; Tshwarelo Mohlala; Relebohile Molaoa; Marasi Monyau; Asogan Moodaly; Jothi Moodley; Robert Moon; Calvin Moore; Bhavani Morarjee; Kholofelo Moyaba; Nina Gitau Muchunu; Christopher Muller; Helgard Muller; Johan Muller; Caroline Munyonga; Alban Murewi; Kate Murphy; Emmanuel Musonza; Tom Mutabazi; David Myburgh; Johann Myburgh; Kamie Naidu; Nolene Naidu; Gokul Nair; Vafa Naraghi; Bridget Nash; Eduan NaudÈ; Tyrone Negus; Theresa Nel; Huw Newton-Hill; Buntu Ngcebetsha; Towan Nothling; Dr. Markus Oldenburg; Adekunle Oyewo; Thomas OíDonnell; Dr. Jaynie Padayachee; Poveshen Padayachee; Masimba Paradza; Quinton Paulse; Dave Pawson; Justin Pead; Carli Pengilly; Nicolette Pekeur; Joan Pienaar; Petrus Pieter; Sirika Pillay; Jacques Plaut; Jaco du Plessis; Barry Povey; Barry Povey; Andrea Prinsloo; David Prinsloo; Joseph Raimondo; Sanya Rajani; Alastair Ramlakan; Thinus Ras; Dr. Matina J. Rassias; Ona Rautenbach; Dr. Jocelyn Read; Jonathan Reader; Jane Reddick; Robert Reddick; Dr. Matthew Reece; Chris Reeders; Razvan Remsing; Laura Richter; Max Richter; Sean Riddle; Dr. David Roberts; Christopher Roberts; Helen Robertson; Evan Robinson; Christian Roelofse; Raoul Rontsch; Dr. Andrew Rose; Katie Ross; Jeanne-MariÈ Roux; Karen Roux; Mark Roux; Bianca Ruddy; Heinrich Rudman; Nitin Rughoonauth; Katie Russell; Steven Sam; Jason Avron Samuels; Dr. Carl Scheffler; Nathaniel Schwartz; Duncan Scott; Christo van Schalkwyk; Rhoda van Schalkwyk; Helen Seals; Relebohile Sefako; Prof. Sergey Rakityansky; Sandra Serumaga-Zake; Paul Shangase; Cameron Sharp; Ian Sherratt; Dr. James Short; Cho Hee Shrader; Roger Sieloff; Brandon Sim; Bonga Skozana; Clare Slotow; Bradley Smith; Greg Solomon; Nicholas Spaull; Hester Spies; Dr. Andrew Stacey; Dr. Jim Stasheff; Mike Stay; Nicol Steenkamp; Dr. Fred Strassberger; Mike Stringer; Stephanie Strydom; Abdulhuck Suliman; Masixole Swartbooi; Tshenolo Tau; Tim Teatro; Ben Thompson; Shen Tian; Xolani Timbile; Liezel du Toit; Nicola du Toit; Dr. Francois Toerien; RenÈ Toerien; Dr. Johan du Toit; Robert Torregrosa; Jimmy Tseng; Pieter Vergeer; Rizmari Versfeld; Nina Verwey; Mfundo Vezi; Mpilonhle Vilakazi; Wetsie Visser; Alexander Volkwyn; Mia de Vos; Dr. Karen Wallace; John Walmsley; Helen Waugh; Leandra Webb; Dr. Dawn Webber; Michelle Wen; Dr. Rufus Wesi; Francois Wessels; Wessel Wessels; Neels van der Westhuizen; Sabet van der Westhuizen; Dr. Alexander Wetzler; Dr. Spencer Wheaton; Vivian White; Dr. Gerald Wigger; Harry Wiggins; Heather Williams; Wendy Williams; Julie Wilson; Timothy Wilson; Andrew Wood; Emma Wormauld; Dr. Sahal Yacoob; Jean Youssef; Ewald Zietsman; Johan Zietsman; Marina van Zyl

SPONSOR

This text book was developed with corporate social investment funding from MMI Holdings.

MMI FOUNDATION

Well structured, impactful Corporate Social Investment (CSI) has the ability to contribute positively to nation building and drive positive change in the communities. MMI's commitment to social investment means that we are constantly looking for ways in which we can assist some of South Africa's most vulnerable citizens to expand their horizons and gain greater access to life's opportunities. This means that we do not view social investment as a nice to have or as an exercise in marketing or sponsorship but rather as a critical part of our contribution to society.

The merger between Metropolitan and Momentum was lauded for the complementary fit between two companies. This complementary fit is also evident in the focus areas of CSI programmes where Metropolitan and Momentum together cover and support the most important sectors and where the greatest need is in terms of social participation.

HIV/AIDS is becoming a manageable disease in many developed countries but in a country such as ours, it remains a disease where people are still dying of this scourge unnecessarily. Metropolitan continues to make a difference in making sure that HIV AIDS moves away from being a death sentence to a manageable disease. Metropolitan's other focus area is education which remains the key to economic prosperity for our country.

Momentum's focus on persons with disabilities ensures that this community is included and allowed to make their contribution to society. Orphaned and vulnerable children are another focus area for Momentum and projects supported ensure that children are allowed to grow up safely, to assume their role along with other children in inheriting a prosperous future.

momentum METROPOLITAN

EVERYTHING MATHS AND SCIENCE

The Everything Mathematics and Science series covers Mathematics, Physical Sciences, Life Sciences and Mathematical Literacy.

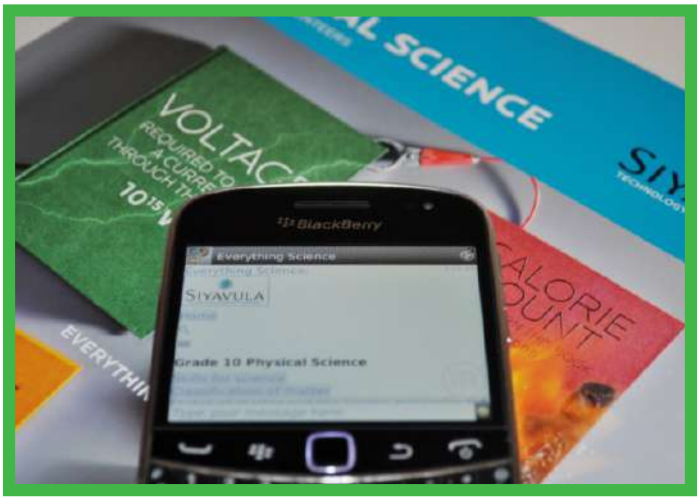
The Siyavula Everything Science textbooks

The Siyavula Everything Maths textbooks

READ ON MOBILE

MOBI-WEB

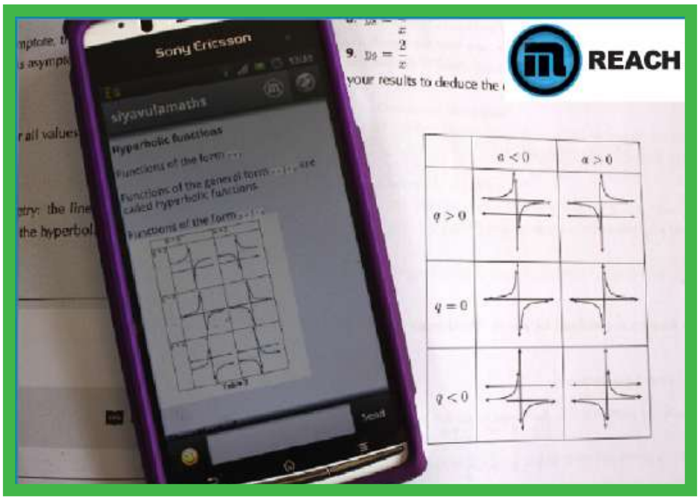
You can read all of the Everything Series textbooks on your mobile phone. Visit the Everything Maths and Everything Science mobi sites at:



m.everythingmaths.co.za and ***m.everythingscience.co.za***

MXIT

All Mxit users can read their Everything Series textbooks on Mxit Reach. Add Everything Maths and Everything Science as Mxit contacts or browse to the books on Mxit Reach.

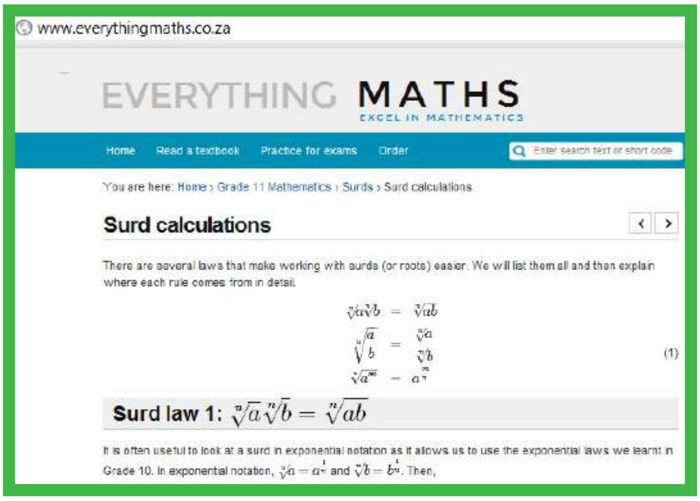


mxit>tradepost>reach>education>everything maths or everything science

DIGITAL TEXTBOOKS

• *READ ONLINE*

The on-line books feature videos, presentations, simulations and fully worked solutions to the questions and exercises found in the book.



www.everythingmaths.co.za and ***www.everythingscience.***

• *DOWNLOAD FOR TABLETS*

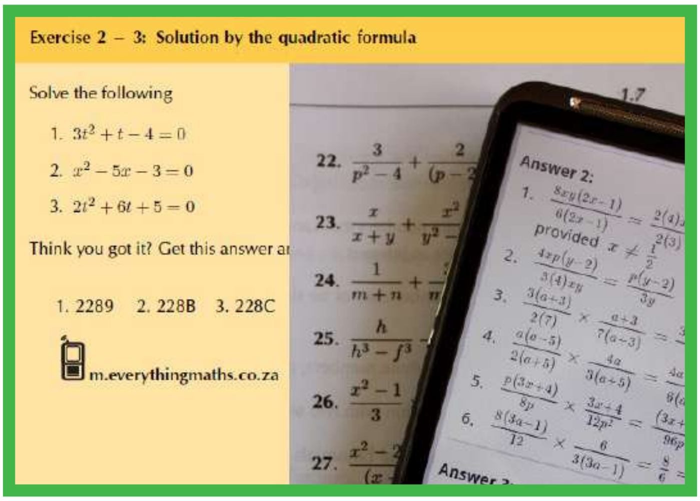
For off-line reading on your PC, tablet, iPad and Kindle you can download a digital copy of the Everything Series textbooks. Visit the Everything Maths and Everything Science websites and download the books.

www.everythingmaths.co.za and ***www.everythingscience.co.za***

PRACTISE INTELLIGENTLY

• *CHECK YOUR ANSWERS ON YOUR PHONE*

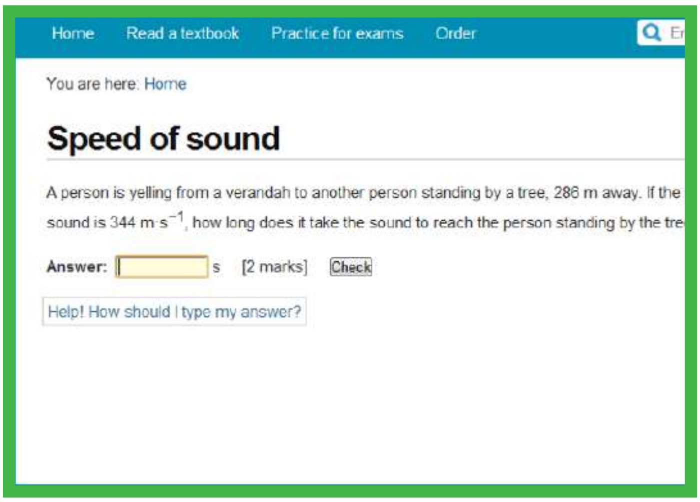
You can check your answer to any question in this textbook on your mobile phone by entering the shortcode found in the textbook into the search box on the mobi-site.



m.everythingmaths.co.za and ***m.everythingscience.co.za***

• *PRACTISE FOR TESTS AND EXAMS ON YOUR PHONE*

To do well in tests and exams you need practice. Practise the exercises from this text-book, additional exercises and questions from past exam papers on ***m.everythingmaths.co.za*** and ***m.everythingscience.co.za*** and Mxit Reach.

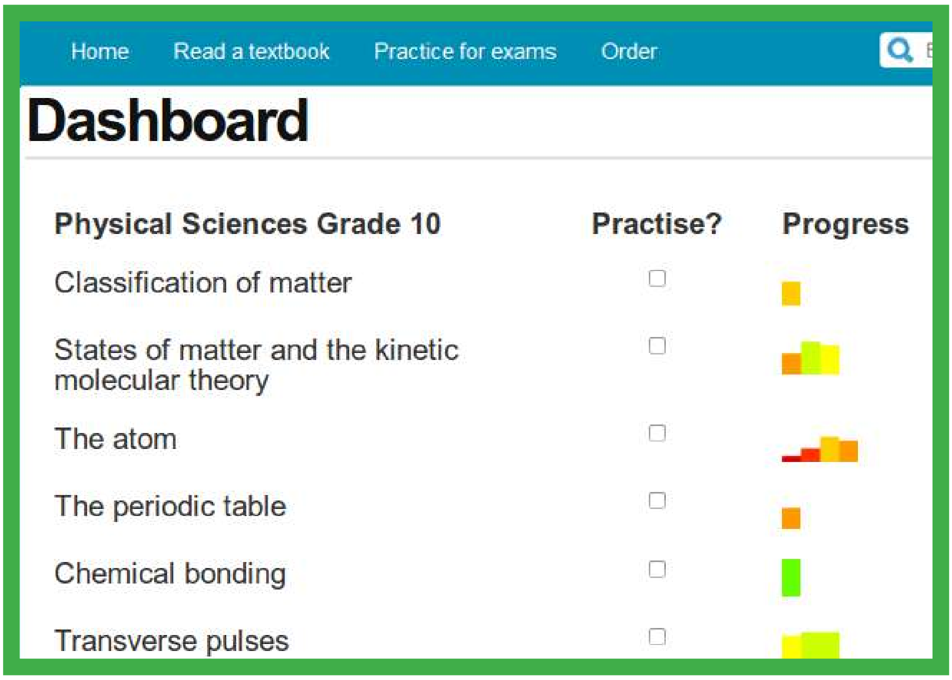


m.everythingmaths.co.za and ***m.everythingscience.co.za***

MANAGE YOUR STUDIES

• *YOUR DASHBOARD*

If you complete you practice homework and test questions at ***m.everythingmaths.co.za*** or ***m.everythingscience.co.za***, you can track of your work. Your dashboard will show you your progress and mastery for every topic in the book and help you to manage your studies. You can use your dashboard to show your teachers, parents, universities or bursary institutions what you have done during the year.



EVERYTHING SCIENCE

When we look outside at everything in nature, look around us at everything manufactured or look up at everything in space we cannot but be struck by the incredible diversity and complexity of life; so many things, that look so different, operating in such unique ways. The physical universe really contains incredible complexity.

Yet, what is even more remarkable than this seeming complexity is the fact that things in the physical universe are knowable. We can investigate them, analyse them and understand them. It is this ability to understand the physical universe that allows us to transform elements and make technological progress possible.

If we look back at some of the things that developed over the last century ñ space travel, advances in medicine, wireless communication (from television to mobile phones) and materials a thousand times stronger than steel we see they are not the consequence of magic or some inexplicable phenomena. They were all developed through the study and systematic application of the physical sciences. So as we look forward at the 21st century and some of the problems of poverty, disease and pollution that face us, it is partly to the physical sciences we need to turn.

For however great these challenges seem, we know that the physical universe is knowable and that the dedicated study thereof can lead to the most remarkable advances. There can hardly be a more exciting challenge than laying bare the seeming complexity of the physical universe and working with the incredible diversity therein to develop products and services that add real quality to peopleís lives.

Physical sciences is far more wonderful, exciting and beautiful than magic! It is everywhere.

Contents

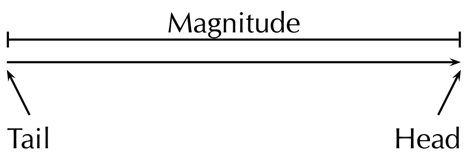
CHAPTER 1

Vectors in two dimensions

1.1 Introduction ESBK2

In grade 10 you learnt about vectors in one dimension. Now we will take these concepts further and learn about vectors in two dimensions as well as components of vectors.

As a very short recap, a vector has both a magnitude and a direction. There are many physical quantities, like forces, that are well described by vectors (called or known as vector quantities). We often use arrows to represent vectors visually because the length of the arrow can be related to the magnitude and the arrowhead can indicate the direction. We will talk about the head, tail and magnitude of a vector when using arrows to represent them. Below is a diagram showing a vector (the arrow). The magnitude is indicated by the length and the labels show the the tail and the head of the vector. The direction of the vector is indicated by the direction in which the arrow is pointing.



When we write the symbol for a physical quantity represented by a vector we draw an arrow over it to signify that it is a vector. If the arrow is left out then we are referring only to the magnitude of the vector quantity.

Key Mathematics Concepts

- Theorem of Pythagoras — Mathematics, Grade 10, Analytical geometry
- Units and unit conversions — Physical Sciences, Grade 10, Science skills
- Equations — Mathematics, Grade 10, Equations and inequalities
- Trigonometry — Mathematics, Grade 10, Trigonometry
- Graphs — Mathematics, Grade 10, Functions and graphs

1.2 Resultant of perpendicular vectors ESBK3

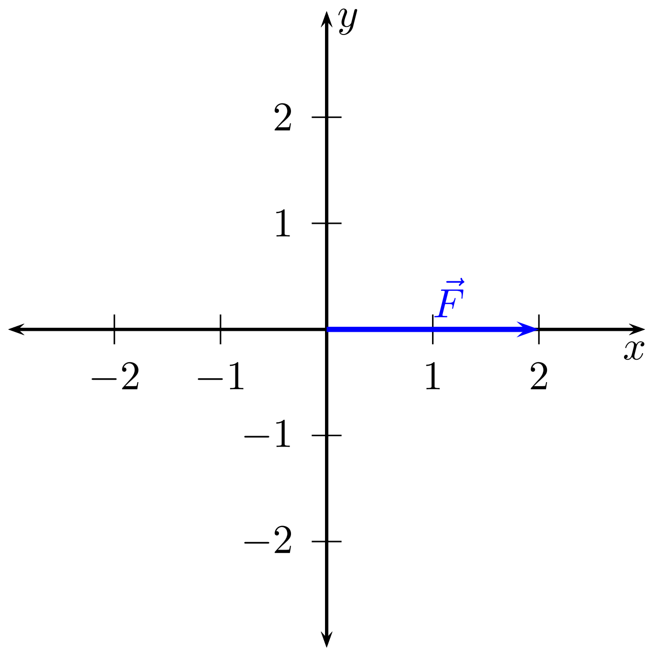
In grade 10 you learnt about the resultant vector in one dimension, we are going to extend this to two dimensions. As a reminder, if you have a number of vectors (think forces for now) acting at the same time you can represent the result of all of them together with a single vector known as the resultant. The resultant vector will have the **same** effect as all the vectors adding together.

We will focus on examples involving forces but it is very **important** to remember that this applies to all physical quantities that can be described by vectors, forces, displacements, accelerations, velocities and more.

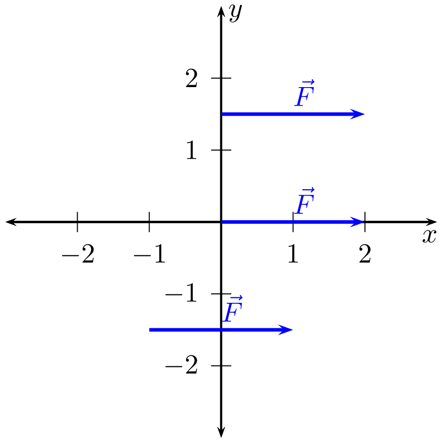
Vectors on the Cartesian plane ESBK4

The first thing to make a note of is that in Grade 10 we worked with vectors all acting in a line, on a single axis. We are now going to go further and start to deal with two dimensions. We can represent this by using the Cartesian plane which consists of two perpendicular (at a right angle) axes. The axes are a x-axis and a y-axis. We normally draw the x-axis from left to right (horizontally) and the y-axis up and down (vertically).

We can draw vectors on the Cartesian plane. For example, if we have a force, $\vec{F}$, of magnitude 2 N acting in the positive x-direction we can draw it as a vector on the Cartesian plane.

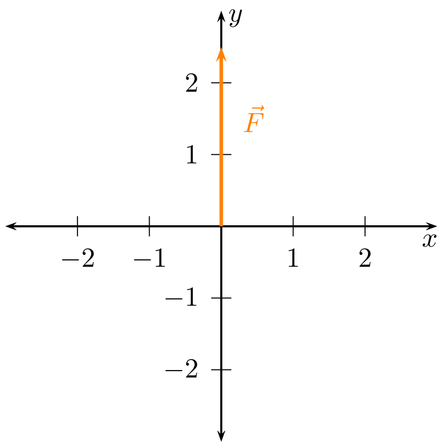


Notice that the length of the vector as measured using the axes is 2, the magnitude specified. A vector doesn't have to start at the origin but can be placed anywhere on the Cartesian plane. Where a vector starts on the plane doesn't affect the physical quantity as long as the magnitude and direction remain the same. That means that all of the vectors in the diagram below can represent the same force. This property is know as *equality of vectors*.

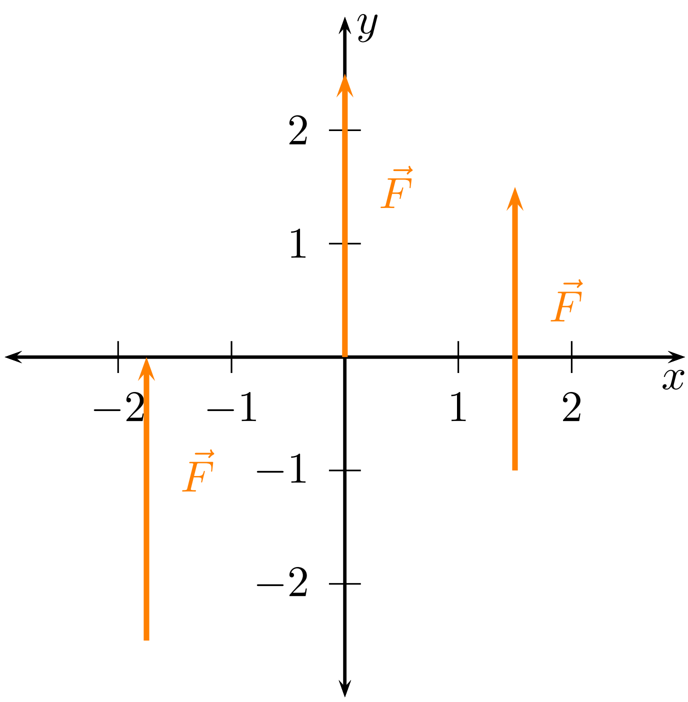


In the diagram the vectors have the same magnitude because the arrows are the same **length** and they have the same **direction**. They are all parallel to the x-direction and parallel to each other.

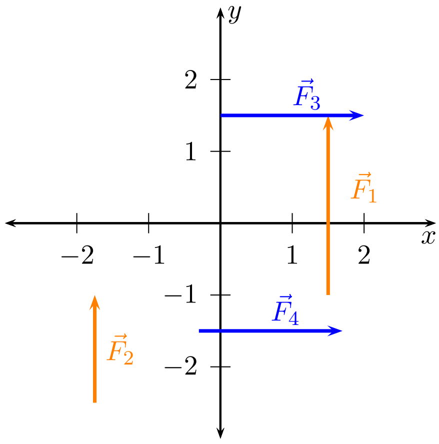
This applies equally in the y-direction. For example, if we have a force, $\vec{F}$, of magnitude 2,5 N acting in the positive y-direction we can draw it as a vector on the Cartesian plane.



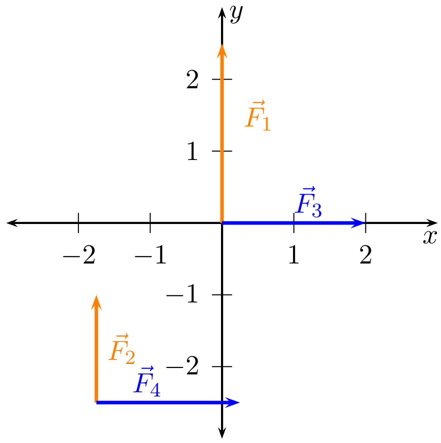
Just as in the case of the x-direction, a vector doesn't have to start at the origin but can be placed anywhere on the Cartesian plane. All of the vectors in the diagram below can represent the same force.



The following diagram shows an example of four force vectors, two vectors that are parallel to each other and the y-axis as well as two that are parallel to each other and the x-axis.



To emphasise that the vectors are perpendicular you can see in the figure below that when originating from the same point the vector are at right angles.



Exercise 1 – 1:

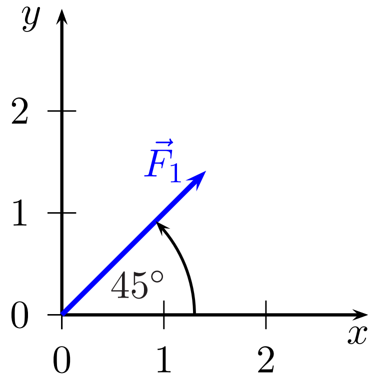
1. Draw the following forces as vectors on the Cartesian plane originating at the origin:
   - $\vec{F}_1 =$ 1,5 N in the positive x-direction
   - $\vec{F}_2 =$ 2 N in the positive y-direction
2. Draw the following forces as vectors on the Cartesian plane:
   - $\vec{F}_1 =$ 3 N in the positive x-direction
   - $\vec{F}_2 =$ 1 N in the negative x-direction
   - $\vec{F}_3 =$ 3 N in the positive y-direction
3. Draw the following forces as vectors on the Cartesian plane:
   - $\vec{F}_1 =$ 3 N in the positive x-direction
   - $\vec{F}_2 =$ 1 N in the positive x-direction
   - $\vec{F}_3 =$ 2 N in the negative x-direction
   - $\vec{F}_4 =$ 3 N in the positive y-direction
4. Draw the following forces as vectors on the Cartesian plane:
   - $\vec{F}_1 =$ 2 N in the positive y-direction
   - $\vec{F}_2 =$ 1,5 N in the negative y-direction
   - $\vec{F}_3 =$ 2,5 N in the negative x-direction
   - $\vec{F}_4 =$ 3 N in the positive y-direction

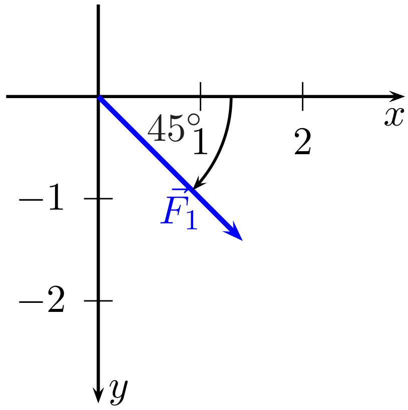
Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23F3 2. 23F4 3. 23F5 4. 23F6

www.everythingscience.co.za m.everythingscience.co.za

Vectors in two dimensions are not always parallel to an axis. We might know that a force acts at an angle to an axis so we still know the direction of the force and if we know the magnitude we can draw the force vector. For example, we can draw $\vec{F}_1 = 2$ N acting at 45° to the positive x-direction:





We always specify the angle as being anti-clockwise from the positive x-axis. So if we specified an negative angle we would measure it clockwise from the x-axis. For example, $\vec{F}_1 = 2$ N acting at −45° to the positive x-direction:

We can use many other ways of specifying the direction of a vector. The direction just needs to be unambiguous. We have used the Cartesian coordinate system and an angle with the x-axis so far but there are other common ways of specifying direction that you need to be aware of and comfortable to handle.

Compass directions ESBK5

We can use compass directions when appropriate to specify the direction of a vector. For example, if we were describing the forces of tectonic plates (the sections of the earth's crust that move) to talk about the forces involved in earthquakes we could talk the force that the moving plates exert on each other.

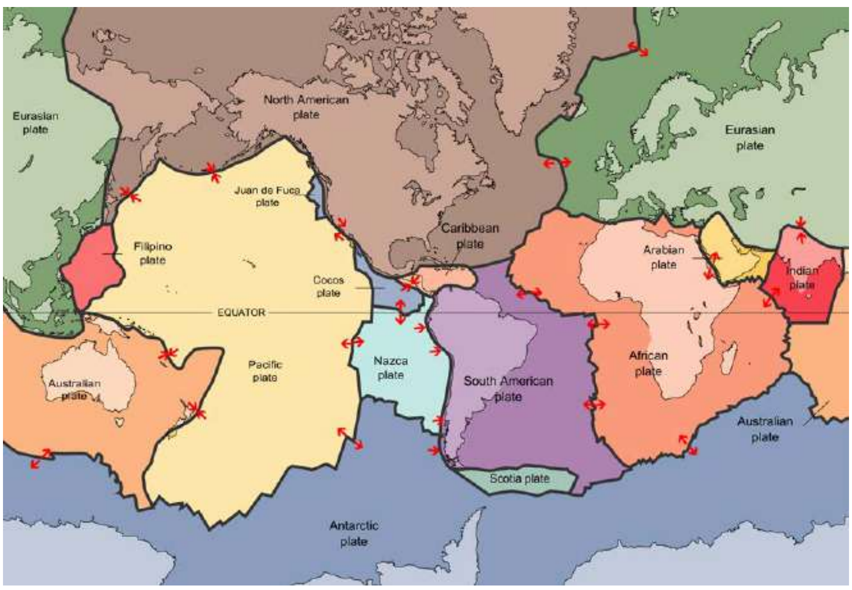


Figure 1.1: A map of the 15 major tectonic plates that make up the Earth's crust.

The four cardinal directions are North, South, East and West when using a compass. They are shown in this figure:

When specifying a direction of a vector using a compass directions are given by name, North or South. If the direction is directly between two directions we can combine the names, for example North-East is half-way between North and East. This can only happen for directions at right angles to each other, you cannot say North-South as it is ambiguous.

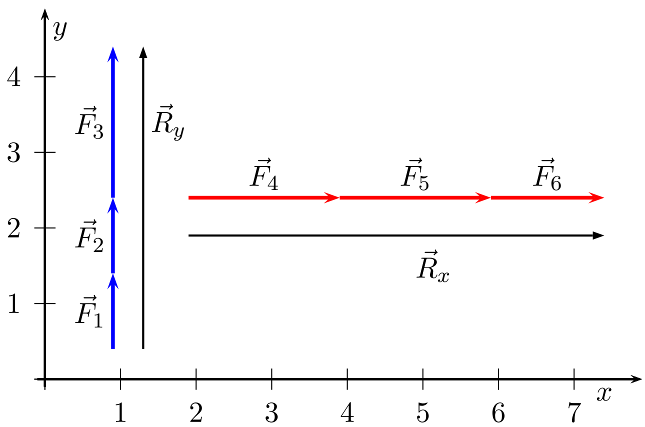
Figure 1.2: A sketch of the compass directions.

Bearings ESBK6

Another way of using the compass to specify direction in a numerical way is to use bearings. A bearing is an angle, usually measured clockwise from North. **Note** that this is different to the Cartesian plane where angles are anti- or counter-clockwise from the positive x-direction.

The resultant vector ESBK7

In grade 10 you learnt about adding vectors together in one dimension. The same principle can be applied for vectors in two dimensions. The following examples show addition of vectors. Vectors that are parallel can be shifted to fall on a line. Vectors falling on the same line are called *co-linear* vectors. To add co-linear vectors we use the tail-to-head method you learnt in Grade 10. In the figure below we remind you of the approach of adding co-linear vectors to get a resultant vector.



In the above figure the blue vectors are in the y-direction and the red vectors are in the x-direction. The two black vectors represent the resultants of the co-linear vectors graphically.

What we have done is implement the tail-to-head method of vector addition for the vertical set of vectors and the horizontal set of vectors.

Worked example 1: Revision: head-to-tail addition in one dimension

QUESTION

Use the graphical head-to-tail method to determine the resultant force on a rugby player if two players on his team are pushing him forwards with forces of $\vec{F_1}$ = 600 N and $\vec{F_2}$ = 900 N respectively and two players from the opposing team are pushing him backwards with forces of $\vec{F_3}$ = 1000 N and $\vec{F_4}$ = 650 N respectively.

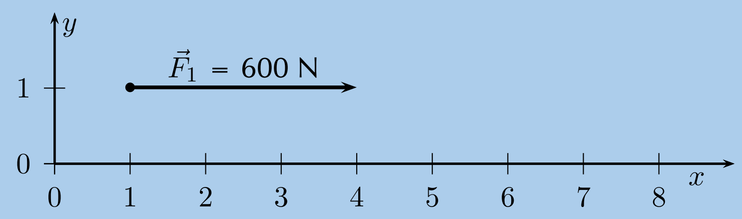
SOLUTION

Step 1: Choose a scale and a reference direction

Let's choose a scale of 100 N: 0,5 cm and for our diagram we will define the positive direction as *to the right*.

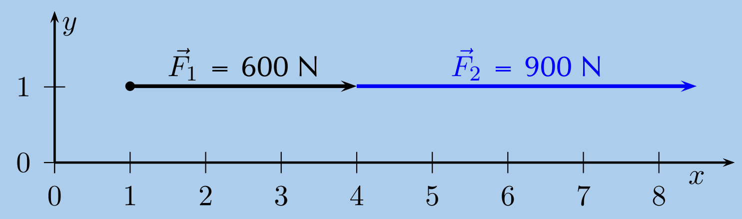
Step 2: Choose one of the vectors and draw it as an arrow of the correct length in the correct direction

We will start with drawing the vector $\vec{F_1}$ = 600 N pointing in the positive direction. Using our scale of 0,5 cm : 100 N, the length of the arrow must be 3 cm pointing to the right.



Step 3: Take the next vector and draw it starting at the arrowhead of the previous vector

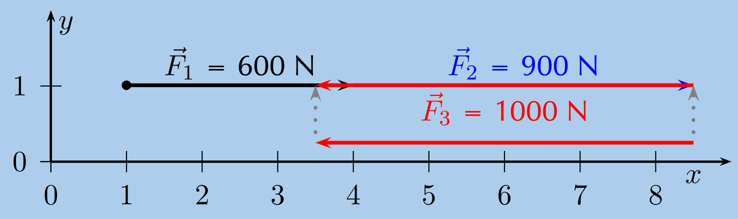
The next vector is $\vec{F_2}$ = 900 N in the same direction as $\vec{F_1}$. Using the scale, the arrow should be 4,5 cm long and pointing to the right.



Step 4: Take the next vector and draw it starting at the arrowhead of the previous vector

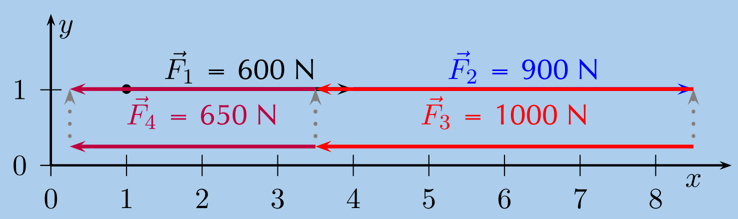
The next vector is $\vec{F_3}$ = 1000 N in the *opposite* direction. Using the scale, this arrow should be 5 cm long and point to the *left*.

Note: We are working in one dimension so this arrow would be drawn on top of the first vectors to the left. This will get confusing so we'll draw it next to the actual line as well to show you what it looks like.



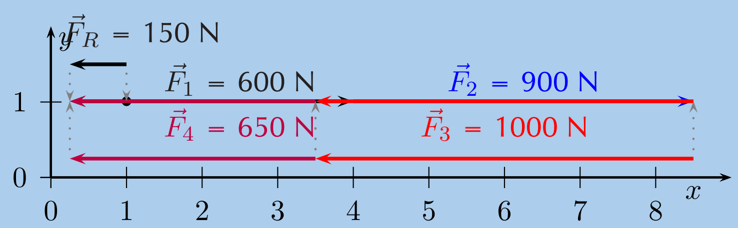
Step 5: Take the next vector and draw it starting at the arrowhead of the previous vector

The fourth vector is $\vec{F_4}$ = 650 N in the opposite direction. Using the scale, this arrow must be 3,25 cm long and point to the left.



Step 6: Draw the resultant, measure its length and find its direction

We have now drawn all the force vectors that are being applied to the player. The resultant vector is the arrow which starts at the tail of the first vector and ends at the head of the last drawn vector.



The resultant vector measures 0,75 cm which, using our scale is equivalent to 150 N and points to the left (*or* the negative direction *or* the direction the opposing team members are pushing in).

Exercise 1 – 2:

1. Find the resultant in the x-direction, R_x, and y-direction, R_y for the following forces:
   - $\vec{F}_1 = 1{,}5$ N in the positive x-direction
   - $\vec{F}_2 = 1{,}5$ N in the positive x-direction
   - $\vec{F}_3 = 2$ N in the negative x-direction
2. Find the resultant in the x-direction, $\vec{R}_x$, and y-direction, $\vec{R}_y$ for the following forces:
   - $\vec{F}_1 = 2{,}3$ N in the positive x-direction
   - $\vec{F}_2 = 1$ N in the negative x-direction
   - $\vec{F}_3 = 2$ N in the positive y-direction
   - $\vec{F}_4 = 3$ N in the negative y-direction
3. Find the resultant in the x-direction, $\vec{R}_x$, and y-direction, $\vec{R}_y$ for the following forces:
   - $\vec{F}_1 = 3$ N in the positive x-direction
   - $\vec{F}_2 = 1$ N in the positive x-direction
   - $\vec{F}_3 = 2$ N in the negative x-direction
   - $\vec{F}_4 = 3$ N in the positive y-direction
4. Find the resultant in the x-direction, $\vec{R}_x$, and y-direction, $\vec{R}_y$ for the following forces:
   - $\vec{F}_1 = 2$ N in the positive y-direction
   - $\vec{F}_2 = 1{,}5$ N in the negative y-direction
   - $\vec{F}_3 = 2{,}5$ N in the negative x-direction
   - $\vec{F}_4 = 3$ N in the positive y-direction
5. Find a force in the x-direction, F_x, and y-direction, F_y, that you can add to the following forces to make the resultant in the x-direction, R_x, and y-direction, R_y zero:
   - $\vec{F}_1 = 2{,}4$ N in the positive y-direction
   - $\vec{F}_2 = 0{,}7$ N in the negative y-direction
   - $\vec{F}_3 = 2{,}8$ N in the negative x-direction
   - $\vec{F}_4 = 3{,}3$ N in the positive y-direction

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23F7   2. 23F8   3. 23F9   4. 23FB   5. 23FC

www.everythingscience.co.za   m.everythingscience.co.za

We apply the same principle to vectors that are at right angles or perpendicular to each other.

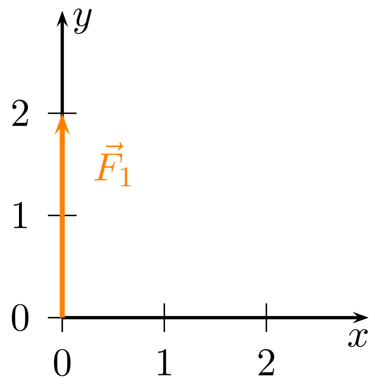
Sketching tail-to-head method

The tail of the one vector is placed at the head of the other but in two dimensions the vectors may not be co-linear. The approach is to draw all the vectors, one at a time. For the first vector begin at the origin of the Cartesian plane, for the second vector draw it from the head of the first vector. The third vector should be drawn from the head of the second and so on. Each vector is drawn from the head of the vector that preceded it. The order doesn't matter as the resultant will be the same if the order is different.

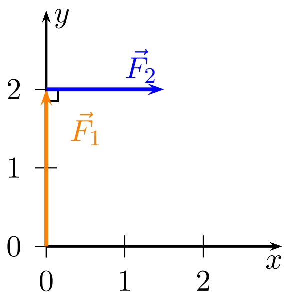
Let us apply this procedure to two vectors:

- $\vec{F}_1 = 2$ N in the positive y-direction
- $\vec{F}_2 = 1{,}5$ N in the positive x-direction

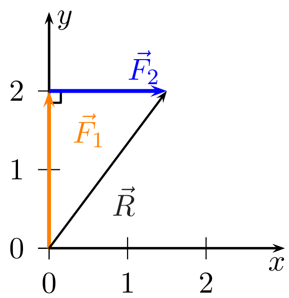
First, we first draw a Cartesian plane with the first vector originating at the origin as shown on the right.



The second step is to take the second vector and draw it from the head of the first vector:

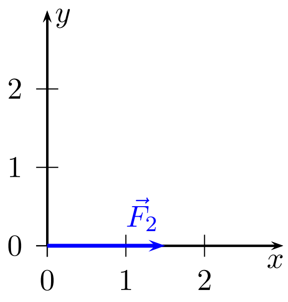


The resultant, $\vec{R}$, is the vector connecting the tail of the first vector drawn to the head of the last vector drawn:

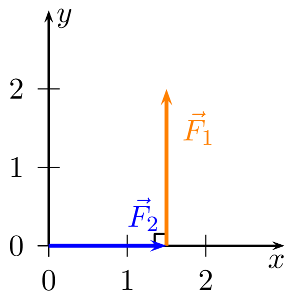


It is important to remember that the order in which we draw the vectors doesn't matter. If we had drawn them in the opposite order we would have the same resultant, $\vec{R}$. We can repeat the process to demonstrate this:

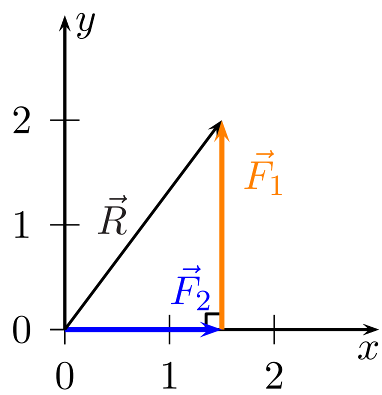
We first draw a Cartesian plane with the second vector originating at the origin:



The next step is to take the other vector and draw it from the head of the vector we have already drawn:



The resultant, $\vec{R}$, is the vector connecting the tail of the first vector drawn to the head of the last vector drawn (the vector from the start point to the end point):



Worked example 2: Sketching vectors using tail-to-head

QUESTION

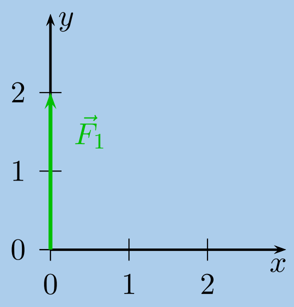
Sketch the resultant of the following force vectors using the tail-to-head method:

- $\vec{F}_1 = 2$ N in the positive y-direction
- $\vec{F}_2 = 1{,}5$ N in the positive x-direction
- $\vec{F}_3 = 1{,}3$ N in the negative y-direction
- $\vec{F}_4 = 1$ N in the negative x-direction

SOLUTION

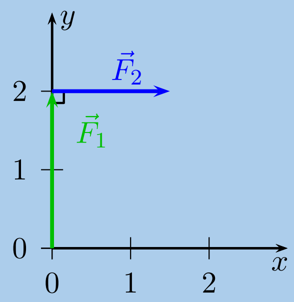
Step 1: Draw the Cartesian plane and the first vector

First draw the Cartesian plane and force, $\vec{F}_1$ starting at the origin:



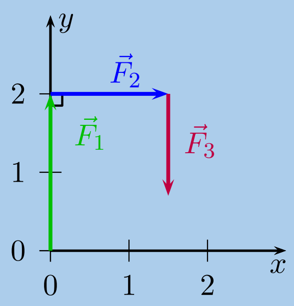
Step 2: Draw the second vector

Starting at the head of the first vector we draw the tail of the second vector:



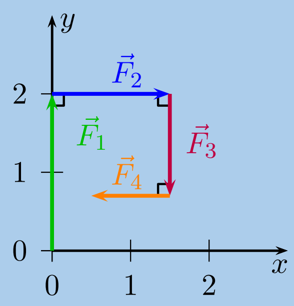
Step 3: Draw the third vector

Starting at the head of the second vector we draw the tail of the third vector:



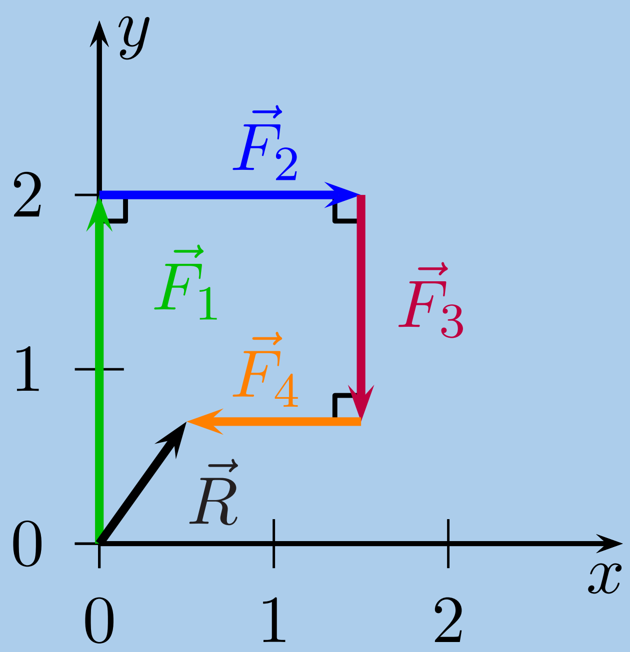
Step 4: Draw the fourth vector

Starting at the head of the third vector we draw the tail of the fourth vector:



Step 5: Draw the resultant vector

Starting at the origin draw the resultant vector to the head of the fourth vector:



Worked example 3: Sketching vectors using tail-to-head

QUESTION

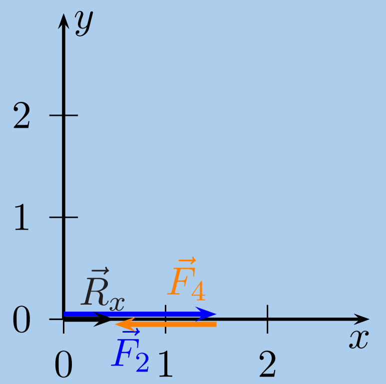
Sketch the resultant of the following force vectors using the tail-to-head method by first determining the resultant in the x- and y-directions:

- $\vec{F}_1 = 2$ N in the positive y-direction
- $\vec{F}_2 = 1{,}5$ N in the positive x-direction
- $\vec{F}_3 = 1{,}3$ N in the negative y-direction
- $\vec{F}_4 = 1$ N in the negative x-direction

SOLUTION

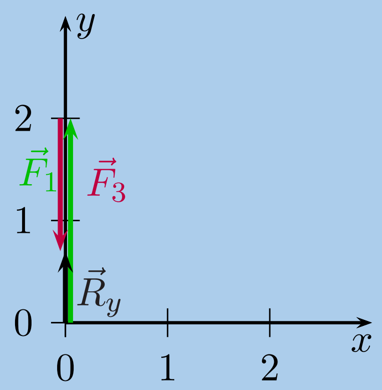
Step 1: First determine $\vec{R}_x$

First draw the Cartesian plane with the vectors in the x-direction:

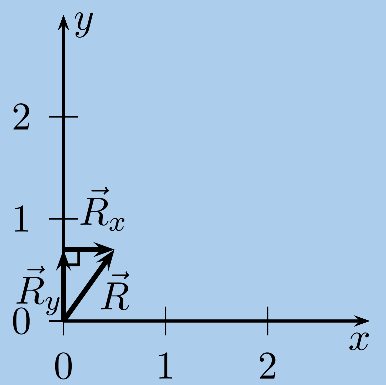


Step 2: Secondly determine $\vec{R}_y$

Next we draw the Cartesian plane with the vectors in the y-direction:

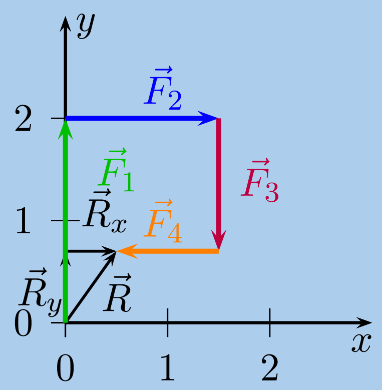


Step 3: Draw the resultant vectors, $\vec{R}_y$ and $\vec{R}_x$ head-to-tail



Step 4: Comparison of results

To double check, we can replot all the vectors again as we did in the previous worked example to see that the outcome is the same:



Exercise 1 – 3:

1. Sketch the resultant of the following force vectors using the tail-to-head method:
   - $\vec{F}_1 = 2{,}1$ N in the positive y-direction
   - $\vec{F}_2 = 1{,}5$ N in the negative x-direction
2. Sketch the resultant of the following force vectors using the tail-to-head method:
   - $\vec{F}_1 = 12$ N in the positive y-direction
   - $\vec{F}_2 = 10$ N in the positive x-direction
   - $\vec{F}_3 = 5$ N in the negative y-direction
   - $\vec{F}_4 = 5$ N in the negative x-direction
3. Sketch the resultant of the following force vectors using the tail-to-head method by first determining the resultant in the x- and y-directions:
   - $\vec{F}_1 = 2$ N in the positive y-direction
   - $\vec{F}_2 = 1{,}5$ N in the negative y-direction
   - $\vec{F}_3 = 1{,}3$ N in the negative y-direction
   - $\vec{F}_4 = 1$ N in the negative x-direction
4. Sketch the resultant of the following force vectors using the tail-to-head method by first determining the resultant in the x- and y-directions:
   - $\vec{F}_1 = 6$ N in the positive y-direction
   - $\vec{F}_2 = 3{,}5$ N in the negative x-direction
   - $\vec{F}_3 = 8{,}7$ N in the negative y-direction
   - $\vec{F}_4 = 3$ N in the negative y-direction

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23FD 2. 23FF 3. 23FG 4. 23FH

www.everythingscience.co.za m.everythingscience.co.za

FACT
When dealing with more than two vectors the procedure is repetitive. First find the resultant of any two of the vectors to be added. Then use the same method to add the resultant from the first two vectors with a third vector. This new resultant is then added to the fourth vector and so on, until there are no more vectors to be added.

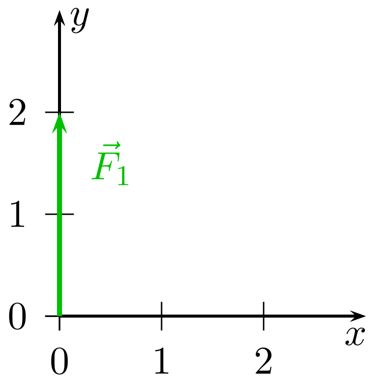
Sketching tail-to-tail method

In this method we draw the two vectors with their tails on the origin. Then we draw a line parallel to the first vector from the head of the second vector and vice versa. Where the parallel lines intersect is the head of the resultant vector that will also start at the origin. We will only deal with perpendicular vectors but this procedure works for any vectors.

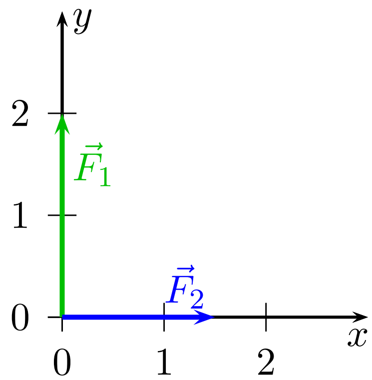
Let us apply this procedure to the same two vectors we used to illustrate the head-to-tail method:

- $\vec{F}_1 = 2$ N in the positive y-direction
- $\vec{F}_2 = 1{,}5$ N in the positive x-direction

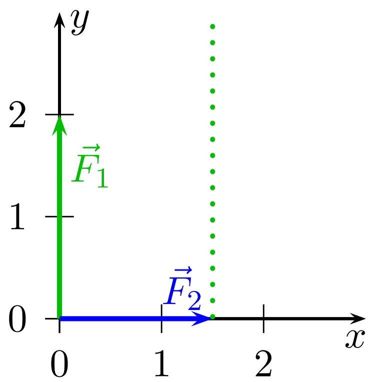
1. We first draw a Cartesian plane with the first vector originating at the origin:



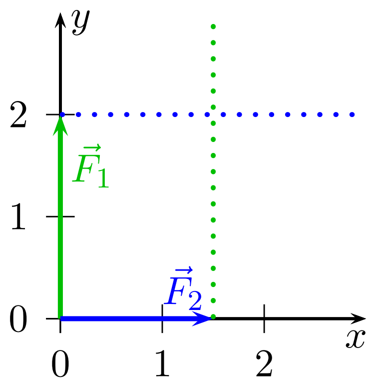
2. Then we add the second vector but also originating from the origin so that the vectors are drawn tail-to-tail:



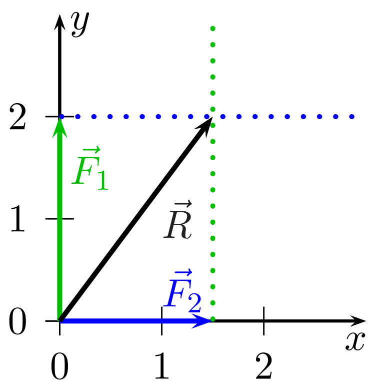
3. Now we draw a line parallel to $\vec{F}_1$ from the head of $\vec{F}_2$:



4. Next we draw a line parallel to $\vec{F}_2$ from the head of $\vec{F}_1$:



5. Where the two lines intersect is the head of the resultant vector which will originate at the origin so:



You might be asking what you would do if you had more than 2 vectors to add together. In this case all you need to do is first determine $\vec{R}_x$ by adding all the vectors that are parallel to the x-direction and $\vec{R}_y$ by adding all the vectors that are parallel to the y-direction. Then you use the tail-to-tail method to find the resultant of $\vec{R}_x$ and $\vec{R}_y$.

Exercise 1 – 4:

1. Sketch the resultant of the following force vectors using the tail-to-tail method:
   - $\vec{F}_1 = 2{,}1$ N in the positive y-direction
   - $\vec{F}_2 = 1{,}5$ N in the negative x-direction
2. Sketch the resultant of the following force vectors using the tail-to-tail method by first determining the resultant in the x- and y-directions:
   - $\vec{F}_1 = 2$ N in the positive y-direction
   - $\vec{F}_2 = 1{,}5$ N in the negative y-direction
   - $\vec{F}_3 = 1{,}3$ N in the negative y-direction
   - $\vec{F}_4 = 1$ N in the negative x-direction
3. Sketch the resultant of the following force vectors using the tail-to-tail method by first determining the resultant in the x- and y-directions:
   - $\vec{F}_1 = 6$ N in the positive y-direction
   - $\vec{F}_2 = 3{,}5$ N in the negative x-direction
   - $\vec{F}_3 = 8{,}7$ N in the negative y-direction
   - $\vec{F}_4 = 3$ N in the negative y-direction

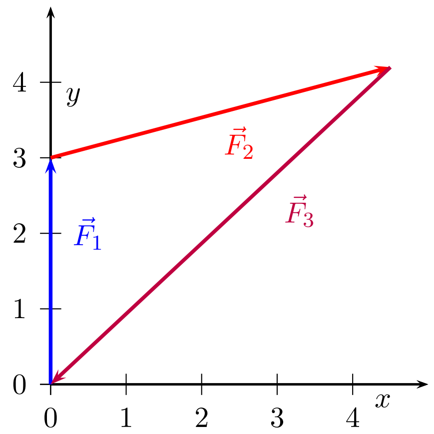
Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23FJ 2. 23FK 3. 23FM

www.everythingscience.co.za m.everythingscience.co.za

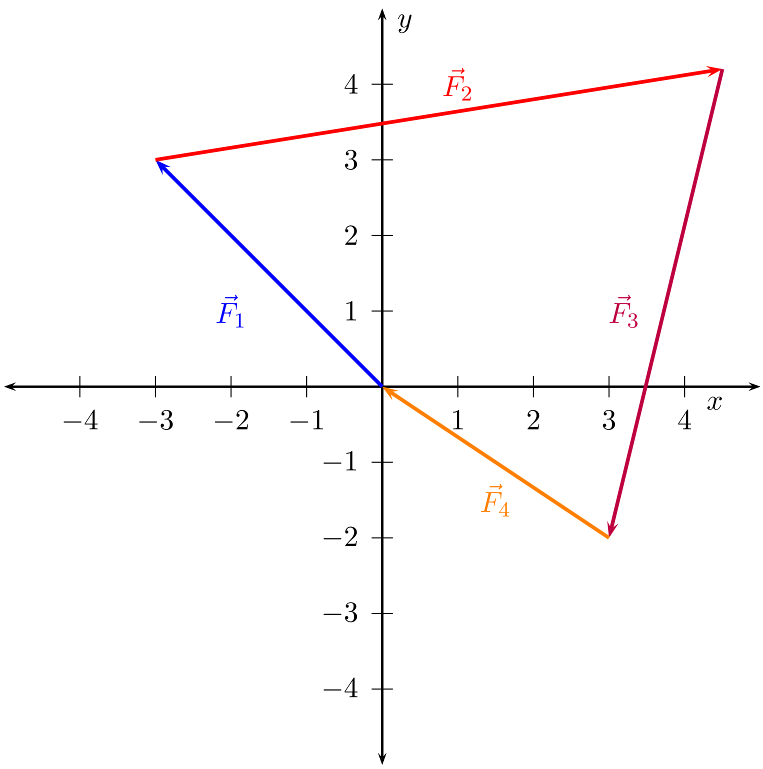
Closed vector diagrams

A closed vector diagram is a set of vectors drawn on the Cartesian using the tail-to-head method and that has a resultant with a magnitude of zero. This means that if the first vector starts at the origin the last vector drawn must end at the origin. The vectors form a closed polygon, no matter how many of them are drawn. Here are a few examples of closed vector diagrams:



In this case there were 3 force vectors. When drawn tail-to-head with the first force starting at the origin the last force drawn ends at the origin. The resultant would have a magnitude of zero. The resultant is drawn from the tail of the first vector to the head of the final vector.

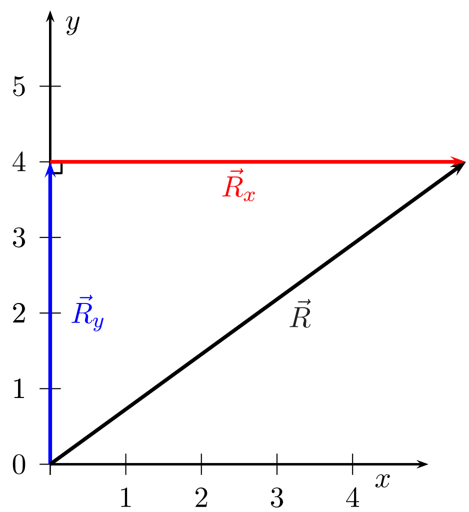
In the diagram below there are 4 vectors that also form a closed vector diagram.



In this case with 4 vectors, the shape is a 4-sided polygon. Any polygon made up of vectors drawn tail-to-head will be a closed vector diagram because a polygon has no gaps.

Using Pythagoras' theorem to find magnitude

If we wanted to know the resultant of the three blue vectors and the three red vectors in Figure 1.2 we can use the resultant vectors in the x- and y-directions to determine this.
The black arrow represents the resultant of the vectors $\vec{R}_x$ and $\vec{R}_y$. We can find the magnitude of this vector using the theorem of Pythagoras because the three vectors form a right angle triangle. If we had drawn the vectors to scale we would be able to measure the magnitude of the resultant as well.



What we've actually sketched out already is our approach to finding the resultant of many vectors using components so remember this example when we get there a little later.

Worked example 4: Finding the magnitude of the resultant

QUESTION

The force vectors in Figure 1.2 have the following magnitudes: 1 N, 1 N, 2 N for the blue ones and 2 N, 2 N and 1,5 N for the red ones. Determine the magnitude of the resultant.

SOLUTION

Step 1: Determine the resultant of the vectors parallel to the y-axis

The resultant of the vectors parallel to the y-axis is found by adding the magnitudes (lengths) of three vectors because they all point in the same direction. The answer is $\vec{R}_y$ = 1 N + 1 N + 2 N = 4 N in the positive y-direction.

Step 2: Determine the resultant of the vectors parallel to the x-axis

The resultant of the vectors parallel to the x-axis is found by adding the magnitudes (lengths) of three vectors because they all point in the same direction. The answer is $\vec{R}_x$ = 2 N + 2 N + 1,5 N = 5,5 N in the positive x-direction.

Step 3: Determine the magnitude of the resultant

We have a right angled triangle. We also know the length of two of the sides. Using Pythagoras we can find the length of the third side. From what we know about resultant vectors this length will be the magnitude of the resultant vector.

The resultant is:

$$\begin{aligned} R_x^2 + R_y^2 &= R^2 \text{ (Pythagoras' theorem)} \\ (5{,}5)^2 + (4)^2 &= R^2 \\ R &= 6{,}8 \end{aligned}$$

Step 4: Quote the final answer

Magnitude of the resultant: 6,8 N

Note: we did not determine the resultant vector in the worked example above because we only determined the magnitude. A vector needs a **magnitude** and a **direction**. We did not determine the direction of the resultant vector.

Graphical techniques

In grade 10 you learnt how to add vectors in one dimension graphically.

We can expand these ideas to include vectors in two-dimensions. The following worked example shows this.

Worked example 5: Finding the magnitude of the resultant in two dimensions graphically

QUESTION

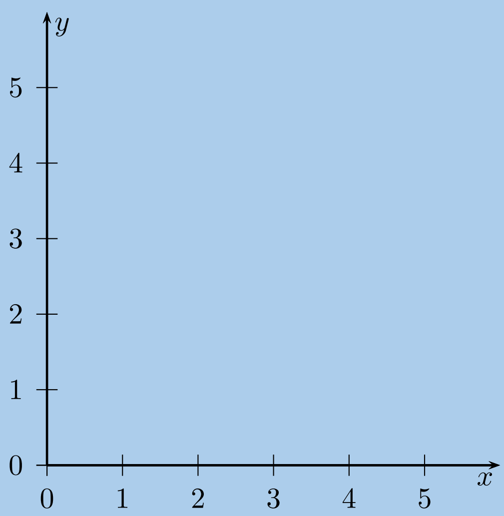
Given two vectors, $\vec{R}_y = 4$ N in the positive y-direction and $\vec{R}_x = 3$ N in the positive x-direction, use the tail-to-head method to find the resultant of these vectors graphically.

SOLUTION

Step 1: Choose a scale and draw axes

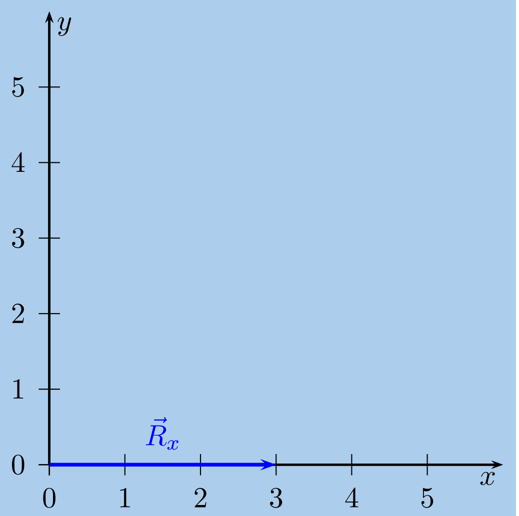
The vectors we have do not have very big magnitudes so we can choose simple scale, we can use 1 N : 1 cm as our scale for the drawing.

Then we draw axes that the vector diagram should fit in. The largest vector has length 4 N and both vectors are in the positive direction so we can draw axes from the origin to 5 and expect the vectors to fit.



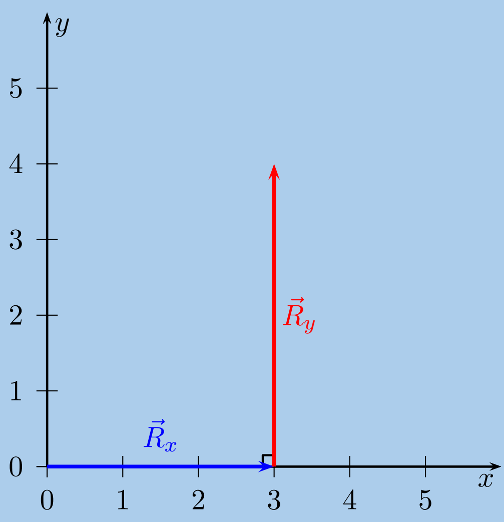
Step 2: Draw $\vec{R}_x$

The magnitude of $\vec{R}_x$ is 3 N so the arrow we need to draw must be 3 cm long. The arrow must point in the positive x-direction.



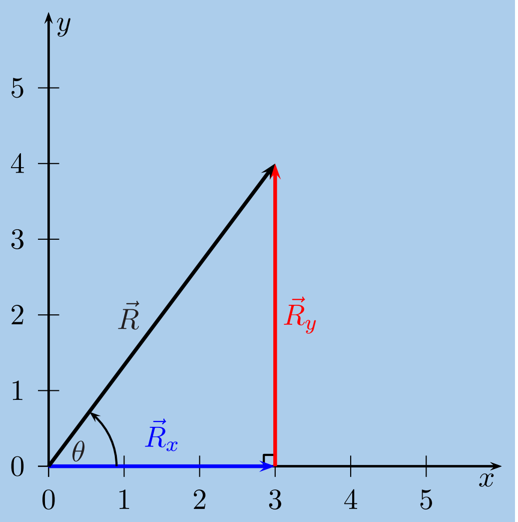
Step 3: Draw $\vec{R}_y$

The length of $\vec{R}_y$ is 4 so the arrow we need to draw must be 4 cm long. The arrow must point in the positive y-direction. The important fact to note is that we are implementing the head-to-tail method so the vector must start at the end (head) of $\vec{R}_x$.



Step 4: Draw the resultant vector, $\vec{R}$

The resultant vector is the vector from the tail of the first vector we drew directly to the head of the last vector we drew. This means we need to draw a vector from the tail of $\vec{R}_x$ to the head of $\vec{R}_y$.



Step 5: Measure the resultant, $\vec{R}$

We are solving the problem graphically so we now need to measure the magnitude of the vector and use the scale we chose to convert our answer from the diagram to the actual result. In the last diagram the resultant, $\vec{R}$ is 5 cm long therefore the magnitude of the vector is 5 N.

The direction of the resultant, θ, we need to measure from the diagram using a protractor. The angle that the vector makes with the x-axis is 53°.

Step 6: Quote the final answer

$\vec{R}$ is 5 N at 53° from the positive x-direction.

In the case where you have to find the resultant of more than two vectors first apply the tail-to-head method to all the vectors parallel to the one axis and then all the vectors parallel to the other axis. For example, you would first calculate $\vec{R}_y$ from all the vectors parallel to the y-axis and then $\vec{R}_x$ from all the vectors parallel to the x-axis. After that you apply the same procedure as in the previous worked example to the get the final resultant.

Worked example 6: Finding the magnitude of the resultant in two dimensions graphically

QUESTION

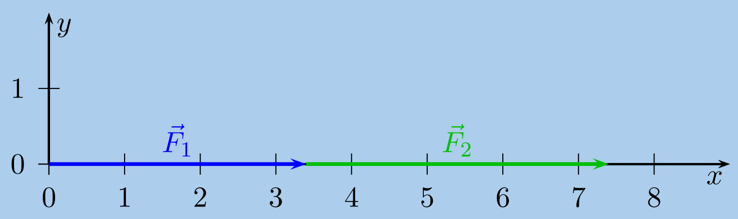
Given the following three force vectors, determine the resultant force:

- $\vec{F}_1 = 3{,}4$ N in the positive x-direction
- $\vec{F}_2 = 4$ N in the positive x-direction
- $\vec{F}_3 = 3$ N in the negative y-direction

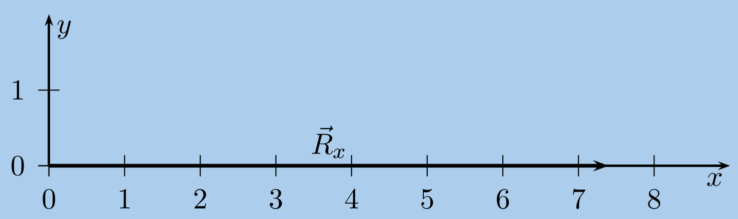
SOLUTION

Step 1: Determine $\vec{R}_x$

First we determine the resultant of all the vectors that are parallel to the x-axis. There are two vectors $\vec{F}_1$ and $\vec{F}_2$ that we need to add. We do this using the tail-to-head method for co-linear vectors.



The single vector, $\vec{R}_x$, that would give us the same outcome is:



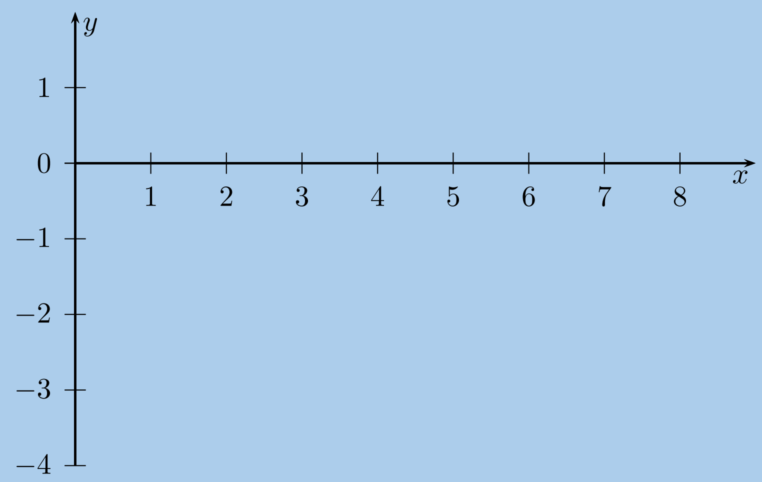
Step 2: Determine $\vec{R}_y$

There is only one vector in the y-direction, $\vec{F}_3$, therefore $\vec{R}_y = \vec{F}_3$.

Step 3: Choose a scale and draw axes

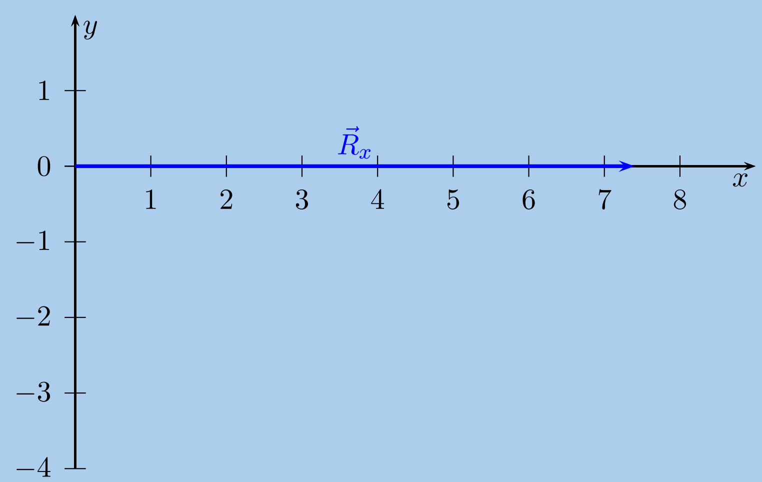
The vectors we have do not have very big magnitudes so we can choose simple scale, we can use 1 N : 1 cm as our scale for the drawing.

Then we draw axes that the diagram should fit on. The longest vector has length 7,4 N. We need our axes to extend just further than the vectors aligned with each axis. Our axes need to start at the origin and go beyond 7,4 N in the positive x-direction and further than 3 N in the negative y-direction. Our scale choice of 1 N : 1 cm means that our axes actually need to extend 7,4 cm in the positive x-direction and further than 3 cm in the negative y-direction



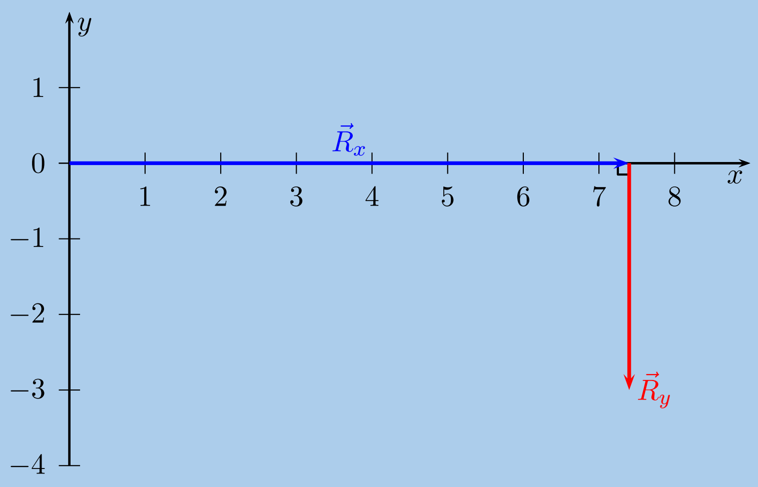
Step 4: Draw $\vec{R}_x$

The magnitude of $\vec{R}_x$ is 7,4 N so the arrow we need to draw must be 7,4 cm long. The arrow must point in the positive x-direction.



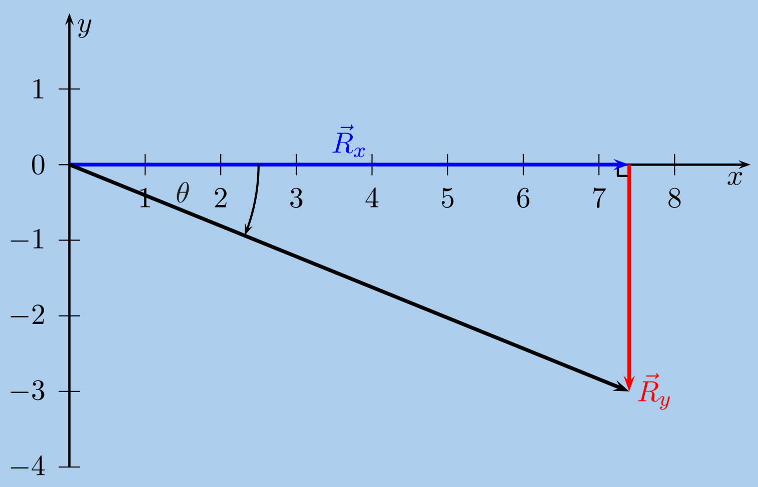
Step 5: Draw $\vec{R}_y$

The magnitude of $\vec{R}_y$ is 3 N so the arrow we need to draw must be 3 cm long. The arrow must point in the negative y-direction. The important fact to note is that we are implementing the head-to-tail method so the vector must start at the end (head) of $\vec{R}_x$.



Step 6: Draw the resultant vector, $\vec{R}$

The resultant vector is the vector from the tail of the first vector we drew directly to the head of the last vector we drew. This means we need to draw a vector from the tail of $\vec{R}_x$ to the head of $\vec{R}_y$.



Step 7: Measure the resultant, $\vec{R}$

We are solving the problem graphically so we now need to measure the magnitude of the vector and use the scale we chose to convert our answer from the diagram to the actual result. In the last diagram the resultant, $\vec{R}$ is 8,0 cm long therefore the magnitude of the vector is 8,0 N.

The direction of the resultant we need to measure from the diagram using a protractor. The angle that the vector makes with the x-axis is 22°.

Step 8: Quote the final answer

$\vec{R}$ is 8,0 N at $-22°$ from the positive x-direction.

Worked example 7: Finding the resultant in two dimensions graphically

QUESTION

Given the following three force vectors, determine the resultant force:

- $\vec{F}_1$ = 2,3 N in the positive x-direction
- $\vec{F}_2$ = 4 N in the positive y-direction
- $\vec{F}_3$ = 3,3 N in the negative y-direction
- $\vec{F}_4$ = 2,1 N in the negative y-direction

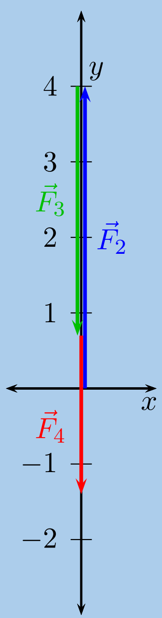
SOLUTION

Step 1: Determine $\vec{R}_x$

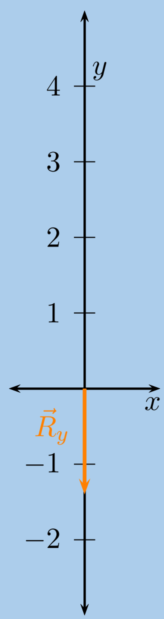
There is only one vector in the x-direction, $\vec{F}_1$, therefore $\vec{R}_x = \vec{F}_1$.

Step 2: Determine $\vec{R}_y$

Then we determine the resultant of all the vectors that are parallel to the y-axis. There are three vectors $\vec{F}_2$, $\vec{F}_3$ and $\vec{F}_4$ that we need to add. We do this using the tail-to-head method for co-linear vectors.



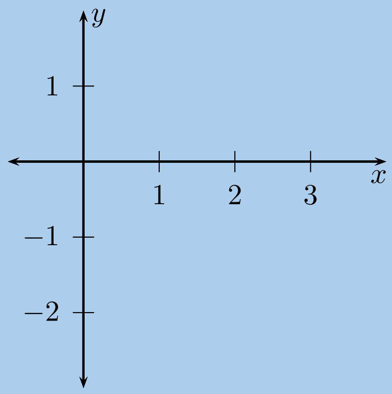
The single vector, $\vec{R}_y$, that would give us the same effect is:



Step 3: Choose a scale and draw axes

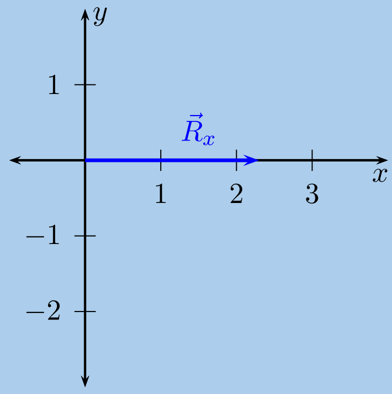
We choose a scale 1 N : 1 cm for the drawing.

Then we draw axes that the diagram should fit in. We need our axes to extend just further than the vectors aligned with each axis. Our axes need to start at the origin and go beyond 2,3 N in the positive x-direction and further than 1,4 N in the negative y-direction. Our scale choice of 1 N = 1 cm means that our axes actually need to extend 2,3 cm in the positive x-direction and further than 1,4 cm in the negative y-direction



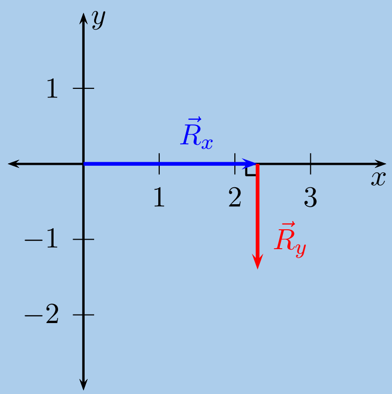
Step 4: Draw $\vec{R}_x$

The magnitude of $\vec{R}_x$ is 2,3 N so the arrow we need to draw must be 2,3 cm long. The arrow must point in the positive x-direction.



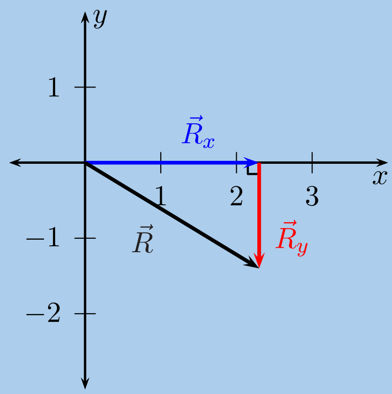
Step 5: Draw $\vec{R}_y$

The magnitude of $\vec{R}_y$ is 1,4 N so the arrow we need to draw must be 1,4 cm long. The arrow must point in the negative y-direction. The important fact to note is that we are implementing the head-to-tail method so the vector must start at the end (head) of $\vec{R}_x$.



Step 6: Draw the resultant vector, $\vec{R}$

The resultant vector is the vector from the tail of the first vector we drew directly to the head of the last vector we drew. This means we need to draw a vector from the tail of $\vec{R}_x$ to the head of $\vec{R}_y$.



Step 7: Measure the resultant, $\vec{R}$

We are solving the problem graphically so we now need to measure the magnitude of the vector and use the scale we chose to convert our answer from the diagram to the magnitude of the vector. In the last diagram the resultant, $\vec{R}$ is 2,7 cm long therefore the magnitude of the vector is 2,7 N.

The direction of the resultant we need to measure from the diagram using a protractor. The angle that the vector makes with the x-axis is 31 degrees.

Step 8: Quote the final answer

$\vec{R}$ is 2,7 N at $-31°$ from the positive x-direction.

Worked example 8: Finding the resultant in two dimensions graphically

QUESTION

A number of tugboats are trying to manoeuvre a submarine in the harbour but they are not working as a team. Each tugboat is exerting a different force on the submarine.

Given the following force vectors, determine the resultant force:

- $\vec{F}_1$ = 3,4 kN in the positive x-direction
- $\vec{F}_2$ = 4000 N in the positive y-direction
- $\vec{F}_3$ = 300 N in the negative y-direction
- $\vec{F}_4$ = 7 kN in the negative y-direction

SOLUTION

Step 1: Convert to consistent S.I. units

To use the graphical method of finding the resultant we need to work in the same units. Strictly speaking in this problem all the vectors are in newtons but they have different factors which will affect the choice of scale. These need to taken into account and the simplest approach is to convert them all to a consistent unit and factor. We could use kN or N, the choice does not matter. We will choose kN. Remember that k represents a factor of $\times 10^3$.

$\vec{F}_1$ and $\vec{F}_4$ do not require any adjustment because they are both in kN. To convert N to kN we use:

$$\begin{aligned}\text{kN} &= \times 10^3 \\ \frac{\text{N}}{\text{kN}} &= \frac{1}{\times 10^3} \\ \text{N} &= \times 10^{-3}\ \text{kN}\end{aligned}$$

To convert the magnitude of $\vec{F}_2$ to kN:

$$\begin{aligned}F_2 &= 4000\ \text{N} \\ F_2 &= 4000 \times 10^{-3}\ \text{kN} \\ F_2 &= 4\ \text{kN}\end{aligned}$$

Therefore $\vec{F}_2$ = 4 kN in the positive y-direction.

To convert the magnitude of $\vec{F}_3$ to kN:

$$\begin{aligned}F_3 &= 300\ \text{N} \\ F_3 &= 300 \times 10^{-3}\ \text{kN} \\ F_3 &= 0{,}3\ \text{kN}\end{aligned}$$

Therefore $\vec{F}_3$ = 0,3 kN in the negative y-direction. So:

- $\vec{F}_1$ = 3,4 kN in the positive x-direction
- $\vec{F}_2$ = 4 kN in the positive y-direction
- $\vec{F}_3$ = 0,3 kN in the negative y-direction
- $\vec{F}_4$ = 7 kN in the negative y-direction

Step 2: Choose a scale and draw axes

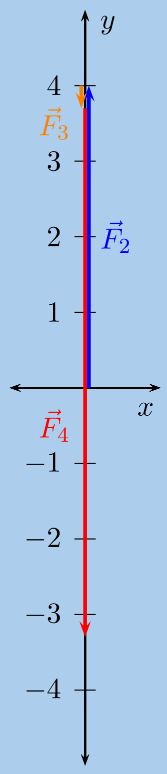
The vectors we have do have very big magnitudes so we need to choose a scale that will allow us to draw them in a reasonable space, we can use 1 kN : 1 cm as our scale for the drawings.

Step 3: Determine $\vec{R}_x$

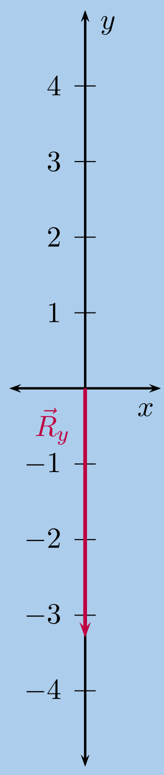
There is only one vector in the x-direction, $\vec{F}_1$, therefore $\vec{R}_x$ = $\vec{F}_1$.

Step 4: Determine $\vec{R}_y$

Then we determine the resultant of all the vectors that are parallel to the y-axis. There are three vectors $\vec{F}_2$, $\vec{F}_3$ and $\vec{F}_4$ that we need to add. We do this using the tail-to-head method for co-linear vectors.

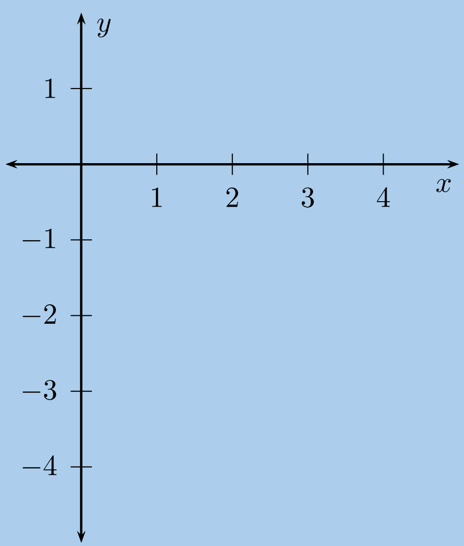


The single vector, $\vec{R}_y$, that would give us the same outcome is:



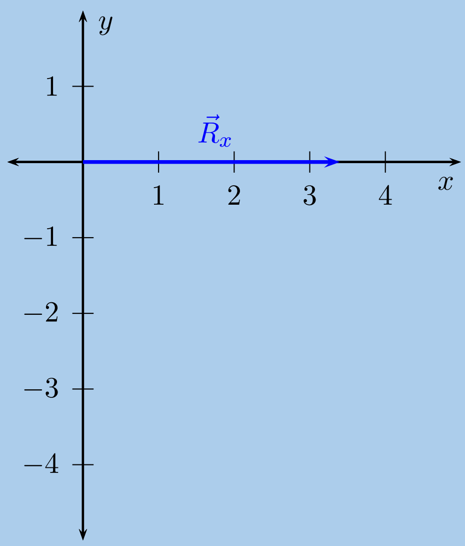
Step 5: Draw axes

Then we draw axes that the diagram should fit on. We need our axes to extend just further than the vectors aligned with each axis. Our axes need to start at the origin and go beyond 3,4 kN in the positive x-direction and further than 3,3 kN in the negative y-direction. Our scale choice of 1 kN : 1 cm means that our axes actually need to extend 3,4 cm in the positive x-direction and further than 3,3 cm in the negative y-direction



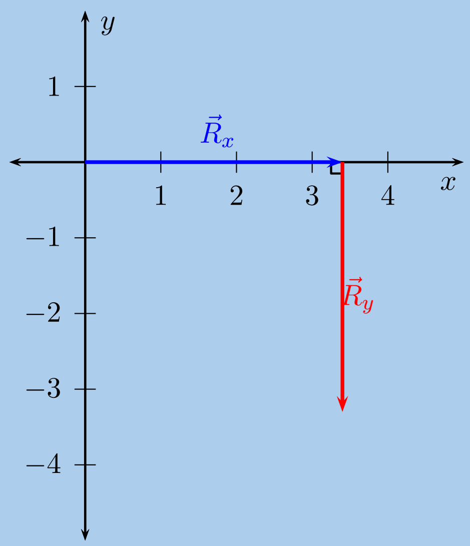
Step 6: Draw $\vec{R}_x$

The length of $\vec{R}_x$ is 3,4 kN so the arrow we need to draw must be 3,4 cm long. The arrow must point in the positive x-direction.



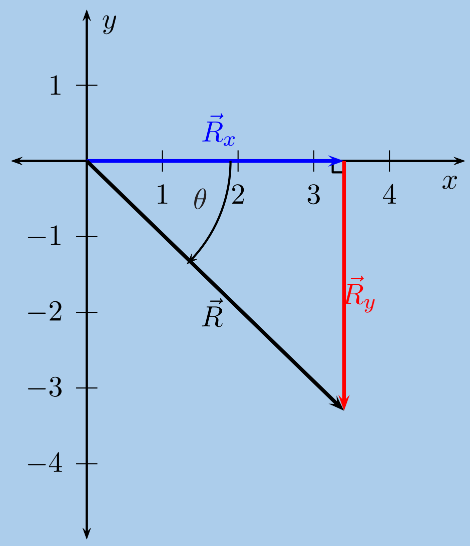
Step 7: Draw $\vec{R}_y$

The length of $\vec{R}_y$ is 3,3 kN so the arrow we need to draw must be 3,3 cm long. The arrow must point in the negative y-direction. The important fact to note is that we are implementing the head-to-tail method so the vector must start at the end (head) of $\vec{R}_x$.



Step 8: Draw the resultant vector, $\vec{R}$

The resultant vector is the vector from the tail of the first vector we drew directly to the head of the last vector we drew. This means we need to draw a vector from the tail of $\vec{R}_x$ to the head of $\vec{R}_y$.



Step 9: Measure the resultant, $\vec{R}$

We are solving the problem graphically so we now need to measure the magnitude of the vector and use the scale we chose to convert our answer from the diagram to the actual result. In the last diagram the resultant, $\vec{R}$ is 4,7 cm long therefore the magnitude of the vector is 4,7 kN.

The direction of the resultant we need to measure from the diagram using a protractor. The angle that the vector makes with the x-axis is 44°.

Step 10: Quote the final answer

$\vec{R}$ is 4,7 kN at $-44°$ from the positive x-direction.

Algebraic methods ESBKB

Algebraic addition and subtraction of vectors

In grade 10 you learnt about addition and subtraction of vectors in one dimension. The following worked example provides a refresher of the concepts.

Worked example 9: Adding vectors algebraically

QUESTION

A force of 5 N to the right is applied to a crate. A second force of 2 N to the left is also applied to the crate. Calculate algebraically the resultant of the forces applied to the crate.

SOLUTION

Step 1: Draw a sketch

A simple sketch will help us understand the problem.

Step 2: Decide which method to use to calculate the resultant

Remember that force is a vector. Since the forces act along a straight line (i.e. the x-direction), we can use the algebraic technique of vector addition.

Step 3: Choose a positive direction

Choose the **positive** direction to be to the right. This means that the **negative** direction is to the left.

Rewriting the problem using the choice of a positive direction gives us a force of 5 N in the positive x-direction and force of 2 N in the negative x-direction being applied to the crate.

Step 4: Now define our vectors algebraically

$$\vec{F}_1 = 5\text{ N}$$
$$\vec{F}_2 = -2\text{ N}$$

Step 5: Add the vectors

Thus, the resultant force is:

$$\begin{aligned}\vec{F}_1 + \vec{F}_2 &= (5) + (-2)\\ &= 3\text{ N}\end{aligned}$$

Step 6: Quote the resultant

Remember that in this case a positive force means to the right: 3 N to the right.

We can now expand on this work to include vectors in two dimensions.

Worked example 10: Algebraic solution in two dimensions

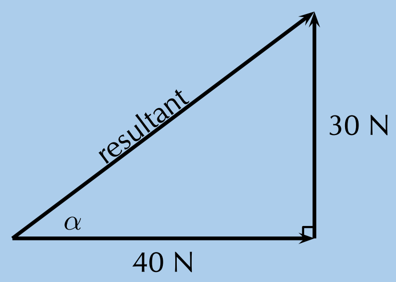
QUESTION

A force of 40 N in the positive x-direction acts simultaneously (at the same time) to a force of 30 N in the positive y-direction. Calculate the magnitude of the resultant force.

SOLUTION

Step 1: Draw a rough sketch

As before, the rough sketch looks as follows:



Step 2: Determine the length of the resultant

Note that the triangle formed by the two force vectors and the resultant vector is a right-angle triangle. We can thus use the Theorem of Pythagoras to determine the length of the resultant. Let R represent the length of the resultant vector. Then:

$$F_x^2 + F_y^2 = R^2 \text{ Pythagoras' theorem}$$
$$(40)^2 + (30)^2 = R^2$$
$$R = 50 \text{ N}$$

Step 3: Quote the resultant

The magnitude of the resultant force is then 50 N.

Direction

For two dimensional vectors we have only covered finding the magnitude of vectors algebraically. We also need to know the direction. For vectors in one dimension this was simple. We chose a positive direction and then the resultant was either in the positive or in the negative direction. In grade 10 you learnt about the different ways to specify direction. We will now look at using trigonometry to determine the direction of the resultant vector.

We can use simple trigonometric identities to calculate the direction. We can calculate the direction of the resultant in the previous worked example.

Worked example 11: Direction of the resultant

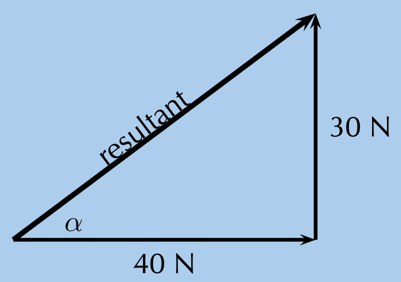
QUESTION

A force of 40 N in the positive x-direction acts simultaneously (at the same time) to a force of 30 N in the positive y-direction. Calculate the magnitude of the resultant force.

SOLUTION

Step 1: Magnitude

We determined the magnitude of the resultant vector in the previous worked example to be 50 N. The sketch of the situation is:



Step 2: Determine the direction of the resultant

To determine the direction of the resultant force, we calculate the angle α between the resultant force vector and the positive x-axis, by using simple trigonometry:

$$\begin{aligned}\tan\alpha &= \frac{\text{opposite side}}{\text{adjacent side}}\\ \tan\alpha &= \frac{30}{40}\\ \alpha &= \tan^{-1}(0{,}75)\\ \alpha &= 36{,}87^{\circ}\end{aligned}$$

Step 3: Quote the resultant

The resultant force is then 50 N at 36,9° to the positive x-axis.

Exercise 1 – 5: Algebraic addition of vectors

1. A force of 17 N in the positive x-direction acts simultaneously (at the same time) to a force of 23 N in the positive y-direction. Calculate the resultant force.
2. A force of 23,7 N in the negative x-direction acts simultaneously to a force of 9 N in the positive y-direction. Calculate the resultant force.
3. Four forces act simultaneously at a point, find the resultant if the forces are:

- $\vec{F_1}$ = 2,3 N in the positive x-direction
- $\vec{F_2}$ = 4 N in the positive y-direction
- $\vec{F_3}$ = 3,3 N in the negative y-direction
- $\vec{F_4}$ = 2,1 N in the negative y-direction

4. The following forces act simultaneously on a pole, if the pole suddenly snaps in which direction will it be pushed:
   - $\vec{F_1}$ = 2,3 N in the negative x-direction
   - $\vec{F_2}$ = 11,7 N in the negative y-direction
   - $\vec{F_3}$ = 6,9 N in the negative y-direction
   - $\vec{F_4}$ = 1,9 N in the negative y-direction

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23FN 2. 23FP 3. 23FQ 4. 23FR

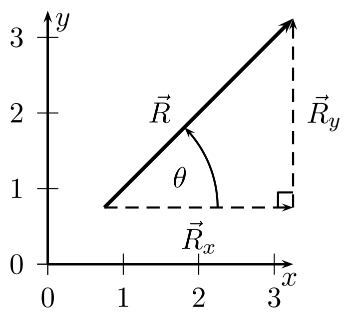
www.everythingscience.co.za m.everythingscience.co.za

1.3 Components of vectors ESBKC

In the discussion of vector addition we saw that a number of vectors acting together can be combined to give a single vector (the resultant). In much the same way a single vector can be broken down into a number of vectors which when added give that original vector. These vectors which sum to the original are called **components** of the original vector. The process of breaking a vector into its components is called **resolving into components**.

In practise it is most useful to resolve a vector into components which are at right angles to one another, usually horizontal and vertical. Think about all the problems we've solved so far. If we have vectors parallel to the x- and y-axes problems are straightforward to solve.

Any vector can be resolved into a horizontal and a vertical component. If $\vec{R}$ is a vector, then the horizontal component of $\vec{R}$ is $\vec{R}_x$ and the vertical component is $\vec{R}_y$.



When resolving into components that are parallel to the x- and y-axes we are always dealing with a right-angled triangle. This means that we can use trigonometric identities to determine the magnitudes of the components (we know the directions because they are aligned with the axes).

From the triangle in the diagram above we know that

$$\cos(\theta) = \frac{R_x}{R}$$
$$\frac{R_x}{R} = \cos(\theta)$$
$$R_x = R\cos(\theta)$$

and

$$\sin\theta = \frac{R_y}{R}$$
$$\frac{R_y}{R} = \sin(\theta)$$
$$R_y = R\sin(\theta)$$

$$\boxed{R_x = R\cos(\theta)}$$
$$\boxed{R_y = R\sin(\theta)}$$

Note that the angle is measured counter-clockwise from the positive x-axis.

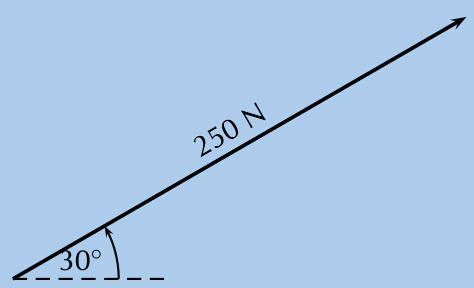
Worked example 12: Resolving a vector into components

QUESTION

A force of 250 N acts at an angle of 30° to the positive x-axis. Resolve this force into components parallel to the x- and y-axes.

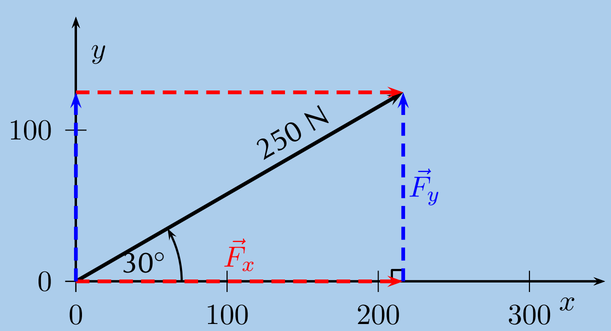
SOLUTION

Step 1: Draw a rough sketch of the original vector



Step 2: Determine the vector components

Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.



Notice how the two components acting together give the original vector as their resultant.

Step 3: Determine the magnitudes of the component vectors

Now we can use trigonometry to calculate the magnitudes of the components of the original displacement:

$$F_y = 250\sin(30°) = 125 \text{ N} \quad \text{and} \quad F_x = 250\cos(30°) = 216{,}5 \text{ N}$$

Remember F_x and F_y are the magnitudes of the components. $\vec{F}_x$ is in the positive x-direction and $\vec{F}_y$ is in the positive y-direction.

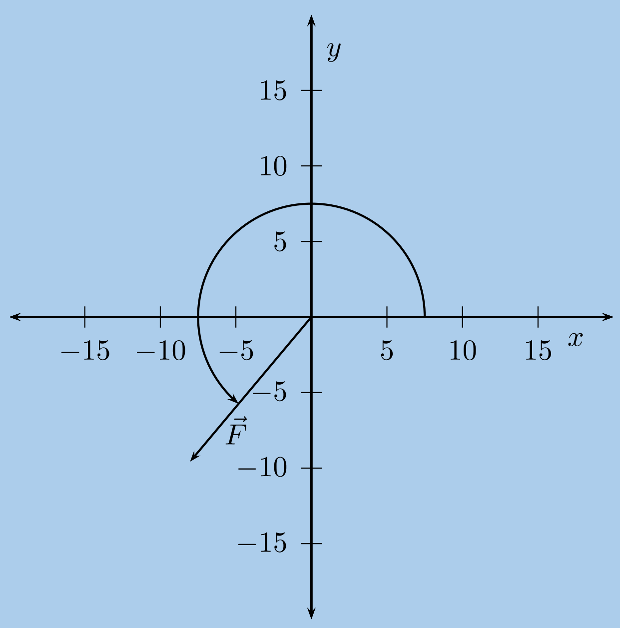
Worked example 13: Resolving a vector into components

QUESTION

A force of 12,5 N acts at an angle of 230° to the positive x-axis. Resolve this force into components parallel to the x- and y-axes.

SOLUTION

Step 1: Draw a rough sketch of the original vector



Step 2: Determine the vector components

Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 12{,}5\sin(230^\circ) = -9{,}58 \text{ N} \quad \text{and} \quad F_x = 12{,}5\cos(230^\circ) = -8{,}03 \text{ N}$$

Notice that by using the full angle we actually get the correct signs for the components if we use the standard Cartesian coordinates. $\vec{F}_x$ is in the negative x-direction and $\vec{F}_y$ is in the negative y-direction.

Exercise 1 – 6:

1. Resolve each of the following vectors into components:
   - $\vec{F}_1 = 5$ N at 45° to the positive x-axis.
   - $\vec{F}_2 = 15$ N at 63° to the positive x-axis.
   - $\vec{F}_3 = 11{,}3$ N at 127° to the positive x-axis.
   - $\vec{F}_4 = 125$ N at 245° to the positive x-axis.
2. Resolve each of the following vectors into components:
   - $\vec{F}_1 = 11 \times 10^4$ N at 33° to the positive x-axis.
   - $\vec{F}_2 = 15$ GN at 28° to the positive x-axis.
   - $\vec{F}_3 = 11{,}3$ kN at 193° to the positive x-axis.
   - $\vec{F}_4 = 125 \times 10^5$ N at 317° to the positive x-axis.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23FS 2. 23FT

www.everythingscience.co.za m.everythingscience.co.za

Vector addition using components ESBKD

Components can also be used to find the resultant of vectors. This technique can be applied to both graphical and algebraic methods of finding the resultant. The method is straightforward:

1. make a rough sketch of the problem;
2. find the horizontal and vertical components of each vector;
3. find the sum of all horizontal components, $\vec{R}_x$;

4. find the sum of all the vertical components, $\vec{R}_y$;
5. then use them to find the resultant, $\vec{R}$.

Consider the two vectors, $\vec{F}_1$ and $\vec{F}_2$, in Figure 1.3, together with their resultant, $\vec{R}$.

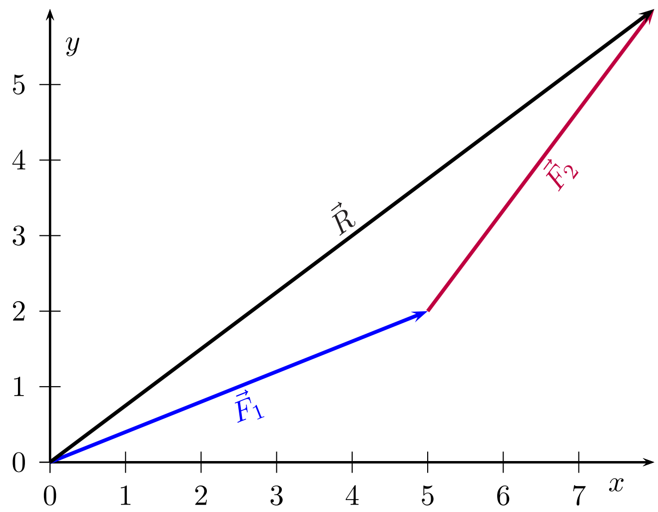


Figure 1.3: An example of two vectors being added to give a resultant.

Each vector in Figure 1.3 can be broken down into one component in the x-direction (horizontal) and one in the y-direction (vertical). These components are two vectors which when added give you the original vector as the resultant. This is shown in Figure 1.4 below:

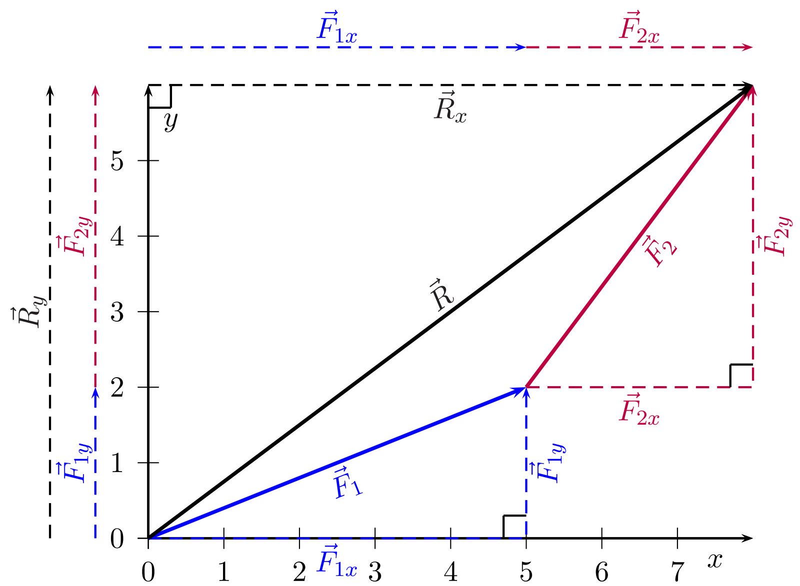


Figure 1.4: Adding vectors using components.

We can see that:

$$\vec{F}_1 = \vec{F}_{1x} + \vec{F}_{1y}$$
$$\vec{F}_2 = \vec{F}_{2x} + \vec{F}_{2y}$$
$$\vec{R} = \vec{R}_x + \vec{R}_y$$

$$\text{But, } \vec{R}_x = \vec{F}_{1x} + \vec{F}_{2x}$$
$$\text{and } \vec{R}_y = \vec{F}_{1y} + \vec{F}_{2y}$$

In summary, addition of the x-components of the two original vectors gives the x-component of the resultant. The same applies to the y-components. So if we just added all the components together we would get the **same answer!** This is another important property of vectors.

Worked example 14: Adding vectors using components

QUESTION

If in Figure 1.4, $\vec{F}_1 = 5{,}385$ N at an angle of 21,8° to the horizontal and $\vec{F}_2 = 5$ N at an angle of 53,13° to the horizontal, find the resultant force, $\vec{R}$.

SOLUTION

Step 1: Decide how to tackle the problem

The first thing we must realise is that the order that we add the vectors does not matter. Therefore, we can work through the vectors to be added in any order. We also draw up the following table to help us work through the problem:

Vector	x-component	y-component	Total
$\vec{F}_1$			
$\vec{F}_2$			
Resultant			

Step 2: Resolve $\vec{F}_1$ into components

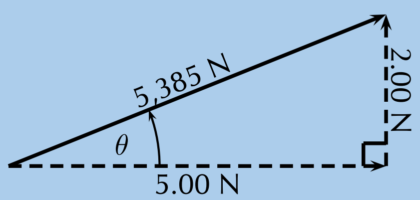
We find the components of $\vec{F}_1$ by using known trigonometric ratios.

First we find the magnitude of the vertical component, $F_{1\text{y}}$:

$$\begin{aligned}
\sin(\theta) &= \frac{F_{1y}}{F_1} \\
\sin(21{,}8°) &= \frac{F_{1y}}{5{,}385} \\
F_{1y} &= (\sin(21{,}8°))\,(5{,}385) \\
&= 2{,}00 \text{ N}
\end{aligned}$$

Secondly we find the magnitude of the horizontal component, $F_{1\text{x}}$:

$$\begin{aligned}
\cos(\theta) &= \frac{F_{1x}}{F_1} \\
\cos(21{,}8°) &= \frac{F_{1x}}{5{,}385} \\
F_{1x} &= (\cos(21{,}8°))\,(5{,}385) \\
&= 5{,}00 \text{ N}
\end{aligned}$$



The components give the sides of the right angle triangle, for which the original vector, $\vec{F}_1$, is the hypotenuse.

Vector	x-component	y-component	Resultant
$\vec{F}_1$	5,00 N	2,00 N	5,385 N
$\vec{F}_2$			
Resultant			

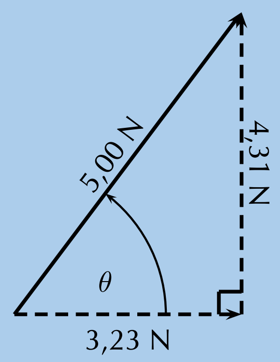
Step 3: Resolve $\vec{F}_2$ into components

We find the components of $\vec{F}_2$ by using known trigonometric ratios. First we find the magnitude of the vertical component, $F_{2\text{y}}$:

$$\begin{aligned}\sin(\theta) &= \frac{F_{2y}}{F_2}\\ \sin(53{,}13°) &= \frac{F_{2y}}{5}\\ F_{2y} &= (\sin(53{,}13°))\,(5)\\ &= 4{,}00\text{ N}\end{aligned}$$

Secondly we find the magnitude of the horizontal component, $F_{2\text{x}}$:

$$\begin{aligned}\cos(\theta) &= \frac{F_{2x}}{F_2}\\ \cos(53{,}13°) &= \frac{F_{2x}}{5}\\ F_{2x} &= (\cos(53{,}13°))\,(5)\\ &= 3{,}00\text{ N}\end{aligned}$$



Vector	x-component	y-component	Total
$\vec{F}_1$	5,00 N	2,00 N	5,385 N
$\vec{F}_2$	3,00 N	4,00 N	5 N
Resultant			

Step 4: Determine the components of the resultant vector

Now we have all the components. If we add all the horizontal components then we will have the x-component of the resultant vector, $\vec{R}_x$. Similarly, we add all the vertical components then we will have the y-component of the resultant vector, $\vec{R}_y$.

$$\begin{aligned}R_x &= F_{1x} + F_{2x}\\ &= 5{,}00\text{ N} + 3{,}00\text{ N}\\ &= 8{,}00\text{ N}\end{aligned}$$

Therefore, $\vec{R}_x$ is 8 N to the right.

$$\begin{aligned}R_y &= F_{1y} + F_{2y}\\ &= 2{,}00\text{ N} + 4{,}00\text{ N}\\ &= 6{,}00\text{ N}\end{aligned}$$

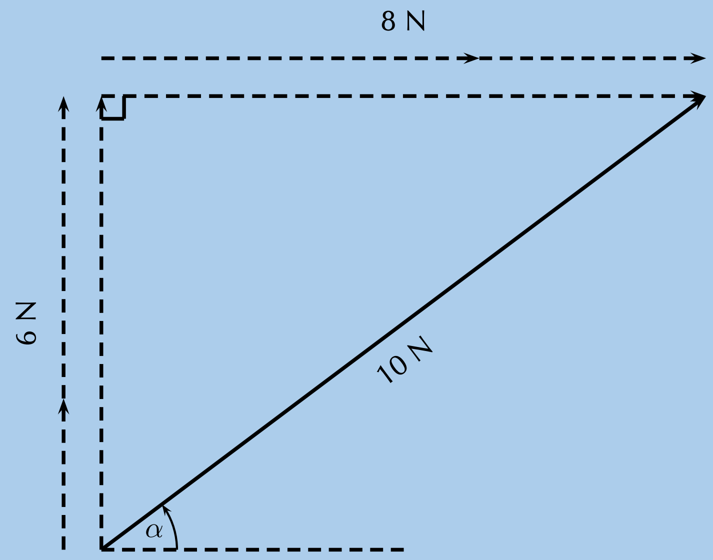
Therefore, $\vec{R}_y$ is 6 N up.

Vector	x-component	y-component	Total
$\vec{F}_1$	5,00 N	2,00 N	5,385 N
$\vec{F}_2$	3,00 N	4,00 N	5 N
Resultant	8,00 N	6,00 N	

Step 5: Determine the magnitude and direction of the resultant vector

Now that we have the components of the resultant, we can use the Theorem of Pythagoras to determine the magnitude of the resultant, R.

$$\begin{aligned} R^2 &= (R_y)^2 + (R_x)^2 \\ &= (6{,}00)^2 + (8{,}00)^2 \\ &= 100{,}00 \\ R &= 10{,}00 \text{ N} \end{aligned}$$



The magnitude of the resultant, R is 10,00 N. So all we have to do is calculate its direction. We can specify the direction as the angle the vectors makes with a known direction. To do this you only need to visualise the vector as starting at the origin of a coordinate system. We have drawn this explicitly below and the angle we will calculate is labelled α.

Using our known trigonometric ratios we can calculate the value of α:

$$\begin{aligned} \tan\alpha &= \frac{6{,}00}{8{,}00} \\ \alpha &= \tan^{-1}\frac{6{,}00}{8{,}00} \\ \alpha &= 36{,}9^\circ \end{aligned}$$

Step 6: Quote the final answer

$\vec{R}$ is 10 m at an angle of $36,9^\circ$ to the positive x-axis.

Worked example 15: Resultant using components

QUESTION

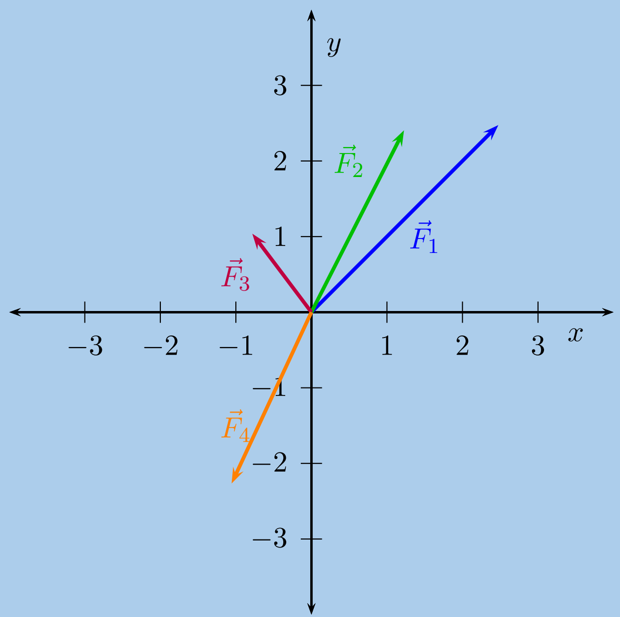
Determine, by resolving into components, the resultant of the following four forces acting at a point:

- $\vec{F}_1 = 3{,}5$ N at 45° to the positive x-axis.
- $\vec{F}_2 = 2{,}7$ N at 63° to the positive x-axis.
- $\vec{F}_3 = 1{,}3$ N at 127° to the positive x-axis.
- $\vec{F}_4 = 2{,}5$ N at 245° to the positive x-axis.

SOLUTION

Step 1: Sketch the problem

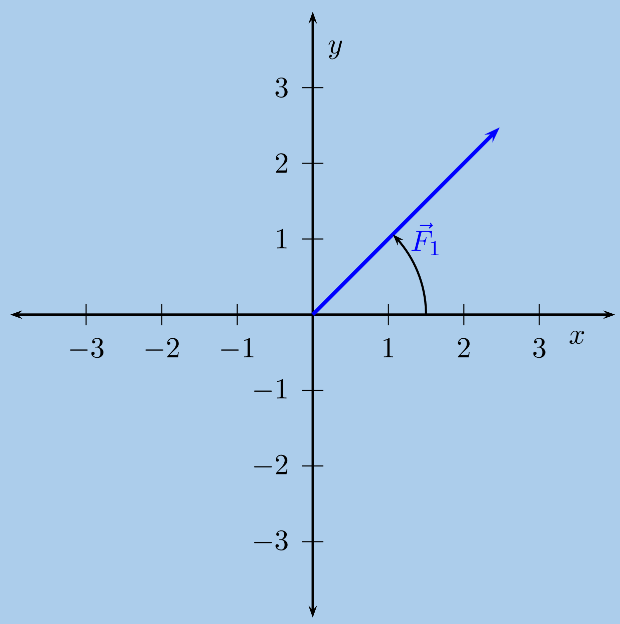
Draw all of the vectors on the Cartesian plane. This does not have to be precisely accurate because we are solving algebraically but vectors need to be drawn in the correct quadrant and with the correct relative positioning other.



We are going to record the various components in a table to help us manage keep track of the calculation. For each vector we need to determine the components in the x- and y-directions.

Vector	x-component	y-component	Total
$\vec{F}_1$			3,5 N
$\vec{F}_2$			2,7 N
$\vec{F}_3$			1,3 N
$\vec{F}_4$			2,5 N
$\vec{R}$			

Step 2: Determine components of $\vec{F}_1$



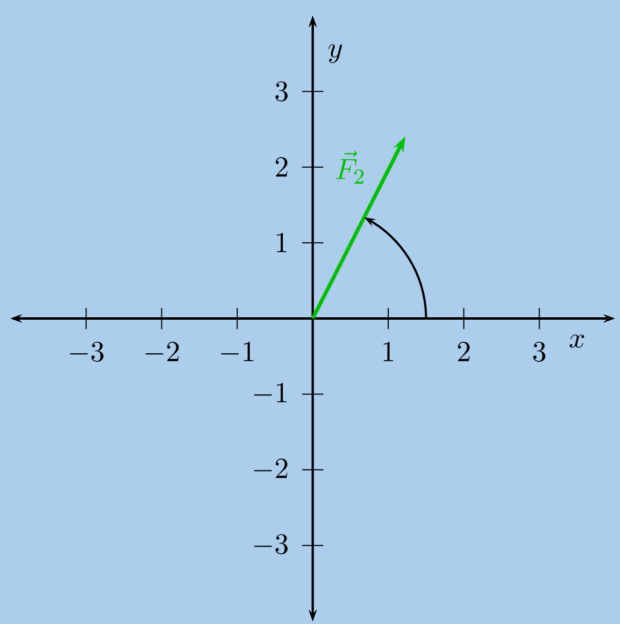
Firstly we find the magnitude of the vertical component, $F_{1\text{y}}$:

$$\begin{aligned}\sin(\theta) &= \frac{F_{1y}}{F_1}\\ \sin(45°) &= \frac{F_{1y}}{3{,}5}\\ F_{1y} &= (\sin(45°))\,(3{,}5)\\ &= 2{,}47\ \text{N}\end{aligned}$$

Secondly we find the magnitude of the horizontal component, $F_{1\text{x}}$:

$$\begin{aligned}\cos(\theta) &= \frac{F_{1x}}{F_1}\\ \cos(45°) &= \frac{F_{1x}}{3{,}5}\\ F_{1x} &= (\cos(45°))\,(3{,}5)\\ &= 2{,}47\ \text{N}\end{aligned}$$

Step 3: Determine components of $\vec{F}_2$



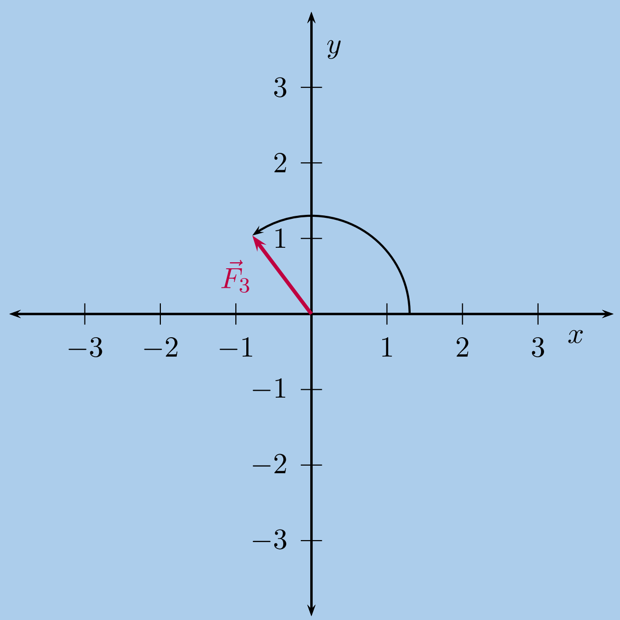
Firstly we find the magnitude of the vertical component, $F_{2\text{y}}$:

$$\begin{aligned} \sin(\theta) &= \frac{F_{2y}}{F_2} \\ \sin(63°) &= \frac{F_{2y}}{2{,}7} \\ F_{2y} &= (\sin(63°))\,(2{,}7) \\ &= 2{,}41\ \text{N} \end{aligned}$$

Secondly we find the magnitude of the horizontal component, $F_{2\text{x}}$:

$$\begin{aligned} \cos\theta &= \frac{F_{2x}}{F_2} \\ \cos(63°) &= \frac{F_{2x}}{2{,}7} \\ F_{2x} &= (\cos(63°))\,(2{,}7) \\ &= 1{,}23\ \text{N} \end{aligned}$$

Step 4: Determine components of $\vec{F}_3$



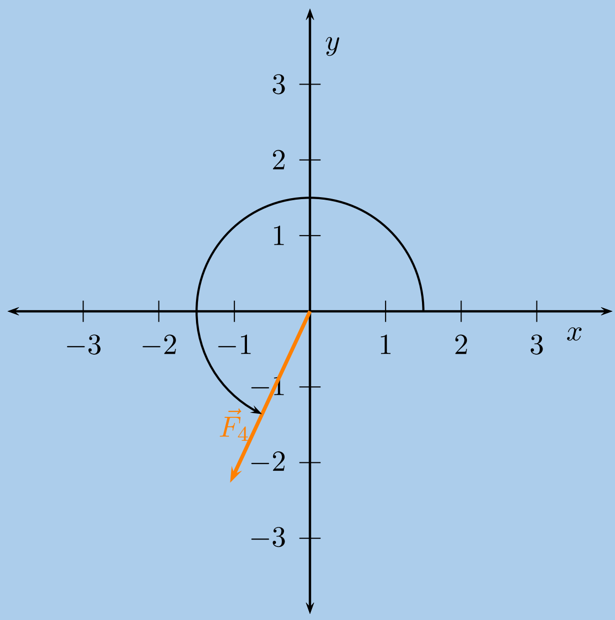
Firstly we find the magnitude of the vertical component, $F_{3\text{y}}$:

$$\begin{aligned} \sin(\theta) &= \frac{F_{3y}}{F_3} \\ \sin(127°) &= \frac{F_{3y}}{1{,}3} \\ F_{3y} &= (\sin 127°)\,(1{,}3) \\ &= 1{,}04\ \text{N} \end{aligned}$$

Secondly we find the magnitude of the horizontal component, $F_{3\text{x}}$:

$$\begin{aligned} \cos(\theta) &= \frac{F_{3x}}{F_3} \\ \cos(127°) &= \frac{F_{3x}}{1{,}3} \\ F_{3x} &= (\cos 127°)\,(1{,}3) \\ &= -0{,}78\ \text{N} \end{aligned}$$

Step 5: Determine components of $\vec{F}_4$



Firstly we find the magnitude of the vertical component, $F_{3\text{y}}$:

$$\begin{aligned}\sin(\theta) &= \frac{F_{4y}}{F_4} \\ \sin(245°) &= \frac{F_{4y}}{2{,}5} \\ F_{4y} &= (\sin(245°))\,(2{,}5) \\ &= -2{,}27\text{ N}\end{aligned}$$

Secondly we find the magnitude of the horizontal component, $F_{4\text{x}}$:

$$\begin{aligned}\cos(\theta) &= \frac{F_{4x}}{F_4} \\ \cos(245°) &= \frac{F_{4x}}{2{,}5} \\ F_{4x} &= (\cos(245°))\,(2{,}5) \\ &= -1{,}06\text{ N}\end{aligned}$$

Step 6: Determine components of resultant

Sum the various component columns to determine the components of the resultant. Remember that if the component was negative don't leave out the negative sign in the summation.

Vector	x-component	y-component	Total
$\vec{F}_1$	2,47 N	2,47 N	3,5 N
$\vec{F}_2$	1,23 N	2,41 N	2,7 N
$\vec{F}_3$	−0,78 N	1,04 N	1,3 N
$\vec{F}_4$	−1,06 N	−2,27 N	2,5 N
$\vec{R}$	1,86 N	3,65 N	

Now that we have the components of the resultant, we can use the Theorem of Pythagoras to determine the magnitude of the resultant, R.

$$\begin{aligned}R^2 &= (R_y)^2 + (R_x)^2 \\ &= (1{,}86)^2 + (3{,}65)^2 \\ &= 16{,}78 \\ R &= 4{,}10\text{ N}\end{aligned}$$

We can also determine the angle with the positive x-axis.

$$\tan(\alpha) = \frac{1{,}86}{3{,}65}$$
$$\alpha = \tan^{-1}(\frac{3{,}65}{1{,}86})$$
$$\alpha = 27{,}00°$$

Step 7: Quote the final answer

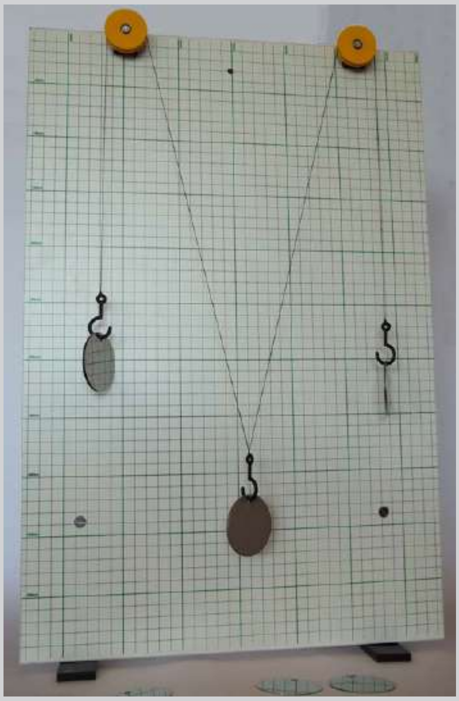
The resultant has a magnitude of 4,10 N at and angle of 27,00° to the positive x-direction.

Informal experiment:Force board

Aim:

Determine the resultant of three non-linear forces using a force board.

Apparatus and materials:



You will need:

- blank paper
- force board
- 4 spring balances
- assortment of weights
- gut or string

Method:

Before beginning the detailed method think about the strategy. By connecting a cord to the ring, running it over a pulley and hanging weights off it you can get a force exerted on the ring. The more weights or heavier the weight you hang the greater the force. The force is in the direction of the cord. If you run a number of cords over pulleys you are exerting more forces, all in different directions, on the ring. So we have a system where we can change the magnitude and direction of the forces acting on the ring. By putting a spring balance between the cord and the ring, we can measure the force. Putting a piece of paper under the ring allows us to draw the directions and the readings on the spring balances allows us to measure the magnitude.

We are going to use this information to measure the forces acting on the rings and then we can determine the resultants graphically.

1. Set up the forceboard and place a piece of paper under the ring.
2. Set up four different forces by connecting a spring balance to the ring on the one side and some cord on the other side. Run the cord over a pulley and attach some weights to it. Work in a group to do this effectively.
3. Draw a line along each cord being careful not to move any of them.
4. Make a note of the reading on each spring balance.
5. Now remove the paper.
6. Working on the paper, draw each line back towards the where the centre of the ring had been. The lines should all intersect at a point. Make this point the centre of your Cartesian coordinate system.
7. Now choose an appropriate scale to relate the length of arrows to the readings on the spring balances. Using the appropriate spring balance and correct line on the paper, draw an arrow to represent each of the forces.

Results:

For two different choices of 3 of the force vectors we will determine the resultant. To determine the resultant we need to add the vectors together. The easiest way to do this is to replicate the vectors using a ruler and protractor and draw them tail-to-head.

Conclusions and questions:

Note the direction and the magnitudes of the resultants of the various combinations.

1. How does the calculated resultant compare to the vector that wasn't used to calculate the resultant in each case?
2. What general relationship should exist between the resultant and the fourth vector and why do you think this is the case?
3. Would this be the same if we had more or less forces in the problem? Justify your answer.

See simulation: 23FV at www.everythingscience.co.za

See video: 23FW at www.everythingscience.co.za

1.4 Chapter summary

ESBKF

See presentation: 23FX at www.everythingscience.co.za

- A vector has a magnitude and direction.
- Vectors can be used to represent many physical quantities that have a magnitude and direction, like forces.
- Vectors may be represented as arrows where the length of the arrow indicates the magnitude and the arrowhead indicates the direction of the vector.
- Vectors in two dimensions can be drawn on the Cartesian plane.
- Vectors can be added graphically using the head-to-tail method or the tail-to-tail method.
- A closed vector diagram is a set of vectors drawn on the Cartesian using the tail-to-head method and that has a resultant with a magnitude of zero.
- Vectors can be added algebraically using Pythagoras' theorem or using components.
- The direction of a vector can be found using simple trigonometric calculations.
- The components of a vector are a series of vectors that, when combined, give the original vector as their resultant.
- Components are usually created that align with the Cartesian coordinate axes. For a vector $\vec{F}$ that makes an angle of θ with the positive x-axis the x-component is $\vec{R_x} = R\cos(\theta)$ and the y-component is $\vec{R_y} = R\sin(\theta)$.

Exercise 1 – 7:

1. Draw the following forces as vectors on the Cartesian plane originating at the origin:
   - $\vec{F_1} = 3{,}7$ N in the positive x-direction
   - $\vec{F_2} = 4{,}9$ N in the positive y-direction
2. Draw the following forces as vectors on the Cartesian plane:
   - $\vec{F_1} = 4{,}3$ N in the positive x-direction
   - $\vec{F_2} = 1{,}7$ N in the negative x-direction
   - $\vec{F_3} = 8{,}3$ N in the positive y-direction
3. Find the resultant in the x-direction, R_x, and y-direction, R_y for the following forces:

- $\vec{F}_1 = 1{,}5$ N in the positive x-direction
- $\vec{F}_2 = 1{,}5$ N in the positive x-direction
- $\vec{F}_3 = 2$ N in the negative x-direction

4. Find the resultant in the x-direction, R_x, and y-direction, R_y for the following forces:

    - $\vec{F}_1 = 4{,}8$ N in the positive x-direction
    - $\vec{F}_2 = 3{,}2$ N in the negative x-direction
    - $\vec{F}_3 = 1{,}9$ N in the positive y-direction
    - $\vec{F}_4 = 2{,}1$ N in the negative y-direction

5. Find the resultant in the x-direction, R_x, and y-direction, R_y for the following forces:

    - $\vec{F}_1 = 2{,}7$ N in the positive x-direction
    - $\vec{F}_2 = 1{,}4$ N in the positive x-direction
    - $\vec{F}_3 = 2{,}7$ N in the negative x-direction
    - $\vec{F}_4 = 1{,}7$ N in the negative y-direction

6. Sketch the resultant of the following force vectors using the tail-to-head method:

    - $\vec{F}_1 = 4{,}8$ N in the positive y-direction
    - $\vec{F}_2 = 3{,}3$ N in the negative x-direction

7. Sketch the resultant of the following force vectors using the tail-to-head method:

    - $\vec{F}_1 = 0{,}7$ N in the positive y-direction
    - $\vec{F}_2 = 6$ N in the positive x-direction
    - $\vec{F}_3 = 3{,}8$ N in the negative y-direction
    - $\vec{F}_4 = 11{,}9$ N in the negative x-direction

8. Sketch the resultant of the following force vectors using the tail-to-head method by first determining the resultant in the x- and y-directions:

    - $\vec{F}_1 = 5{,}2$ N in the positive y-direction
    - $\vec{F}_2 = 7{,}5$ N in the negative y-direction
    - $\vec{F}_3 = 4{,}8$ N in the positive y-direction
    - $\vec{F}_4 = 6{,}3$ N in the negative x-direction

9. Sketch the resultant of the following force vectors using the tail-to-head method by first determining the resultant in the x- and y-directions:

    - $\vec{F}_1 = 6{,}7$ N in the positive y-direction
    - $\vec{F}_2 = 4{,}2$ N in the negative x-direction
    - $\vec{F}_3 = 9{,}9$ N in the negative y-direction
    - $\vec{F}_4 = 3{,}4$ N in the negative y-direction

10. Sketch the resultant of the following force vectors using the tail-to-tail method:

- $\vec{F}_1 = 6{,}1$ N in the positive y-direction
- $\vec{F}_2 = 4{,}5$ N in the negative x-direction

11. Sketch the resultant of the following force vectors using the tail-to-tail method by first determining the resultant in the x- and y-directions:

    - $\vec{F}_1 = 2{,}3$ N in the positive y-direction
    - $\vec{F}_2 = 11{,}8$ N in the negative y-direction
    - $\vec{F}_3 = 7{,}9$ N in the negative y-direction
    - $\vec{F}_4 = 3{,}2$ N in the negative x-direction

12. Four forces act simultaneously at a point, find the resultant if the forces are:

    - $\vec{F}_1 = 2{,}3$ N in the positive x-direction
    - $\vec{F}_2 = 4{,}9$ N in the positive y-direction
    - $\vec{F}_3 = 4{,}3$ N in the negative y-direction
    - $\vec{F}_4 = 3{,}1$ N in the negative y-direction

13. Resolve each of the following vectors into components:

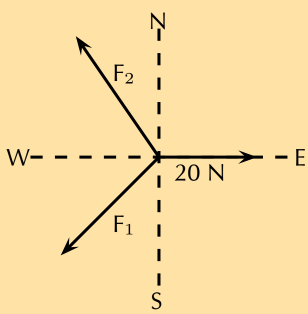
    a) $\vec{F}_1 = 105$ N at $23{,}5°$ to the positive x-axis.
    b) $\vec{F}_2 = 27$ N at $58{,}9°$ to the positive x-axis.
    c) $\vec{F}_3 = 11{,}3$ N at $323°$ to the positive x-axis.
    d) $\vec{F}_4 = 149$ N at $245°$ to the positive x-axis.
    e) $\vec{F}_5 = 15$ N at $375°$ to the positive x-axis.
    f) $\vec{F}_6 = 14{,}9$ N at $75{,}6°$ to the positive x-axis.
    g) $\vec{F}_7 = 11{,}3$ N at $123{,}4°$ to the positive x-axis.
    h) $\vec{F}_8 = 169$ N at $144°$ to the positive x-axis.

14. A point is acted on by two forces and the resultant is zero. The forces

    a) have equal magnitudes and directions.
    b) have equal magnitudes but opposite directions.
    c) act perpendicular to each other.
    d) act in the same direction.

15. A point in equilibrium is acted on by three forces. Force F_1 has components 15 N due south and 13 N due west. What are the components of force F_2?

    a) 13 N due north and 20 N due west
    b) 13 N due north and 13 N due west
    c) 15 N due north and 7 N due west
    d) 15 N due north and 13 N due east



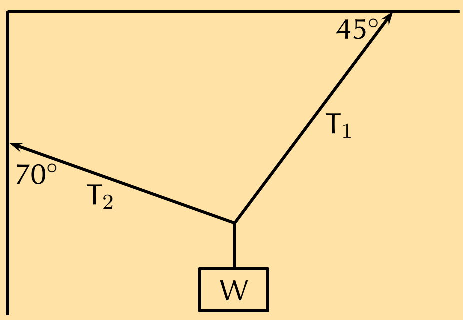
16. Two vectors act on the same point. What should the angle between them be so that a maximum resultant is obtained?

a) $0°$ b) $90°$ c) $180°$ d) cannot tell

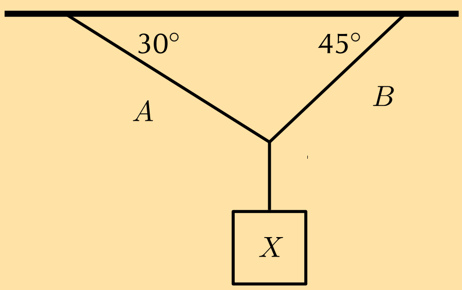
17. Two forces, 4 N and 11 N, act on a point. Which one of the following cannot be the magnitude of a resultant?

a) 4 N b) 7 N c) 11 N d) 15 N

18. An object of weight W is supported by two cables attached to the ceiling and wall as shown. The tensions in the two cables are T_1 and T_2 respectively. Tension $T_1 = 1200$ N. Determine the tension T_2 by accurate construction and measurement or by calculation.



19. An object X is supported by two strings, A and B, attached to the ceiling as shown in the sketch. Each of these strings can withstand a maximum force of 700 N. The weight of X is increased gradually.



a) Draw a rough sketch of the triangle of forces, and use it to explain which string will break first.

b) Determine the maximum weight of X which can be supported.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23FY	2. 23FZ	3. 23G2	4. 23G3	5. 23G4	6. 23G5
7. 23G6	8. 23G7	9. 23G8	10. 23G9	11. 23GB	12. 23GC
13a. 23GD	13b. 23GF	13c. 23GG	13d. 23GH	13e. 23GJ	13f. 23GK
13g. 23GM	13h. 23GN	14. 23GP	15. 23GQ	16. 23GR	17. 23GS
18. 23GT	19. 23GV				

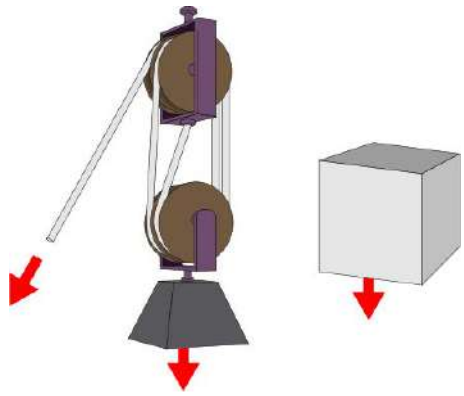
www.everythingscience.co.za **m.everythingscience.co.za**

CHAPTER 2

Newton's laws

2.1 Introduction ESBKG

In this chapter we will learn how a net force is needed to modify the motion of an object. We will recall what a force is and learn about how force and motion are related. We are also introduced to Newton's three laws and we will learn more about the force of gravity.



Key Mathematics Concepts

- Ratio and proportion — Physical Sciences, Grade 10, Science skills
- Equations — Mathematics, Grade 10, Equations and inequalities
- Units and unit conversions — Physical Sciences, Grade 10, Science skills

2.2 Force ESBKH

What is a *force*? ESBKJ

A force is anything that can cause a change to objects. Forces can do things like:

- change the shape of an object,
- accelerate or stop an object, and
- change the direction of a moving object.

A force can be classified as either a *contact force* or a *non-contact force*.

Figure 2.1: Contact forces

Figure 2.2: Contact forces

A contact force must touch or *be in contact* with an object to cause a change. Examples of contact forces are:

- the force that is used to push or pull things, like on a door to open or close it
- the force that a sculptor uses to turn clay into a pot
- the force of the wind to turn a windmill

A non-contact force does not have to touch an object to cause a change. Examples of non-contact forces are the forces due to:

- gravity, like the Earth pulling the Moon towards itself;
- electricity, like a proton and an electron attracting each other; and
- magnetism, like a magnet pulling a paper clip towards itself.

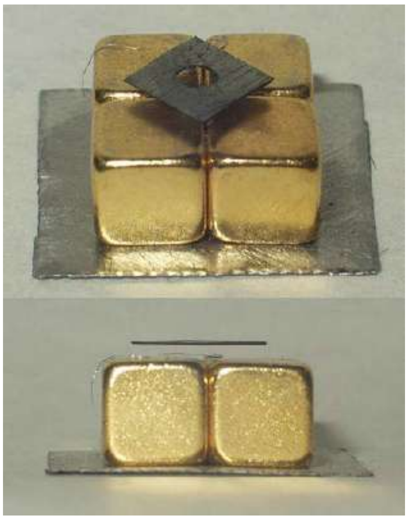


Figure 2.3: Non-contact forces

The unit of force in the international system of units (S.I. units) is the *newton* (symbol *N*). This unit is named after Sir Isaac Newton who first defined force. Force is a vector quantity and so it has a magnitude and a direction. We use the symbol $\vec{F}$ for force.

This chapter will often refer to the *resultant force* acting on an object. The resultant force is simply the vector sum of all the forces acting on the object. It is very important to remember that all the forces must be acting on the *same* object. The resultant force is the force that has the same effect as all the other forces added together.

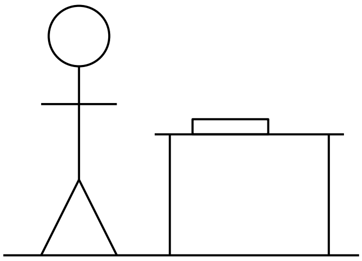
A lot of the physics topics you will study revolve around the impact or effect of forces. Although there are many different forces, will learn some fundamental principles for approaching the problems and applications in this book no matter which force applies

Physics is the study of the natural world and you probably know a lot more physics than you think. You see things happening everyday that are governed by the laws of physics but you probably aren't thinking about physics at the time. If you throw a stone up in the air it eventually falls to the ground. A lot of physics can be learnt by analysing an everyday situation.

We are going to learn about some forces in the next few sections but before we start lets describe an everyday situation in which they all play a role so you can visualise what is happening. You need a table and a three books that all have different masses. Take any book and put it on the table. Nothing happens, the book just rests on the table if the table is flat. If you slowly lift one side of the table so that the top of the table is tilted the book doesn't move immediately. As you lift the table more and more the book suddenly starts to slide off the table. You can repeat this with all three books and see how much you have to tilt the table before the books start to slide.

This real world situation illustrates a lot of the physics we want to learn about in this chapter.

The normal force



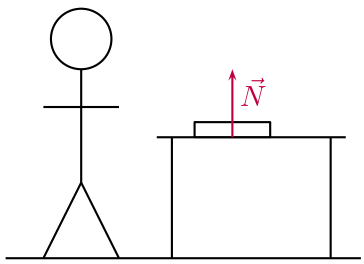
When an object is placed on a surface, for example think of the case of putting a book on a table, there are a number of forces acting. Firstly, if the table were not there the book would fall to the floor. The force that causes this is gravity. The table stops the book falling to the floor. The only way this can happen is for the table to exert a force on the book. The force that the table exerts on the book must balance out the force of gravity. This tells us a few things immediately! Gravity is a force pulling the book down, it is a vector. The force that the table exerts must balance this out and it can only do this if it has the same magnitude and acts in the opposite direction.

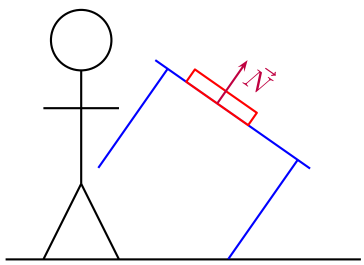
This occurs often, gravity pulls a person towards the earth but when you are standing on the ground something must be balancing it, if you put a heavy box on the ground the gravitational force is balanced. If you put a brick on water it will sink because nothing balances the gravitational force. We give the force that a surface (any surface) exerts to balance the forces on an object in contact with that surface the **normal** force.

The normal force is a force that acts on the object as a result of the interaction with the surface and is perpendicular to the surface. This last part might be seem unexpected (counter-intuitive) because if we tilt the table slightly the direction of the gravitational force hasn't changed but the direction of the normal force has a little (the normal is not always directly opposite gravity). Don't panic, this will all make sense before the end of this chapter. Remember: the normal force is **always perpendicular (at a right angle)** to the surface.

DEFINITION: *Normal force*

The normal force, $\vec{N}$, is the force exerted by a surface on an object in contact with it.





Friction forces

Why does a box sliding on a surface eventually come to a stop? The answer is **friction**. Friction arises where two surfaces are in contact and moving relative to each other.

For an everyday example, press your hands together and move one backwards and forwards, we have two surfaces in contact with one moving relative to the other. Your

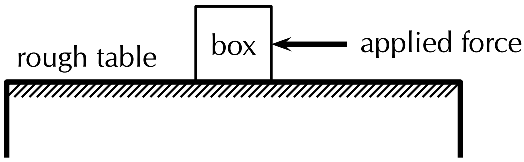
hands get warm, you will have experienced this before and probably rub your hands together in winter to warm them up. The heat is generated through friction.

Friction arises because the surfaces interact with each other. Think about sandpaper with lots of bumps on the surface. If you rub sandpaper the bumps will slot into any groove .

When the surface of one object slides over the surface of another, each body exerts a frictional force on the other. For example if a book slides across a table, the table exerts a frictional force onto the book and the book exerts a frictional force onto the table. Frictional forces act **parallel to surfaces**.

DEFINITION: *Frictional force*

Frictional force is the force that opposes the motion of an object in contact with a surface and it acts parallel to the surface the object is in contact with.



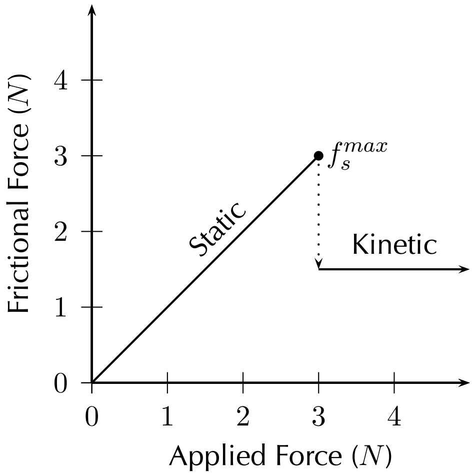
The magnitude of the frictional force depends on the surface and the magnitude of the normal force. Different surfaces will give rise to different frictional forces, even if the normal force is the same. Frictional forces are proportional to the magnitude of the normal force.

$$F_{\text{friction}} \propto N$$

For every surface we can determine a constant factor, the coefficient of friction, that allows us to calculate what the frictional force would be if we know the magnitude of the normal force. We know that static friction and kinetic friction have different magnitudes so we have different coefficients for the two types of friction:

- μ_s is the coefficient of **static** friction
- μ_k is the coefficient of **kinetic** friction

A force is not always large enough to make an object move, for example a small applied force might not be able to move a heavy crate. The frictional force opposing the motion of the crate is equal to the applied force but acting in the opposite direction. This frictional force is called *static friction*. When we increase the applied force (push harder), the frictional force will also increase until it reaches a maximum value. When the applied force is larger than the maximum force of static friction the object will move. The static frictional force can vary from zero (when no other forces are present and the object is stationary) to a maximum that depends on the surfaces.

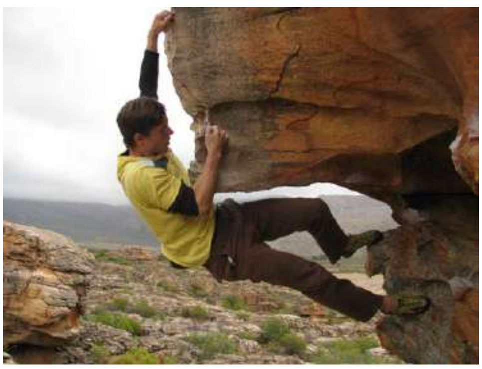


For static friction the force can vary up to some maximum value after which friction has been overcome and the object starts to move. So we define a maximum value for the static friction: $f_s^{max} = \mu_s N$.

When the applied force is greater than the maximum, static frictional force, the object moves but still experiences friction. This is called *kinetic friction*. For kinetic friction the value remains the same regardless of the magnitude of the applied force. The magnitude of the kinetic friction is: $f_k = \mu_k N$.

Remember that static friction is present when the object is not moving and kinetic friction while the object is moving. For example when you drive at constant velocity in a car on a tar road you have to keep the accelerator pushed in slightly to overcome the friction between the tar road and the wheels of the car. However, while moving at a constant velocity the wheels of the car are rolling, so this is not a case of two surfaces "rubbing" against each other and we are in fact looking at static friction. If you should break hard, causing the car to skid to a halt, we would be dealing with two surfaces rubbing against each other and hence kinetic friction. The higher the value for the coefficient of friction, the more 'sticky' the surface is and the lower the value, the more 'slippery' the surface is.

Friction is very useful. If there was no friction and you tried to prop a ladder up against a wall, it would simply slide to the ground. Rock climbers use friction to maintain their grip on cliffs. The brakes of cars would be useless if it wasn't for friction!



Early humans made use of friction to create fire. Friction can create a lot of heat and

the early humans used this fact when they rubbed two sticks together to start a fire. When you rub your hands together fast and pressing hard you will feel that they get warm. This is heat created by the friction. You can use this to start a fire.

To start a fire you need two pieces of wood, one long straight, round piece approximately the same thickness as your finger and about 40 cm long as well as a thicker flat piece of wood. The flat thick piece of wood needs a hole that the long straight one can fit into. Then you put the flat piece on the ground, the long straight one in the hole and rub it between your hands applying downwards pressure to increase the normal force and the amount of friction. Where the two pieces of wood rub against each other the friction results in many tiny pieces of wood being rubbed off and getting hot.

See video: 23GW at www.everythingscience.co.za

After a while the hole will start to smoke. At this point the smoking wood bits, called the ember, need to be gently tipped out of the hole into a small bed of dry grass. You cover the ember completely and blow gently. The grass should start to burn. Then you use the burning grass to light some dry twigs, and keep working your way up to bigger pieces of wood.

To make it even easier, a bow from wood with a string can be used to cause the wood to turn. By twisting the string from the bow around the long piece of wood it can be driven without requiring a person use their hands.

Worked example 1: Static friction

QUESTION

A box resting on a surface experiences a normal force of magnitude 30 N and the coefficient of static friction tween the surface and the box, μ_s, is 0,34. What is the maximum static frictional force?

SOLUTION

Step 1: Maximum static friction

We know that the relationship between the maximum static friction, f_s^{max}, the coefficient of static friction, μ_s and the normal, N, to be:

$$f_s^{max} = \mu_s N$$

We have been given that $\mu_s = 0{,}34$ and $N = 30$ N. This is all of the information required to do the calculation.

Step 2: Calculate the result

$$
\begin{aligned}
f_s^{max} &= \mu_s N \\
&= (0{,}34)(30) \\
&= 10{,}2
\end{aligned}
$$

The maximum magnitude of static friction is 10,2 N.

Worked example 2: Static friction

QUESTION

The forwards of your school's rugby team are trying to push their scrum machine. The normal force exerted on the scrum machine is 10 000 N. The machine isn't moving at all. If the coefficient of static friction is 0,78 what is the minimum force they need to exert to get the scrum machine to start moving?

SOLUTION

Step 1: Minimum or maximum

The question asks what the minimum force required to get the scrum machine moving will be. We don't know a relationship for this but we do know how to calculate the maximum force of static friction. The forwards need to exert a force greater than this so the minimum amount they can exert is in fact equal to the maximum force of static friction.

Step 2: Maximum static friction

We know that the relationship between the maximum static friction, f_s^{max}, the coefficient of static friction, μ_s and the normal, N, to be:

$$f_s^{max} = \mu_s N$$

We have been given that $\mu_s = 0{,}78$ and $N = 10\,000$ N. This is all of the information required to do the calculation.

Step 3: Calculate the result

$$
\begin{aligned}
f_s^{max} &= \mu_s N \\
&= (0{,}78)(10\,000) \\
&= 7800 \text{ N}
\end{aligned}
$$

The maximum magnitude of static friction is 7800 N.

Worked example 3: Kinetic friction

QUESTION

The normal force exerted on a pram is 100 N. The pram's brakes are locked so that the wheels cannot turn. The owner tries to push the pram but it doesn't move. The owner pushes harder and harder until it suddenly starts to move when the applied force is three quarters of the normal force. After that the owner is able to keep it moving with a force that is half of the force at which it started moving. What is the magnitude of the applied force at which it starts moving and what are the coefficients of static and kinetic friction?

SOLUTION

Step 1: Maximum static friction

The owner of the pram increases the force he is applying until suddenly the pram starts to move. This will be equal to the maximum static friction which we know is given by:

$$f_s^{max} = \mu_s N$$

We are given that the magnitude of the applied force is three quarters of the normal force magnitude, so:

$$\begin{aligned} f_s^{max} &= \frac{3}{4}N \\ &= \frac{3}{4}(100) \\ &= 75 \text{ N} \end{aligned}$$

Step 2: Coefficient of static friction

We now know both the maximum magnitude of static friction and the magnitude of the normal force so we can find the coefficient of static friction:

$$\begin{aligned} f_s^{max} &= \mu_s N \\ 75 &= \mu_s(100) \\ \mu_s &= 0{,}75 \end{aligned}$$

Step 3: Coefficient of kinetic friction

The magnitude of the force required to keep the pram moving is half of the magnitude of the force required to get it to start moving so we can determine it from:

$$\begin{aligned} f_k &= \frac{1}{2}f_s^{max} \\ &= \frac{1}{2}(75) \\ &= 37{,}5 \text{ N} \end{aligned}$$

We know the relationship between the magnitude of the kinetic friction, magnitude of the normal force and coefficient of kinetic friction. We can use it to solve for the

coefficient of kinetic friction:

$$\begin{aligned} f_k &= \mu_k N \\ 37{,}5 &= \mu_k(100) \\ \mu_k &= 0{,}375 \end{aligned}$$

Worked example 4: Coefficient of static friction

QUESTION

A block of wood experiences a normal force of 32 N from a rough, flat surface. There is a rope tied to the block. The rope is pulled parallel to the surface and the tension (force) in the rope can be increased to 8 N before the block starts to slide. Determine the coefficient of static friction.

SOLUTION

Step 1: Analyse the question and determine what is asked

The normal force is given (32 N) and we know that the block does not move until the applied force is 8 N.

We are asked to find the coefficient for static friction μ_s.

Step 2: Find the coefficient of static friction

$$\begin{aligned} F_f &= \mu_s N \\ 8 &= \mu_s(32) \\ \mu_s &= 0{,}25 \end{aligned}$$

Note that the coefficient of friction does not have a unit as it shows a ratio. The value for the coefficient of friction friction can have any value up to a maximum of 0,25. When a force less than 8 N is applied, the coefficient of friction will be less than 0,25.

Worked example 5: Static friction

QUESTION

A box resting on an inclined plane experiences a normal force of magnitude 130 N and the coefficient of static friction, μ_s, between the box and the surface is 0,47. What

is the maximum static frictional force?

SOLUTION

Step 1: Maximum static friction

We know that the relationship between the maximum static friction, f_s^{max}, the coefficient of static friction, μ_s and the normal, N, to be:

$$f_s^{max} = \mu_s N.$$

This does not depend on whether the surface is inclined or not. Changing the inclination of the surface will affect the magnitude of the normal force but the method of determining the frictional force remains the same.

We have been given that $\mu_s = 0{,}47$ and $N = 130$ N. This is all of the information required to do the calculation.

Step 2: Calculate the result

$$\begin{aligned} f_s^{max} &= \mu_s N \\ &= (0{,}47)(130) \\ &= 61{,}1 \text{ N} \end{aligned}$$

The maximum magnitude of static friction is 61,1 N.

Informal experiment:Normal forces and friction

Aim:

To investigate the relationship between normal force and friction.

Apparatus:

Spring balance, several blocks, of the same material, with hooks attached to one end, several rough and smooth surfaces, bricks or blocks to incline the surfaces

Method:

- Attach each of the blocks to the spring balance in turn and note the reading.
- Take one block and attach it to the spring balance. Now slide the block along each of the surfaces in turn. Note the readings. Repeat for the other blocks.
- Repeat the above step but incline the surfaces at different angles.

Results:

Record your results in a table

Tension

Tension is the magnitude of the force that exists in objects like ropes, chains and struts that are providing support. For example, there are tension forces in the ropes supporting a child's swing hanging from a tree.

Exercise 2 – 1:

1. A box is placed on a rough surface. It has a normal force of magnitude 120 N. A force of 20 N applied to the right cannot move the box. Calculate the magnitude and direction of the friction forces.
2. A block rests on a horizontal surface. The normal force is 20 N. The coefficient of static friction between the block and the surface is 0,40 and the coefficient of dynamic friction is 0,20.
    a) What is the magnitude of the frictional force exerted on the block while the block is at rest?
    b) What will the magnitude of the frictional force be if a horizontal force of magnitude 5 N is exerted on the block?
    c) What is the minimum force required to start the block moving?
    d) What is the minimum force required to keep the block in motion once it has been started?
    e) If the horizontal force is 10 N, determine the frictional force.
3. A lady injured her back when she slipped and fell in a supermarket. She holds the owner of the supermarket accountable for her medical expenses. The owner claims that the floor covering was not wet and meets the accepted standards. He therefore cannot accept responsibility. The matter eventually ends up in court. Before passing judgement, the judge approaches you, a science student, to determine whether the coefficient of static friction of the floor is a minimum of 0,5 as required. He provides you with a tile from the floor, as well as one of the shoes the lady was wearing on the day of the incident.
    a) Write down an expression for the coefficient of static friction.
    b) Plan an investigation that you will perform to assist the judge in his judgement. Follow the steps outlined below to ensure that your plan meets the requirements.
        i. Formulate an investigation question.
        ii. Apparatus: List *all* the other apparatus, except the tile and the shoe, that you will need.
        iii. A stepwise method: How will you perform the investigation? Include a relevant, labelled diagram.
        iv. Results: What will you record?
        v. Conclusion: How will you interpret the results to draw a conclusion?

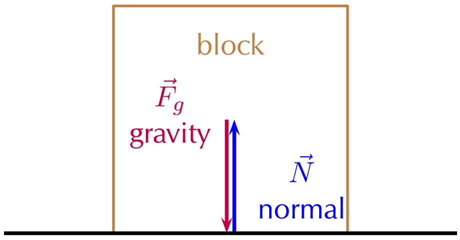
Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23GX 2. 23GY 3. 23GZ

www.everythingscience.co.za m.everythingscience.co.za

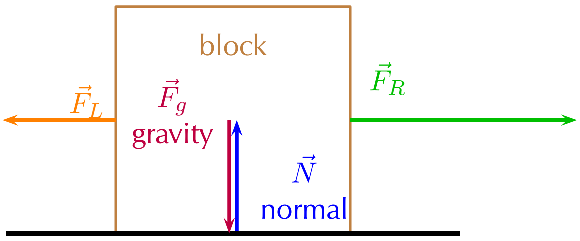
Force diagrams ESBKM

Force diagrams are sketches of the physical situation you are dealing with, with arrows for all the forces acting drawn on the system. For example, if a block rests on a surface then there is a force from gravity pulling the block down and there is a normal force acting on the block from the surface. The normal force and the force of gravity have the same magnitude in this situation. The force diagram for this situation is:



The length of the arrows are the same to indicate that the forces have the same magnitude.

Another example could be a the same block on a surface but with an applied force, $\vec{F}_L$, to the left of 10 N and an applied force, $\vec{F}_R$, to the right of 20 N. The weight and normal also have magnitudes of 10 N.



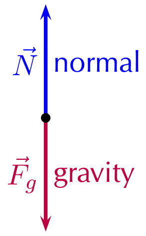
It is important to keep the following in mind when you draw force diagrams:

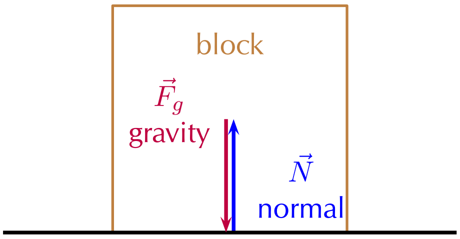
- Make your drawing large and clear.
- You must use arrows and the direction of the arrow will show the direction of the force.
- The length of the arrow will indicate the size of the force, in other words, the longer arrows in the diagram (F_R for example) indicates a bigger force than a shorter arrow (F_L). Arrows of the same length indicate forces of equal size (F_N and F_g). Use "little lines" like in maths to show this.

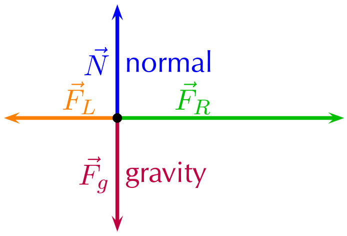
- Draw neat lines using a ruler. The arrows must touch the system or object.
- All arrows must have labels. Use letters with a key on the side if you do not have enough space on your drawing.
- The labels must indicate what is applying the force (the force of the car) on what the force is applied (on the trailer) and in which direction (to the right)
- If the values of the forces are known, these values can be added to the diagram or key.

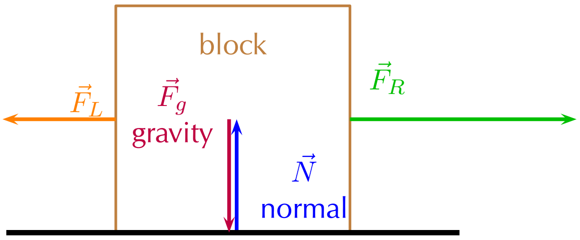
Free body diagrams ESBKN

In a free-body diagram, the object of interest is drawn as a dot and all the forces acting on it are drawn as arrows pointing away from the dot. We can redraw the two force diagrams above as free body diagrams:





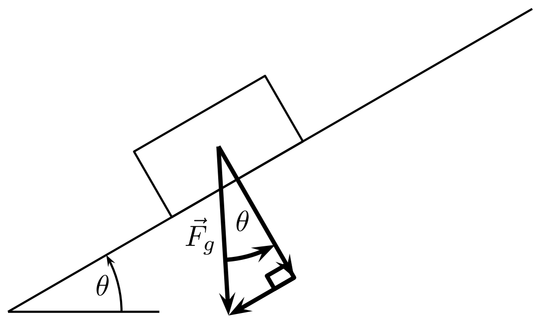




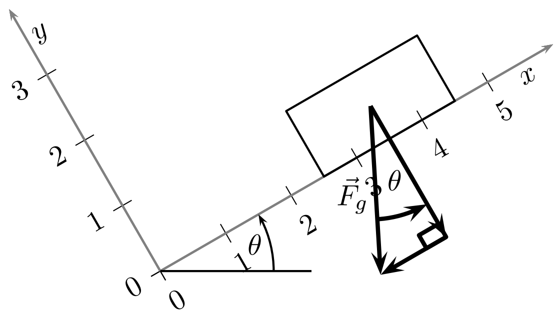
Resolving forces into components ESBKP

We have looked at resolving forces into components. There is one situation we will consider where this is particularly useful, problems involving an inclined plane. It is important because the normal force depends on the component of the gravitational force that is perpendicular to the slope.

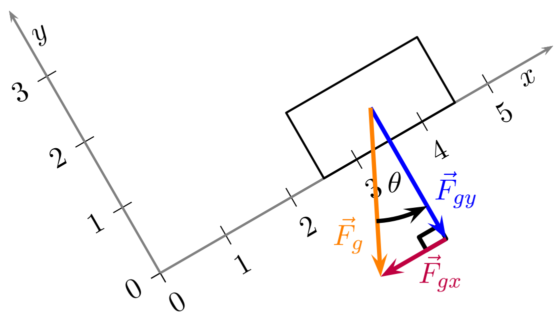
Let us consider a block on an inclined plane. The plane is inclined at an angle θ to the horizontal. It feels a gravitational force, $\vec{F}_g$, directly downwards. This force can be broken into components that are perpendicular to the plane and parallel to it. This is shown here:



We can use any coordinate system to describe the situation and the simplest thing to do is to make the x-axis of the Cartesian coordinate system align with the inclined plane. Here is the same physical situation with the coordinate system drawn in:



This means that the components of gravitational force are aligned (parallel) with one of the axes of the coordinate system. We have shown in the figures that the angle between the horizontal and the incline is also the angle between the gravitational force and its component perpendicular to the inclined plane (this is normal to the plane). Using this angle and the fact that the components form part of a right-angled triangle we can calculate the components using trigonometry:



Using trigonometry the components are given by:

$$F_{gx} = F_g \sin(\theta)$$
$$F_{gy} = F_g \cos(\theta)$$

Worked example 6: Components of force due to gravity

QUESTION

A block on an inclined plane experiences a force due to gravity, $\vec{F}_g$ of 137 N straight down. If the slope is inclined at 37° to the horizontal, what is the component of the force due to gravity perpendicular and parallel to the slope?

SOLUTION

Step 1: Components

We know that for a block on a slope we can resolve the force due to gravity, $\vec{F}_g$ into components parallel and perpendicular to the slope.

$$F_{gx} = F_g \sin(\theta)$$
$$F_{gy} = F_g \cos(\theta)$$

Step 2: Calculations

This problem is straightforward as we know that the slope is inclined at an angle of 37°. This is the same angle we need to use to calculate the components, therefore:

$$\begin{aligned} F_{gx} &= F_g \sin(\theta) \\ &= (137) \sin(37°) \\ &= 82{,}45 \text{ N} \end{aligned}$$

$$\begin{aligned} F_{gy} &= F_g \cos(\theta) \\ &= (137) \cos(37°) \\ &= 109{,}41 \text{ N} \end{aligned}$$

Step 3: Final answer

The component of $\vec{F}_g$ that is perpendicular to the slope is $\vec{F}_{gy} = 109{,}41$ N in the negative y-direction.

The component of $\vec{F}_g$ that is perpendicular to the slope is $\vec{F}_{gx} = 82{,}45$ N in the negative x-direction.

Exercise 2 – 2:

1. A block on an inclined plane experiences a force due to gravity, $\vec{F}_g$ of 456 N straight down. If the slope is inclined at 67,8° to the horizontal, what is the component of the force due to gravity perpendicular and parallel to the slope?
2. A block on an inclined plane is subjected to a force due to gravity, $\vec{F}_g$ of 456 N straight down. If the component of the gravitational force parallel to the slope is $\vec{F}_{gx} = 308{,}7$ N in the negative x-direction (down the slope), what is the incline of the slope?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23H2 2. 23H3

Finding the resultant force ESBKQ

The easiest way to determine a resultant force is to draw a free body diagram. Remember from Chapter 1 that we use the length of the arrow to indicate the vector's magnitude and the direction of the arrow to show which direction it acts in.

After we have done this, we have a diagram of vectors and we simply find the sum of the vectors to get the resultant force.

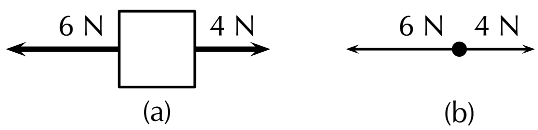


Figure 2.4: (a) Force diagram of 2 forces acting on a box. (b) Free body diagram of the box.

For example, two people push on a box from opposite sides with forces of 4 N and 6 N respectively as shown in Figure 2.4 (a). The free body diagram in Figure 2.4 (b) shows the object represented by a dot and the two forces are represented by arrows with their tails on the dot.

As you can see, the arrows point in opposite directions and have different lengths. The resultant force is 2 N to the left. This result can be obtained algebraically too, since the two forces act along the same line. First, as in motion in one direction, choose a frame of reference. Secondly, add the two vectors taking their directions into account.

For the example, assume that the positive direction is to the right, then:

$$\begin{aligned} F_R &= (4) + (-6) \\ &= -2 \\ &= -2 \text{ N to the left.} \end{aligned}$$

Remember that a negative answer means that the force acts in the *opposite* direction to the one that you chose to be positive. You can *choose* the positive direction to be any way you want, but once you have chosen it you *must* use it consistently for that problem.

As you work with more force diagrams in which the forces exactly balance, you may notice that you get a zero answer (e.g. 0 N). This simply means that the forces are balanced and the resultant is zero.

Once a force diagram has been drawn the techniques of vector addition introduced in Chapter 1 can be used. Depending on the situation you might choose to use a graphical technique such as the tail-to-head method or the parallelogram method, or else an algebraic approach to determine the resultant. Since force is a vector quantity all of these methods apply.

A good strategy is:

- resolve all forces into components parallel to the x- and y-directions;
- calculate the resultant in each direction, $\vec{R}_x$ and $\vec{R}_y$, using co-linear vectors; and
- use $\vec{R}_x$ and $\vec{R}_y$ to calculate the resultant, $\vec{R}$.

Worked example 7: Finding the resultant force

QUESTION

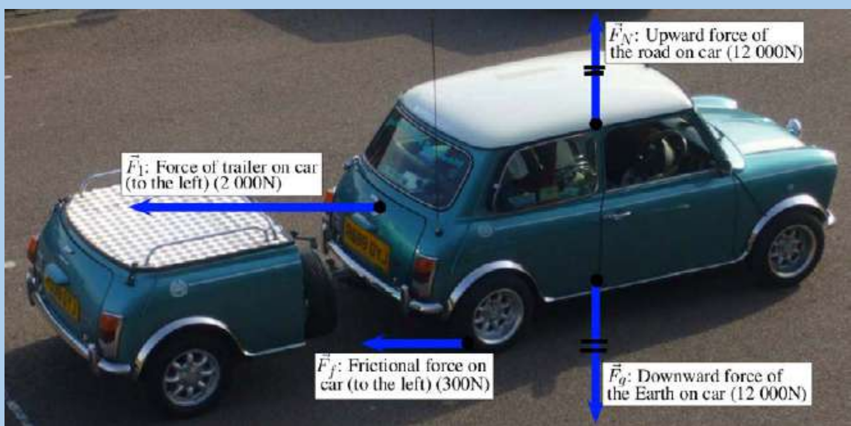
A car (experiencing a gravitational force of magnitude 12 000 N and a normal force of the same magnitude) applies a force of 2000 N on a trailer (experiencing a gravitational force of magnitude 2500 N and normal force of the same magnitude). A constant frictional force of magnitude 200 N is acting on the trailer, and a constant frictional force of magnitude 300 N is acting on the car.

1. Draw a force diagram of all the forces acting on the car.
2. Draw a free body diagram of all the forces acting on the trailer.
3. Use the force diagram to determine the resultant force on the trailer.

SOLUTION

Step 1: Draw the force diagram for the car.

The question asks us to draw all the forces on the car. This means that we must include horizontal and vertical forces.



Step 2: Determine the resultant force on the trailer.

To find the resultant force we need to add all the horizontal forces together. We do not add vertical forces as the movement of the car and trailer will be in a horizontal direction, and not up or down. $F_R = 2000 + (-200) = 1800$N to the right.

See simulation: 23H4 at www.everythingscience.co.za

Exercise 2 – 3: Forces and motion

1. A boy pushes a shopping trolley (weight due to gravity of 150 N) with a constant force of 75 N. A constant frictional force of 20 N is present.

   a) Draw a labelled force diagram to identify all the forces acting on the shopping trolley.

   b) Draw a free body diagram of all the forces acting on the trolley.

   c) Determine the resultant force on the trolley.

2. A donkey (experiencing a gravitational force of 2500 N) is trying to pull a cart (force due to gravity of 800 N) with a force of 400 N. The rope between the donkey and the cart makes an angle of 30° with the cart. The cart does not move.

   a) Draw a free body diagram of all the forces acting on the donkey.

   b) Draw a force diagram of all the forces acting on the cart.

   c) Find the magnitude and direction of the frictional force preventing the cart from moving.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23H5 2. 23H6

www.everythingscience.co.za **m.everythingscience.co.za**

2.3 Newton's laws ESBKR

In this section we will look at the effect of forces on objects and how we can make things move. This will link together what you have learnt about motion and what you have learnt about forces.

Newton's first law ESBKS

Sir Isaac Newton was a scientist who lived in England (1642-1727) who was interested in the motion of objects under various conditions. He suggested that a stationary object will remain stationary unless a force acts on it and that a moving object will continue moving unless a force slows it down, speeds it up or changes its direction of motion. From this he formulated what is known as Newton's first law of motion:

DEFINITION: *Newton's first law of motion*

An object continues in a state of rest or uniform motion (motion with a constant velocity) unless it is acted on by an unbalanced (net or resultant) force.

This property of an object, to continue in its current state of motion unless acted upon by a net force, is called *inertia*.

Let us consider the following situations:

An ice skater pushes herself away from the side of the ice rink and skates across the ice. She will continue to move in a straight line across the ice unless something stops her. Objects are also like that. If we kick a soccer ball across a soccer field, according to Newton's first law, the soccer ball should keep on moving forever! However, in real life this does not happen.

Is Newton's Law wrong? Not really. Newton's first law applies to situations where there aren't any external forces present. This means that friction is not present. In the case of the ice skater, the friction between the skates and the ice is very little and she will continue moving for quite a distance. In the case of the soccer ball, air resistance (friction between the air and the ball) and friction between the grass and the ball is present and this will slow the ball down.

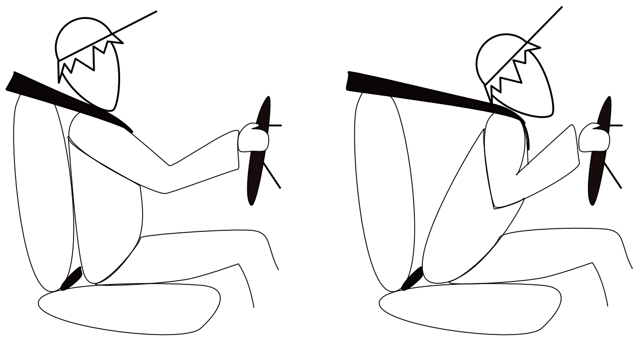
See video: 23H7 at www.everythingscience.co.za

Newton's first law in action

Let us look at the following two examples. We will start with a familiar example:

Seat belts:

We wear seat belts in cars. This is to protect us when the car is involved in an accident. If a car is travelling at 120 $km \cdot h^{-1}$, the passengers in the car is also travelling at 120 $km \cdot h^{-1}$. When the car suddenly stops a force is exerted on the car (making it slow down), but not on the passengers. The passengers will carry on moving forward at 120 $km \cdot h^{-1}$ according to Newton's first law. If they are wearing seat belts, the seat belts will stop them by exerting a force on them and so prevent them from getting hurt.



See video: 23H8 at www.everythingscience.co.za

Rockets:

A spaceship is launched into space. The force of the exploding gases pushes the rocket through the air into space. Once it is in space, the engines are switched off and it will keep on moving at a constant velocity. If the astronauts want to change the direction of the spaceship they need to fire an engine. This will then apply a force on the rocket and it will change its direction.

Worked example 8: Newton's first law in action

QUESTION

Why do passengers get thrown to the side when the car they are driving in goes around a corner?

SOLUTION

Step 1: What happens before the car turns

Before the car starts turning both the passengers and the car are travelling at the same velocity. (picture A)

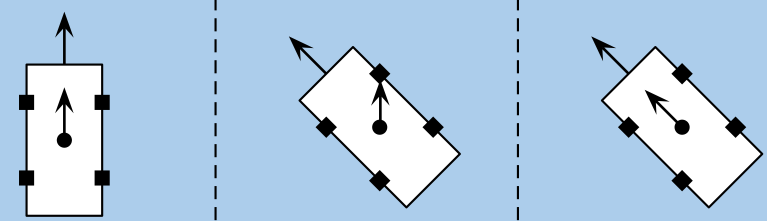
Step 2: What happens while the car turns

The driver turns the wheels of the car, which then exert a force on the car and the car turns. This force acts on the car but not the passengers, hence (by Newton's first law)

the passengers continue moving with the same original velocity. (picture B)

Step 3: Why passengers get thrown to the side?

If the passengers are wearing seat belts they will exert a force on the passengers until the passengers' velocity is the same as that of the car (picture C). Without a seat belt the passenger may hit the side of the car.



A: Both the car and the person travelling at the same velocity

B: The car turns but not the person

C: Both the car and the person are travelling at the same velocity again

Exercise 2 – 4:

1. If a passenger is sitting in a car and the car turns round a bend to the right, what happens to the passenger? What happens if the car turns to the left?
2. Helium is less dense than the air we breathe. Discuss why a helium balloon in a car driving around a corner appears to violate Newton's first law and moves towards the inside of the turn not the outside like a passenger.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23H9 2. 23HB

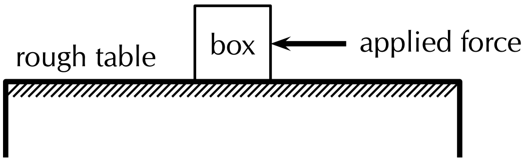
www.everythingscience.co.za **m.everythingscience.co.za**

Newton's second law of motion ESBKT

According to Newton's first law, things 'like to keep on doing what they are doing'. In other words, if an object is moving, it tends to continue moving (in a straight line and at the same speed) and if an object is stationary, it tends to remain stationary. So how do objects start moving?

Let us look at the example of a 10 kg box on a rough table. If we push lightly on the

box as indicated in the diagram, the box won't move. Let's say we applied a force of 100 N, yet the box remains stationary. At this point a frictional force of 100 N is acting on the box, preventing the box from moving. If we increase the force, let's say to 150 N and the box almost starts to move, the frictional force is 150 N. To be able to move the box, we need to push hard enough to overcome the friction and then move the box. If we therefore apply a force of 200 N remembering that a frictional force of 150 N is present, the 'first' 150 N will be used to overcome or 'cancel' the friction and the other 50 N will be used to move (accelerate) the block. In order to accelerate an object we must have a resultant force acting on the block.



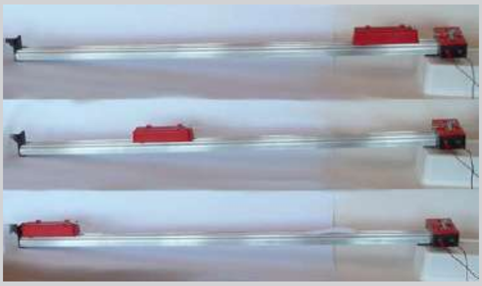
Now, what do you think will happen if we pushed harder, lets say 300 N? Or, what do you think will happen if the mass of the block was more, say 20 kg, or what if it was less? Let us investigate how the motion of an object is affected by mass and force.

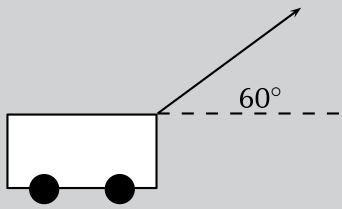
Formal experiment: Newton's second law of motion

Aim:

To investigate the relation between the acceleration of objects and the application of a constant resultant force.

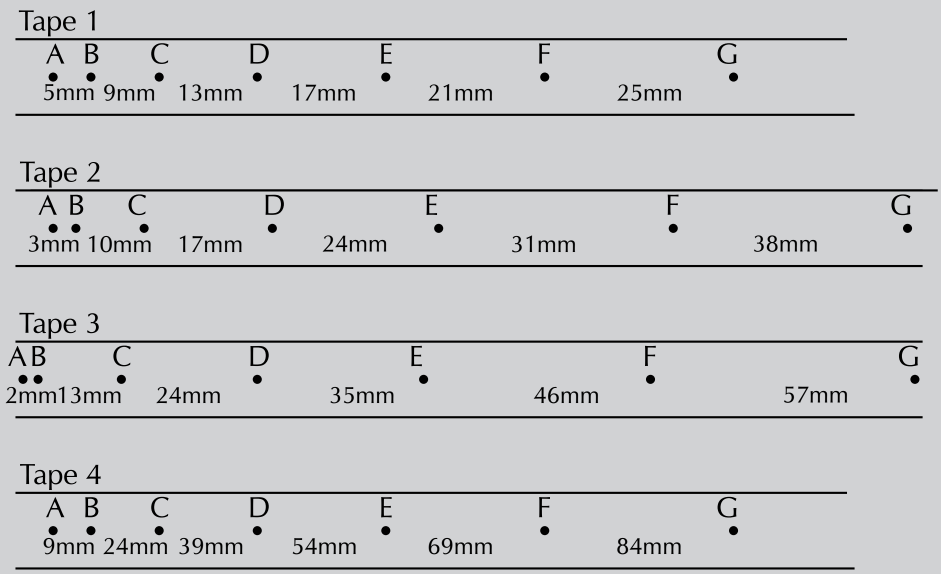
Method:





1. A constant force of 20 N, acting at an angle of 60° to the horizontal, is applied to a dynamics trolley.
2. Ticker tape attached to the trolley runs through a ticker timer of frequency 20 Hz as the trolley is moving on the frictionless surface.
3. The above procedure is repeated 4 times, each time using the same force, but varying the mass of the trolley as follows:
   - Case 1: 6,25 kg
   - Case 2: 3,57 kg
   - Case 3: 2,27 kg
   - Case 4: 1,67 kg

4. Shown below are sections of the four ticker tapes obtained. The tapes are marked with the letters A, B, C, D, etc. A is the first dot, B is the second dot and so on. The distance between each dot is also shown.



Tapes are not drawn to scale.

Instructions:

1. Use each tape to calculate the instantaneous velocity (in m·s^{-1}) of the trolley at points B and F (remember to convert the distances to m first!). Use these velocities to calculate the trolley's acceleration in each case.
2. Tabulate the mass and corresponding acceleration values as calculated in each case. Ensure that each column and row in your table is appropriately labelled.
3. Draw a graph of acceleration vs. mass, using a scale of 1 cm = 1 m·s^{-2} on the y-axis and 1 cm = 1 kg on the x-axis.
4. Use your graph to read off the acceleration of the trolley if its mass is 5 kg.
5. Write down a conclusion for the experiment.

You will have noted in the investigation above that the heavier the trolley is, the slower it moved when the force was constant. The acceleration is *inversely* proportional to the mass. In mathematical terms: $a \propto \frac{1}{m}$

In a similar investigation where the mass is kept constant, but the applied force is varied, you will find that the bigger the force is, the faster the object will move. The acceleration of the trolley is therefore *directly* proportional to the resultant force. In mathematical terms: $a \propto F$.

Rearranging the above equations, we get a $\propto \frac{F}{m}$ **or** $F = ma$.

Remember that both force and acceleration are vectors quantities. The acceleration is

in the same direction as the force that is being applied. If multiple forces are acting simultaneously then we only need to work with the resultant force or net force.

DEFINITION: *Newton's second law of motion*

If a resultant force acts on a body, it will cause the body to accelerate in the direction of the resultant force. The acceleration of the body will be directly proportional to the resultant force and inversely proportional to the mass of the body. The mathematical representation is:

$$\vec{F}_{net} = m\vec{a}$$

Force is a vector quantity. Newton's second law of motion should be applied to the y- and x-directions separately. You can use the resulting y- and x-direction resultants to calculate the overall resultant as we saw in the previous chapter.

See video: 23HC at www.everythingscience.co.za

Applying Newton's second law of motion

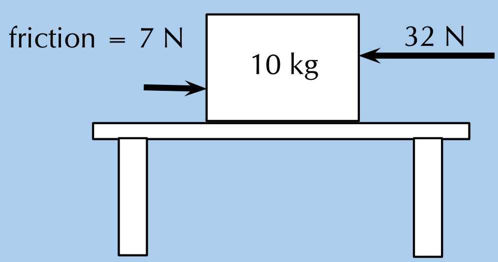
Newton's second law can be applied to a variety of situations. We will look at the main types of examples that you need to study.

Worked example 9: Newton's second law: Box on a surface

QUESTION

A 10 kg box is placed on a table. A horizontal force of magnitude 32 N is applied to the box. A frictional force of magnitude 7 N is present between the surface and the box.

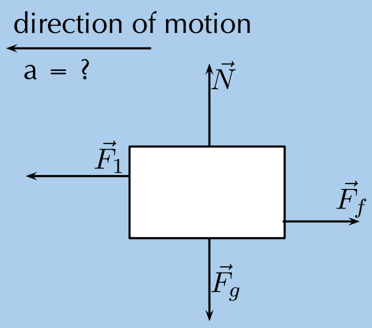
1. Draw a force diagram indicating all of the forces acting on the box.
2. Calculate the acceleration of the box.



SOLUTION

Step 1: Identify the horizontal forces and draw a force diagram

We only look at the forces acting in a horizontal direction (left-right) and not vertical (up-down) forces. The applied force and the force of friction will be included. The force of gravity, which is a vertical force, will not be included.



Step 2: Calculate the acceleration of the box

Remember that we consider the y- and x-directions separately. In this problem we can ignore the y-direction because the box is resting on a table with the gravitational force balanced by the normal force.

We have been given:

Applied force $F_1 = 32$ N

Frictional force $F_f = -7$ N

Mass $m = 10$ kg

To calculate the acceleration of the box we will be using the equation $\vec{F}_R = m\vec{a}$. Therefore:

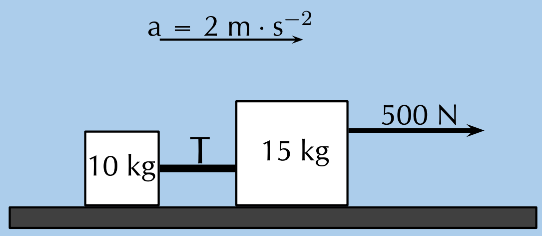
$$\begin{aligned}
\vec{F}_R &= m\vec{a} \\
\vec{F}_1 + \vec{F}_f &= (10)\vec{a} \\
(32 - 7) &= (10)\vec{a} \\
25 &= (10)\vec{a} \\
\vec{a} &= 0{,}25 \text{ m}\cdot\text{s}^{-2} \text{ to the left.}
\end{aligned}$$

Worked example 10: Newton's second law: box on a surface

QUESTION

Two crates, 10 kg and 15 kg respectively, are connected with a thick rope according to the diagram. A force, to the right, of 500 N is applied. The boxes move with an acceleration of 2 m·s^{-2} to the right. One third of the total frictional force is acting on the 10 kg block and two thirds on the 15 kg block. Calculate:

1. the magnitude and direction of the total frictional force present.
2. the magnitude of the tension in the rope at T.



SOLUTION

Step 1: Evaluate what is given

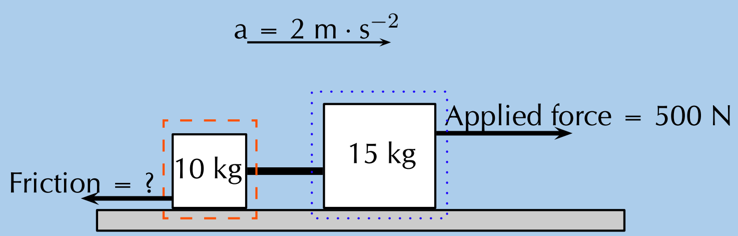
To make things easier lets give the two crates labels, let us call the 10 kg crate number 2 and the 15 kg crate number 1.

We have two crates that have overall has an acceleration that is given. The fact that the crates are tied together with a rope means that they will both have the same acceleration. They will also both feel the same force due to the tension in the rope.

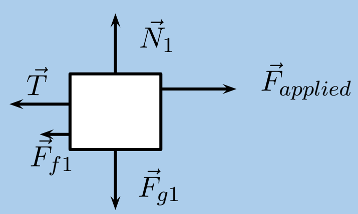
We are told that there is friction but we are only given the relationship between the total frictional force both crates experience and the fraction each one experiences. The total friction, $\vec{F}_{fT}$ will be the sum of the friction on crate 1, $\vec{F}_{f1}$, and the friction on crate 2, $\vec{F}_{f2}$. We are told that $\vec{F}_{f1} = \frac{2}{3}\vec{F}_{fT}$ and $\vec{F}_{f2} = \frac{1}{3}\vec{F}_{fT}$. We know the blocks are accelerating to the right and we know that friction will be in the direction opposite to the direction of motion and parallel to the surface.

Step 2: Draw force diagrams

The diagram for the crates is:



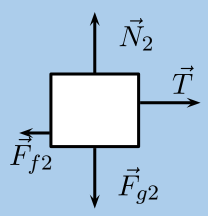
The diagram for crate 1 (indicated by blue dotted lines) will be:



Where:

- $\vec{F}_{g1}$ is the force due to gravity on the first crate
- $\vec{N}_1$ is the normal force from the surface on the first crate
- $\vec{T}$ is the force of tension in the rope
- $\vec{F}_{applied}$ is the external force being applied to the crate
- $\vec{F}_{f1}$ is the force of friction on the first crate

The diagram for crate 2 (indicated by orange dashed lines) will be:



Where:

- $\vec{F}_{g2}$ is the force due to gravity on the second crate
- $\vec{N}_2$ is the normal force from the surface on the second crate
- $\vec{T}$ is the force of tension in the rope
- $\vec{F}_{f2}$ is the force of friction on the second crate

Step 3: Apply Newton's second law of motion

The problem tells us that the crates are accelerating along the x-direction which means that the forces in the y-direction do not result in a net force. We can treat the different directions separately so we only need to consider the x-direction.

We are working with one dimension and can choose a sign convention to indicate the direction of the vectors. We choose vectors to the right (or in the positive x-direction) to be positive.

We can now apply Newton's second law of motion to the first crate because we know the acceleration and we know all the forces acting on the crate. Using positive to indicate a force to the right we know that $F_{res1} = F_{applied} - F_{f1} - T$

$$
\begin{aligned}
\vec{F}_{res1} &= m_1\vec{a} \\
F_{applied} - F_{f1} - T &= m_1 a \\
F_{applied} - \frac{2}{3}F_{fT} - T &= m_1 a \\
(500) - \frac{2}{3}F_{fT} - T &= (15)(2) \\
-T &= (15)(2) - (500) + \frac{2}{3}F_{fT}
\end{aligned}
$$

Now apply Newton's second law of motion to the second crate because we know the acceleration and we know all the forces acting on the crate. We know that $F_{res2} = T - F_{f2}$. **Note that tension is in the opposite direction.**

$$\begin{aligned}\vec{F}_{res2} &= m_2\vec{a}\\ T - F_{f2} &= m_2 a\\ T - \frac{1}{3}F_{fT} &= m_2 a\\ T &= (10)(2) + \frac{1}{3}F_{fT}\end{aligned}$$

Step 4: Solve simultaneously

We have used Newton's second law of motion to create two equations with two unknowns, this means we can solve simultaneously. We solved for T in the equations above but one carries a negative sign so if we add the two equations we will subtract out the value of the tension allowing us to solve for F_{fT}:

$$\begin{aligned}(T) + (-T) &= ((10)(2) + \frac{1}{3}F_{fT}) + ((15)(2) - (500) + \frac{2}{3}F_{fT})\\ 0 &= 20 + 30 - 500 + \frac{1}{3}F_{fT} + \frac{2}{3}F_{fT}\\ 0 &= -450 + F_{fT}\\ F_{fT} &= 450\text{ N}\end{aligned}$$

We can substitute the magnitude of F_{fT} into the equation for crate 2 to determine the magnitude of the tension:

$$\begin{aligned}T &= (10)(2) + \frac{1}{3}F_{fT}\\ T &= (10)(2) + \frac{1}{3}(450)\\ T &= 20 + 150\\ T &= 170\text{ N}\end{aligned}$$

Step 5: Quote final answers

The total force due to friction is 450 N to the left. The magnitude of the force of tension is 170 N.

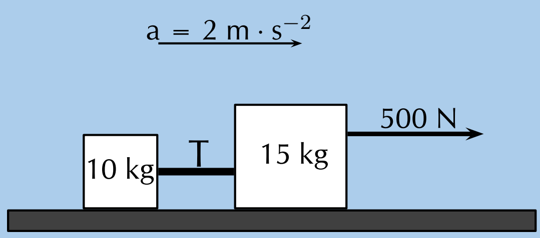
Worked example 11: Newton's second law: box on a surface (Alternative Method)

QUESTION

Two crates, 10 kg and 15 kg respectively, are connected with a thick rope according to the diagram. A force, to the right, of 500 N is applied. The boxes move with an

acceleration of 2 m·s^{-2} to the right. One third of the total frictional force is acting on the 10 kg block and two thirds on the 15 kg block. Calculate:

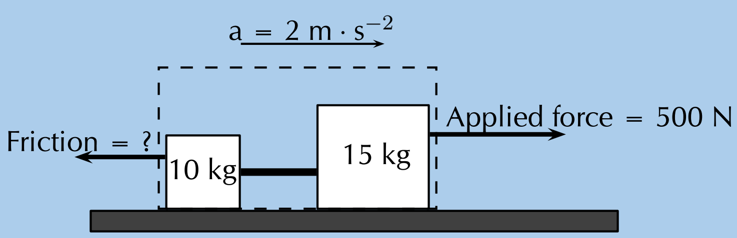
1. the magnitude and direction of the total frictional force present.
2. the magnitude of the tension in the rope at T.



SOLUTION

Step 1: Draw a force diagram

Always draw a force diagram although the question might not ask for it. The acceleration of the whole system is given, therefore a force diagram of the whole system will be drawn. Because the two crates are seen as a unit, the force diagram will look like this:



Step 2: Calculate the frictional force

To find the frictional force we will apply Newton's second law. We are given the mass (10 + 15 kg) and the acceleration (2 m·s^{-2}). Choose the direction of motion to be the positive direction (to the right is positive).

$$\begin{aligned} F_R &= ma \\ F_{\text{applied}} + F_f &= ma \\ 500 + F_f &= (10 + 15)\,(2) \\ F_f &= 50 - 500 \\ F_f &= -450N \end{aligned}$$

The frictional force is 450 N opposite to the direction of motion (to the left).

Step 3: Find the tension in the rope

To find the tension in the rope we need to look at one of the two crates on their own. Let's choose the 10 kg crate. Firstly, we need to draw a force diagram:

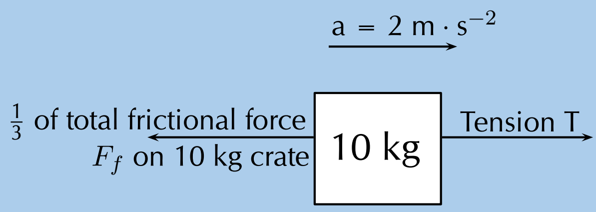


Figure 2.5: Force diagram of 10 kg crate.

The frictional force on the 10 kg block is one third of the total, therefore:

$F_f = \frac{1}{3} \times 450$

$F_f = 150\text{N}$

If we apply Newton's second law:

$$\begin{aligned} F_R &= ma \\ T + F_f &= (10)\,(2) \\ T + (-150) &= 20 \\ T &= 170 \text{ N} \end{aligned}$$

Note: If we had used the same principle and applied it to 15 kg crate, our calculations would have been the following:

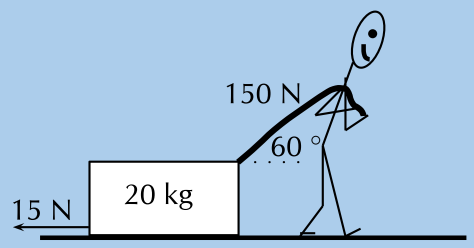
$$\begin{aligned} F_R &= ma \\ F_{\text{applied}} + T + F_f &= (15)\,(2) \\ 500 + T + (-300) &= 30 \\ T &= -170 \text{ N} \end{aligned}$$

The negative answer here means that the force is in the direction opposite to the motion, in other words to the left, which is correct. However, the question asks for the magnitude of the force and your answer will be quoted as 170 N.

Worked example 12: Newton's second law: man pulling a box

QUESTION

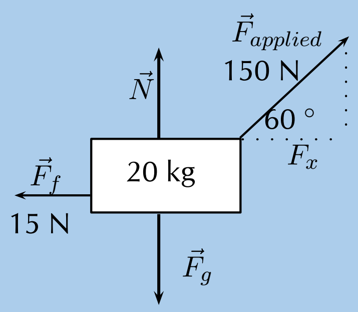
A man is pulling a 20 kg box with a rope that makes an angle of 60° with the horizontal. If he applies a force of magnitude 150 N and a frictional force of magnitude 15 N is present, calculate the acceleration of the box.

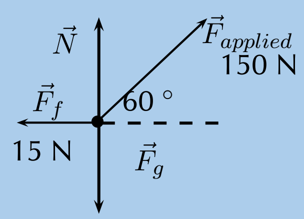


SOLUTION

Step 1: Draw a force diagram or free body diagram

The motion is horizontal and therefore we will only consider the forces in a horizontal direction. Remember that vertical forces do not influence horizontal motion and vice versa.





Step 2: Calculate the horizontal component of the applied force

We first need to choose a direction to be the positive direction in this problem. We choose the positive x-direction (to the right) to be positive.

The applied force is acting at an angle of 60° to the horizontal. We can only consider forces that are parallel to the motion. The horizontal component of the applied force needs to be calculated before we can continue:

$$\begin{aligned} F_x &= F_{applied}\cos(\theta) \\ &= 150\cos(60°) \\ &= 75 \text{ N} \end{aligned}$$

Step 3: Calculate the acceleration

To find the acceleration we apply Newton's second law:

$$\begin{aligned} F_R &= ma \\ F_x + F_f &= (20)a \\ (75) + (-15) &= (20)a \\ a &= \frac{60}{20} \\ a &= 3 \text{ m·s}^{-2} \end{aligned}$$

The acceleration is 3 m·s^{-2} to the right.

Worked example 13: Newton's second law: truck and trailer

QUESTION

A 2000 kg truck pulls a 500 kg trailer with a constant acceleration. The engine of the truck produces a thrust of 10 000 N. Ignore the effect of friction. Calculate the:

1. acceleration of the truck; and
2. tension in the tow bar T between the truck and the trailer, if the tow bar makes an angle of 25° with the horizontal.

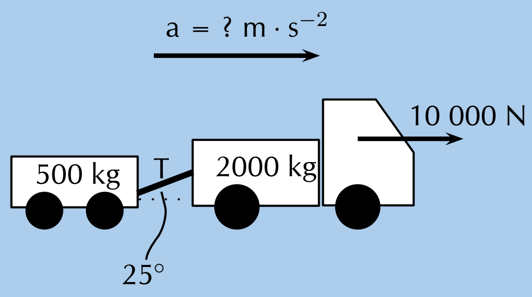


Figure 2.6: Truck pulling a trailer.

SOLUTION

Step 1: Draw a force diagram

Draw a force diagram indicating all the forces on the system as a whole:

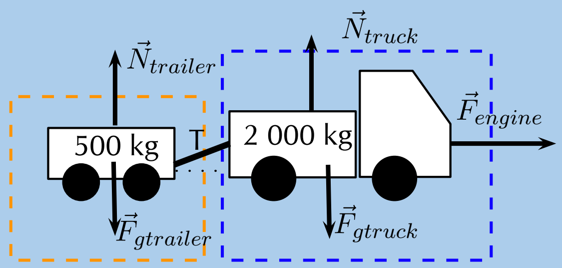
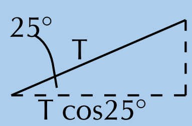


Figure 2.7: Free body diagrams for truck pulling a trailer.

Step 2: Apply Newton's second law of motion

We choose the positive x-direction to be the positive direction. We only need to consider the horizontal forces. Using only the horizontal forces means that we first need to note that the tension acts at an angle to the horizontal and we need to use the horizontal component of the tension in our calculations.



The horizontal component has a magnitude of $T\cos(25°)$.

In the absence of friction, the only force that causes the system to accelerate is the thrust of the engine. If we now apply Newton's second law of motion to the truck we have:

$$\begin{aligned}
\vec{F}_{Rtruck} &= m_{truck}\vec{a} \text{ (we use signs to indicate direction)}\\
F_{engine} - T\cos(25°) &= (2000)a\\
(10\ 000) - T\cos(25°) &= (2000)a\\
a &= \frac{(10\ 000) - T\cos(25°)}{(2000)}
\end{aligned}$$

We now apply the same principle to the trailer (remember that the direction of the tension will be opposite to the case of the truck):

$$\begin{aligned}
\vec{F}_{Rtrailer} &= m_{trailer}\vec{a} \text{ (we use signs to indicate direction)}\\
T\cos(25°) &= (500)a\\
a &= \frac{T\cos(25°)}{(500)}
\end{aligned}$$

We now have two equations and two unknowns so we can solve simultaneously. We subtract the second equation from the first to get:

$$\begin{aligned}
(a) - (a) &= (\frac{(10\ 000) - T\cos(25°)}{(2000)}) - (\frac{T\cos(25°)}{(500)})\\
0 &= (\frac{(10\ 000) - T\cos(25°)}{(2000)}) - (\frac{T\cos(25°)}{(500)})\\
&\text{(multiply through by 2000)}\\
0 &= (10\ 000) - T\cos(25°) - 4T\cos(25°)\\
5T\cos(25°) &= (10\ 000)\\
T &= \frac{(10\ 000)}{5\cos(25°)}\\
T &= 2206{,}76 \text{ N}
\end{aligned}$$

Now substitute this result back into the second equation to solve for the magnitude of a

$$\begin{aligned}
a &= \frac{T\cos(25°)}{(500)}\\
&= \frac{(2206{,}76)\cos(25°)}{(500)}\\
&= 4{,}00 \text{ m·s}^{-2}
\end{aligned}$$

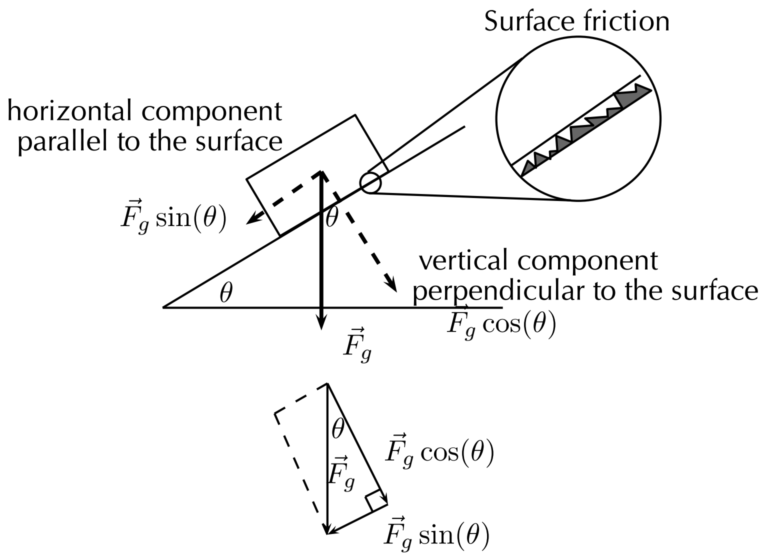
TIP
Do not use the abbreviation W for weight as it is used to abbreviate 'work'. Rather use the force of gravity F_g for weight.

Object on an inclined plane

In an earlier section we looked at the components of the gravitational force parallel and perpendicular to the slope for objects on an inclined plane. When we look at problems on an inclined plane we need to include the component of the gravitational force parallel to the slope.

Think back to the pictures of the book on a table, as one side of the table is lifted higher the book starts to slide. Why? The book starts to slide because the component of the gravitational force parallel to the surface of the table gets larger for the larger angle of inclination. This is like the applied force and it eventually becomes larger than the frictional force and the book accelerates down the table or inclined plane.

The force of gravity will also tend to push an object 'into' the slope. This is the component of the force perpendicular to the slope. There is no movement in this direction as this force is balanced by the slope pushing up against the object. This "pushing force" is the normal force (N) which we have already learnt about and is equal in magnitude to the perpendicular component of the gravitational force, but opposite in direction.

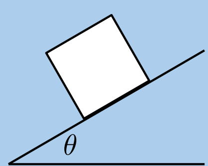


See simulation: 23HD at www.everythingscience.co.za

Worked example 14: Newton's second law: box on inclined plane

QUESTION

A body of mass M is at rest on an inclined plane due to friction.



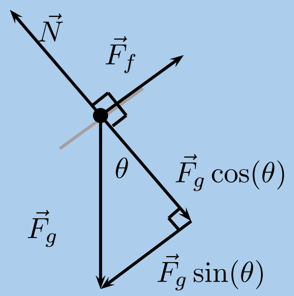
What of the following option is the magnitude of the frictional force acting on the body?

1. F_g
2. $F_g \cos(\theta)$
3. $F_g \sin(\theta)$
4. $F_g \tan(\theta)$

SOLUTION

Step 1: Analyse the situation

The question asks us to determine the magnitude of the frictional force. The body is said to be at rest on the plane, which means that it is not moving and therefore the acceleration is zero. We know that the frictional force will act parallel to the slope. If there were no friction the box would slide down the slope so friction must be acting up the slope. We also know that there will be a component of gravity perpendicular to the slope and parallel to the slope. The free body diagram for the forces acting on the block is:



Step 2: Determine the magnitude of the frictional force

We can apply Newton's second law to this problem. We know that the object is not moving so the resultant acceleration is zero. We choose up the slope to be the positive direction. Therefore:

$$\begin{aligned} \vec{F}_R &= m\vec{a} \text{ using signs for direction} \\ F_f - F_g \sin(\theta) &= m(0) \\ F_f - F_g \sin(\theta) &= m(0) \\ F_f &= F_g \sin(\theta) \end{aligned}$$

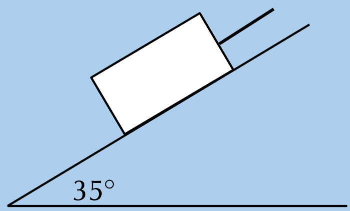
Step 3: Quote your final answer

The force of friction has the same magnitude as the component of the force of gravitation parallel to the slope, $F_g \sin(\theta)$.

Worked example 15: Newton's second law: object on an incline

QUESTION

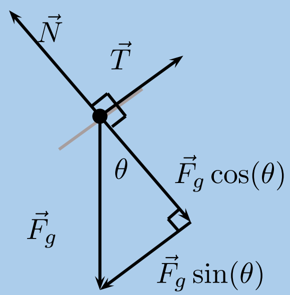
A force of magnitude $T = 312$N up an incline is required to keep a body at rest on a frictionless inclined plane which makes an angle of 35° with the horizontal. Calculate the magnitudes of the force due to gravity and the normal force, giving your answers to three significant figures.



SOLUTION

Step 1: Find the magnitude of $\vec{F}_g$

We are usually asked to find the magnitude of $\vec{T}$, but in this case $\vec{T}$ is given and we are asked to find $\vec{F}_g$. We can use the same equation. T is the force that balances the component of $\vec{F}_g$ parallel to the plane (F_{gx}) and therefore it has the same magnitude.



We can apply Newton's second law to this problem. We know that the object is not moving so the resultant acceleration is zero. We choose up the slope to be the positive direction. Therefore:

$$\begin{aligned}\vec{F}_R &= m\vec{a} \text{ using signs for direction}\\ T - F_g \sin(\theta) &= m(0)\\ F_g &= \frac{T}{\sin(\theta)}\\ &= \frac{312}{\sin(35°)}\\ &= 543{,}955 \text{ N}\end{aligned}$$

Step 2: Find the magnitude of $\vec{N}$

We treat the forces parallel and perpendicular to the slope separately. The block is stationary so the acceleration perpendicular to the slope is zero. Once again we can

apply Newton's second law of motion. We choose the direction of the normal force as the positive direction.

$$\begin{aligned}\vec{F}_R &= m\vec{a} \text{ using signs for direction}\\ N - F_g\cos(\theta) &= m(0)\\ N &= F_g\cos(\theta)\end{aligned}$$

We could substitute in the value of F_g calculated earlier. We would like to illustrate that there is another approach to adopt to ensure you get the correct answer even if you made a mistake calculating F_g. $F_g\cos(\theta)$ can also be determined with the use of trigonometric ratios. We know from the previous part of the question that $T = F_g\sin(\theta)$. We also know that

$$\begin{aligned}\tan(\theta) &= \frac{F_g\sin(\theta)}{F_g\cos(\theta)}\\ &= \frac{T}{N}\\ N &= \frac{T}{\tan(\theta)}\\ &= \frac{312}{\tan(35°)}\\ &= 445{,}58 \text{ N}\end{aligned}$$

Note that the question asks that the answers be given to 3 significant figures. We therefore round $\vec{N}$ from 445,58 N up to 446 N perpendicular to the surface upwards and $\vec{T}$ from 543,955 N up to 544 N parallel to the plane, up the slope.

Lifts and rockets

So far we have looked at objects being pulled or pushed across a surface, in other words motion parallel to the surface the object rests on. Here we only considered forces parallel to the surface, but we can also lift objects up or let them fall. This is vertical motion where only vertical forces are being considered.

Let us consider a 500 kg lift, with no passengers, hanging on a cable. The purpose of the cable is to pull the lift upwards so that it can reach the next floor or lower the lift so that it can move downwards to the floor below. We will look at five possible stages during the motion of the lift and apply our knowledge of Newton's second law of motion to the situation. The 5 stages are:

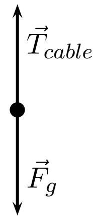
1. A stationary lift suspended above the ground.
2. A lift accelerating upwards.
3. A lift moving at a constant velocity.
4. A lift decelerating (slowing down).
5. A lift accelerating downwards (the cable snaps!).

We choose the upwards direction to be the positive direction for this discussion.

Stage 1:

The 500 kg lift is stationary at the second floor of a tall building.

The lift is not accelerating. There must be a tension $\vec{T}$ from the cable acting on the lift and there must be a force due to gravity, $\vec{F}_g$. There are no other forces present and we can draw the free body diagram:



We apply Newton's second law to the vertical direction:

$$\begin{aligned}\vec{F}_R &= m_{\text{lift}}\vec{a} \text{ (we use signs to indicate direction)}\\ T - F_g &= m_{\text{lift}}(0)\\ T &= F_g\end{aligned}$$

The forces are equal in magnitude and opposite in direction.

Stage 2:

The lift moves upwards at an acceleration of 1 m·s^{-2}.

If the lift is accelerating, it means that there is a resultant force in the direction of the motion. This means that the force acting upwards is now greater than the force due to gravity $\vec{F}_g$ (down). To find the magnitude of the $\vec{T}$ applied by the cable we can do the following calculation: (Remember we have chosen upwards as positive.)

We apply Newton's second law to the vertical direction:

$$\begin{aligned}\vec{F}_R &= m_{\text{lift}}\vec{a} \text{ (we use signs to indicate direction)}\\ T - F_g &= m_{\text{lift}}(1)\\ T &= F_g + m_{\text{lift}}(1)\end{aligned}$$

The answer makes sense as we need a bigger force upwards to cancel the effect of gravity as well as have a positive resultant force.

Stage 3:

The lift moves at a constant velocity.

When the lift moves at a constant velocity, the acceleration is zero,

$$\begin{aligned}\vec{F}_R &= m_{\text{lift}}\vec{a} \text{ (we use signs to indicate direction)}\\ T - F_g &= m_{\text{lift}}(0)\\ T &= F_g\end{aligned}$$

The forces are equal in magnitude and opposite in direction. It is common **mistake** to think that because the lift is moving there is a net force acting on it. It is only if it is **accelerating** that there is a net force acting.

Stage 4:

The lift slows down at a rate of 2 m·s^{-2}. The lift was moving upwards so this means that it is decelerating or accelerating in the direction opposite to the direction of motion. This means that the acceleration is in the negative direction.

$$\begin{aligned}\vec{F}_R &= m_{\text{lift}}\vec{a} \text{ (we use signs to indicate direction)}\\ T - F_g &= m_{\text{lift}}(-2)\\ T &= F_g - 2m_{\text{lift}}\end{aligned}$$

As the lift is now slowing down there is a resultant force downwards. This means that the force acting downwards is greater than the force acting upwards.

This makes sense as we need a smaller force upwards to ensure that the resultant force is downward. The force of gravity is now greater than the upward pull of the cable and the lift will slow down.

Stage 5:

The cable snaps.

When the cable snaps, the force that used to be acting upwards is no longer present. The only force that is present would be the force of gravity. The lift will fall freely and its acceleration.

Apparent weight

Your weight is the magnitude of the gravitational force acting on your body. When you stand in a lift that is stationery and then starts to accelerate upwards you feel you are pressed into the floor while the lift accelerates. You feel like you are heavier and your weight is more. When you are in a stationery lift that starts to accelerate downwards you feel lighter on your feet. You feel like your weight is less.

Weight is measured through normal forces. When the lift accelerates upwards you feel a greater normal force acting on you as the force required to accelerate you upwards in addition to balancing out the gravitational force.

When the lift accelerates downwards you feel a smaller normal force acting on you. This is because a net force downwards is required to accelerate you downwards. This phenomenon is called **apparent weight** because your weight didn't actually change.

Rockets

As with lifts, rockets are also examples of objects in vertical motion. The force of gravity pulls the rocket down while the thrust of the engine pushes the rocket upwards. The force that the engine exerts must overcome the force of gravity so that the rocket can accelerate upwards. The worked example below looks at the application of Newton's second law in launching a rocket.

Worked example 16: Newton's second law: rocket

QUESTION

A rocket (of mass 5000 kg) is launched vertically upwards into the sky at an acceleration of 20 m·s^{-2}. If the magnitude of the force due to gravity on the rocket is 49 000 N, calculate the magnitude and direction of the thrust of the rocket's engines.

SOLUTION

Step 1: Analyse what is given and what is asked

We have the following:

$m = 5000\text{kg}$

$\vec{a}$ = 20 m·s^{-2} upwards.

$\vec{F}_g$ = 49 000 N downwards.

We are asked to find the thrust of the rocket engine $\vec{F}$.

Step 2: Find the thrust of the engine

We will apply Newton's second law:

$$\begin{aligned} \vec{F}_R &= m\vec{a} \text{ (using signs to indicate direction)} \\ F - F_g &= (5000)(20) \\ F - (49\,000) &= (5000)(20) \\ F &= 149\,000 \text{ N} \end{aligned}$$

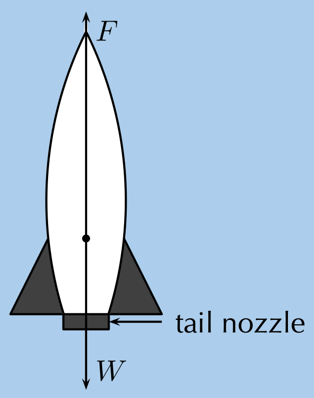
Step 3: Quote your final answer

The force due to the thrust is 149 000 N upwards.

Worked example 17: Rockets

QUESTION

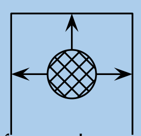
How do rockets accelerate in space?



SOLUTION

Step 1:

- Gas explodes inside the rocket.
- This exploding gas exerts a force on each side of the rocket (as shown in the picture below of the explosion chamber inside the rocket).



Note that the forces shown in this picture are representative. With an explosion there will be forces in all directions.

- Due to the symmetry of the situation, all the forces exerted on the rocket are balanced by forces on the opposite side, except for the force opposite the open side. This force on the upper surface is unbalanced.
- This is therefore the resultant force acting on the rocket and it makes the rocket accelerate forwards.

Exercise 2 – 5:

1. A tug is capable of pulling a ship with a force of 100 kN. If two such tugs are pulling on one ship, they can produce any force ranging from a minimum of 0 kN to a maximum of 200 kN. Give a detailed explanation of how this is possible. Use diagrams to support your result.
2. A car of mass 850 kg accelerates at 2 m·s^{-2}. Calculate the magnitude of the resultant force that is causing the acceleration.
3. Find the force needed to accelerate a 3 kg object at 4 m·s^{-2}.
4. Calculate the acceleration of an object of mass 1000 kg accelerated by a force of magnitude 100 N.

5. An object of mass 7 kg is accelerating at 2,5 m·s^{-2}. What resultant force acts on it?

6. Find the mass of an object if a force of 40 N gives it an acceleration of 2 m·s^{-2}.

7. Find the acceleration of a body of mass 1000 kg that has a force with a magnitude of 150 N acting on it.

8. Find the mass of an object which is accelerated at 3 m·s^{-2} by a force of magnitude 25 N.

9. Determine the acceleration of a mass of 24 kg when a force of magnitude 6 N acts on it. What is the acceleration if the force were doubled and the mass was halved?

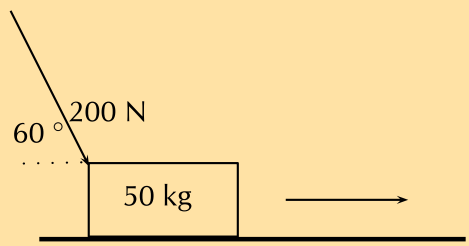
10. A mass of 8 kg is accelerating at 5 m·s^{-2}.

    a) Determine the resultant force that is causing the acceleration.
    b) What acceleration would be produced if we doubled the force and reduced the mass by half?

11. A motorcycle of mass 100 kg is accelerated by a resultant force of 500 N. If the motorcycle starts from rest:

    a) What is its acceleration?
    b) How fast will it be travelling after 20 s?
    c) How long will it take to reach a speed of 35 m·s^{-1}?
    d) How far will it travel from its starting point in 15 s?

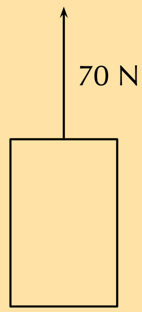
12. A force of 200 N, acting at 60° to the horizontal, accelerates a block of mass 50 kg along a horizontal plane as shown.



    a) Calculate the component of the 200 N force that accelerates the block horizontally.
    b) If the acceleration of the block is 1,5 m·s^{-2}, calculate the magnitude of the frictional force on the block.
    c) Calculate the vertical force exerted by the block on the plane.

13. A toy rocket experiences a force due to gravity of magnitude 4,5 N is supported vertically by placing it in a bottle. The rocket is then ignited. Calculate the force that is required to accelerate the rocket vertically upwards at 8 m·s^{-2}.

14. A constant force of magnitude 70 N is applied vertically to a block as shown. The block experiences a force due to gravity of 49 N. Calculate the acceleration of the block.



15. A student experiences a gravitational force of magnitude 686 N investigates the motion of a lift. While he stands in the lift on a bathroom scale (calibrated in newton), he notes three stages of his journey.

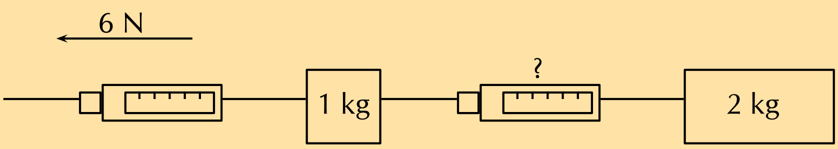
    a) For 2 s immediately after the lift starts, the scale reads 574 N.
    b) For a further 6 s it reads 686 N.
    c) For the final 2 s it reads 854 N.

    Answer the following questions:

    a) Is the motion of the lift upward or downward? Give a reason for your answer.
    b) Write down the magnitude and the direction of the resultant force acting on the student for each of the stages 1, 2 and 3.

16. A car of mass 800 kg accelerates along a level road at 4 $m \cdot s^{-2}$. A frictional force of 700 N opposes its motion. What force is produced by the car's engine?

17. Two objects, with masses of 1 kg and 2 kg respectively, are placed on a smooth surface and connected with a piece of string. A horizontal force of 6 N is applied with the help of a spring balance to the 1 kg object. Ignoring friction, what will the force acting on the 2 kg mass, as measured by a second spring balance, be?



18. A rocket of mass 200 kg has a resultant force of 4000 N upwards on it.

    a) What is its acceleration on the Earth, where it experiences a gravitational force of 1960 N?
    b) What driving force does the rocket engine need to exert on the back of the rocket on the Earth?

19. A car going at 20 $m \cdot s^{-1}$ accelerates uniformly and comes to a stop in a distance of 20 m.

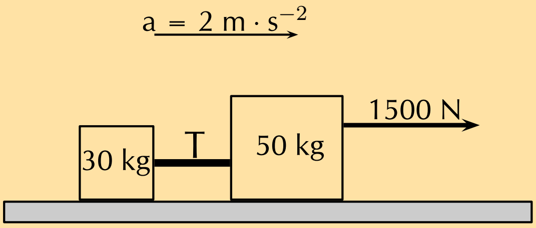
    a) What is its acceleration?
    b) If the car is 1000 kg how much force do the brakes exert?

20. A block on an inclined plane experiences a force due to gravity, $\vec{F}_g$ of 300 N straight down. If the slope is inclined at 67,8° to the horizontal, what is the component of the force due to gravity perpendicular and parallel to the slope? At what angle would the perpendicular and parallel components of the force due to gravity be equal?

21. A block on an inclined plane is subjected to a force due to gravity, $\vec{F}_g$ of 287 N straight down. If the component of the gravitational force parallel to the slope is $\vec{F}_{gx} = 123{,}7$ N in the negative x-direction (down the slope), what is the incline of the slope?

22. A block on an inclined plane experiences a force due to gravity, $\vec{F}_g$ of 98 N straight down. If the slope is inclined at an unknown angle to the horizontal but we are told that the ratio of the components of the force due to gravity perpendicular and parallel to the slope is 7:4. What is the angle the incline makes with the horizontal?

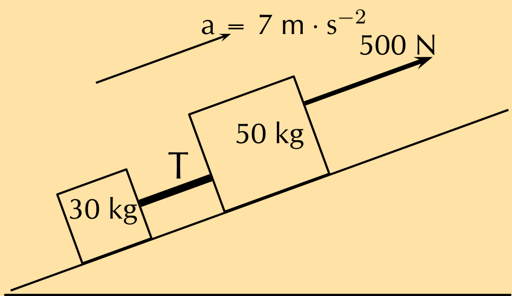
23. Two crates, 30 kg and 50 kg respectively, are connected with a thick rope according to the diagram. A force, to the right, of 1500 N is applied. The boxes move with an acceleration of 7 m·s^{-2} to the right. The ratio of the frictional forces on the two crates is the same as the ratio of their masses. Calculate:



a) the magnitude and direction of the total frictional force present.

b) the magnitude of the tension in the rope at T.

24. Two crates, 30 kg and 50 kg respectively, are connected with a thick rope according to the diagram. If they are dragged up an incline such that the ratio of the parallel and perpendicular components of the gravitational force on each block are 3 : 5. The boxes move with an acceleration of 7 m·s^{-2} up the slope. The ratio of the frictional forces on the two crates is the same as the ratio of their masses. The magnitude of the force due to gravity on the 30 kg crate is 294 N and on the 50 kg crate is 490 N. Calculate:



a) the magnitude and direction of the total frictional force present.

b) the magnitude of the tension in the rope at T.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23HF	2. 23HG	3. 23HH	4. 23HJ	5. 23HK	6. 23HM
7. 23HN	8. 23HP	9. 23HQ	10. 23HR	11. 23HS	12. 23HT
13. 23HV	14. 23HW	15. 23HX	16. 23HY	17. 23HZ	18. 23J2
19. 23J3	20. 23J4	21. 23J5	22. 23J6	23. 23J7	24. 23J8

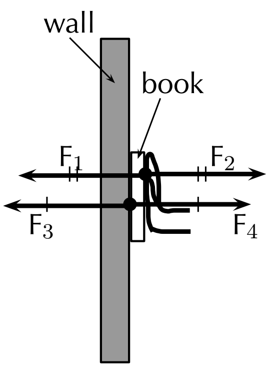
www.everythingscience.co.za m.everythingscience.co.za

Newton's third law of motion ESBKV

Figure 2.8: Newton's action-reaction pairs.

Newton's third law of Motion deals with the interaction between pairs of objects. For example, if you hold a book up against a wall you are exerting a force on the book (to keep it there) and the book is exerting a force back at you (to keep you from falling through the book). This may sound strange, but if the book was not pushing back at you, your hand would push through the book! These two forces (the force of the hand on the book (F_1) and the force of the book on the hand (F_2)) are called an action-reaction pair of forces. They have the same magnitude, but act in opposite directions and act on different objects (the one force is onto the book and the other is onto your hand).

There is another action-reaction pair of forces present in this situation. The book is pushing against the wall (action force) and the wall is pushing back at the book (reaction). The force of the book on the wall (F_3) and the force of the wall on the book (F_4) are shown in the diagram.



F$_1$: force of hand on book
F$_2$: force of book on hand
F$_3$: force of book on wall
F$_4$: force of wall on book

DEFINITION: *Newton's third law of motion*

If body A exerts a force on body B, then body B exerts a force of equal magnitude on body A, but in the opposite direction.

See video: 23J9 at www.everythingscience.co.za

These action-reaction pairs have several properties:

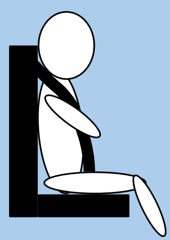
- the same type of force acts on the objects,
- the forces have the same magnitude but opposite direction, and
- the forces act on different objects.

Newton's action-reaction pairs can be found everywhere in life where two objects interact with one another. The following worked examples will illustrate this:

Worked example 18: Newton's third law - seat belt

QUESTION

Dineo is seated in the passenger seat of a car with the seat belt on. The car suddenly stops and he moves forwards (Newton's first law - he continues in his state of motion) until the seat belt stops him. Draw a labelled force diagram identifying two action-reaction pairs in this situation.



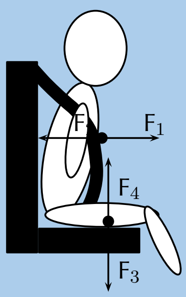
SOLUTION

Step 1: Draw a force diagram

Start by drawing the picture. You will be using arrows to indicate the forces so make your picture large enough so that detailed labels can also be added. The picture needs to be accurate, but not artistic! Use stick-men if you have to.

Step 2: Label the diagram

Take one pair at a time and label them carefully. If there is not enough space on the drawing, then use a key on the side.



F_1: The force of Dineo on the seat belt
F_2: The force of the seat belt on Dineo
F_3: The force of Dineo on the seat (downwards)
F_4: The force of the seat on Dineo (upwards)

Worked example 19: Newton's third law: forces in a lift

QUESTION

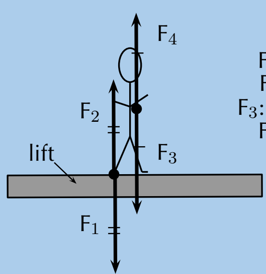
Tammy travels from the ground floor to the fifth floor of a hotel in a lift moving at constant velocity. Which ONE of the following statements is TRUE about the magnitude of the force exerted by the floor of the lift on Tammy's feet? Use Newton's third law to justify your answer.

1. It is greater than the magnitude of Tammy's weight.
2. It is equal in magnitude to the force Tammy's feet exert on the floor of the lift.
3. It is equal to what it would be in a stationary lift.
4. It is greater than what it would be in a stationary lift.

SOLUTION

Step 1: Analyse the situation

This is a Newton's third law question and not Newton's second law. We need to focus on the action-reaction pairs of forces and not the motion of the lift. The following diagram will show the action-reaction pairs that are present when a person is standing on a scale in a lift.



F_1: force of feet on lift (downwards)
F_2: force of lift on feet (upwards)
F_3: force of gravity on person (downwards)
F_4: force of person on lift (upwards)

Figure 2.9: Newton's action-reaction pairs in a lift.

In this question statements are made about the force of the floor (lift) on Tammy's feet. This force corresponds to F_2 in our diagram. The reaction force that pairs up with

this one is F_1, which is the force that Tammy's feet exerts on the floor of the lift. The magnitude of these two forces are the same, but they act in opposite directions.

Step 2: Choose the correct answer

It is important to analyse the question first, before looking at the answers. The answers might confuse you if you look at them first. Make sure that you understand the situation and know what is asked before you look at the options.

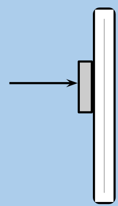
The correct answer is number 2.

Worked example 20: Newton's third law: book and wall

QUESTION

Bridget presses a book against a vertical wall as shown in the photograph.

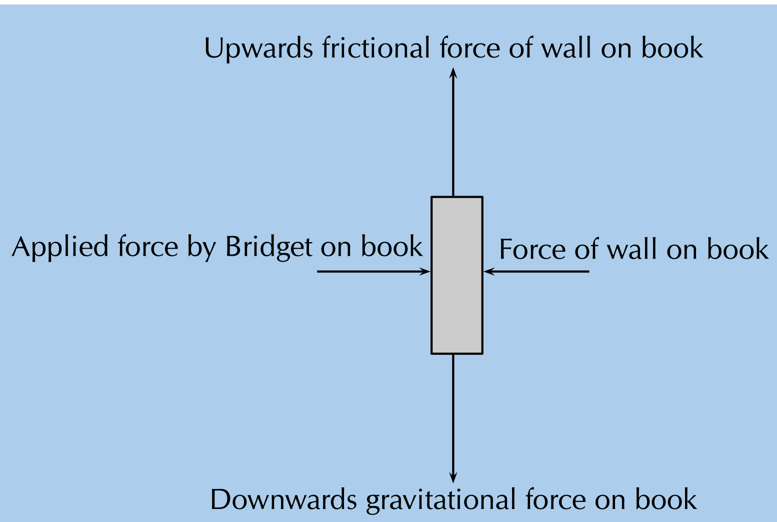
1. Draw a labelled force diagram indicating all the forces acting on the book.
2. State, in words, Newton's third law of Motion.
3. Name the action-reaction pairs of forces acting in the horizontal plane.

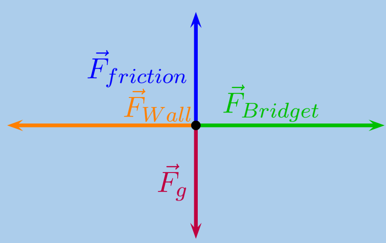


SOLUTION

Step 1: Draw a force diagram

A force diagram will look like this:





Note that we had to draw all the forces acting on the book and not the action-reaction pairs. None of the forces drawn are action-reaction pairs, because they all act on the same object (the book). When you label forces, be as specific as possible, including the direction of the force and both objects involved, for example, do not say gravity (which is an incomplete answer) but rather say 'Downward (direction) gravitational force of the Earth (object) on the book (object)'.

Step 2: State Newton's third law

If body A exerts a force onto body B, then body B will exert a force equal in magnitude, but opposite in direction, onto body A.

Step 3: Name the action-reaction pairs

The question only asks for action-reaction forces in the horizontal plane. Therefore:

Pair 1: Action: Applied force of Bridget on the book; Reaction: The force of the book on the girl.

Pair 2: Action: Force of the book on the wall; Reaction: Force of the wall on the book.

Note that a Newton's third law pair will always involve the same combination of words, like 'book on wall' and 'wall on book'. The objects are 'swapped around' in naming the pairs.

General experiment: Balloon rocket

Aim:

In this experiment for the entire class, you will use a balloon rocket to investigate Newton's third law. A fishing line will be used as a track and a plastic straw taped to the balloon will help attach the balloon to the track.

Apparatus:

You will need the following items for this experiment:

1. balloons (one for each team)
2. plastic straws (one for each team)
3. tape (cellophane or masking)
4. fishing line, 10 metres in length
5. a stopwatch - optional (a cell phone can also be used)
6. a measuring tape - optional

Method:

1. Divide into groups of at least five.
2. Attach one end of the fishing line to the blackboard with tape. Have one teammate hold the other end of the fishing line so that it is taut and roughly horizontal. The line must be held steady and **must not** be moved up or down during the experiment.
3. Have one teammate blow up a balloon and hold it shut with his or her fingers. Have another teammate tape the straw along the side of the balloon. Thread the fishing line through the straw and hold the balloon at the far end of the line.
4. Let go of the rocket and observe how the rocket moves forward.
5. Optionally, the rockets of each group can be timed to determine a winner of the fastest rocket.
   a) Assign one teammate to time the event. The balloon should be let go when the time keeper yells "Go!" Observe how your rocket moves toward the blackboard.
   b) Have another teammate stand right next to the blackboard and yell "Stop!" when the rocket hits its target. If the balloon does not make it all the way to the blackboard, "Stop!" should be called when the balloon stops moving. The timekeeper should record the flight time.
   c) Measure the exact distance the rocket travelled. Calculate the average speed at which the balloon travelled. To do this, divide the distance travelled by the time the balloon was "in flight." Fill in your results for Trial 1 in the Table below.
   d) Each team should conduct two more races and complete the sections in the Table for Trials 2 and 3. Then calculate the average speed for the three trials to determine your team's race entry time.

Results:

	Distance (m)	Time (s)	Speed ($m \cdot s^{-1}$)
Trial 1			
Trial 2			
Trial 3			
		Average:	

Conclusions:

The winner of this race is the team with the fastest average balloon speed.

See video: 23JB at www.everythingscience.co.za

While doing the experiment, you should think about,

1. What made your rocket move?
2. How is Newton's third law of Motion demonstrated by this activity?
3. Draw pictures using labelled arrows to show the forces acting on the inside of the balloon before it was released and after it was released.

See video: 23JC at www.everythingscience.co.za

See video: 23JD at www.everythingscience.co.za

The Saturn V (pronounced "Saturn Five") was an American human-rated expendable rocket used by NASA's Apollo and Skylab programs from 1967 until 1973. A multistage liquid-fuelled launch vehicle, NASA launched 13 Saturn Vs from the Kennedy Space Center, Florida with no loss of crew or payload. It remains the tallest, heaviest, and most powerful rocket ever brought to operational status and still holds the record for the heaviest launch vehicle payload.

Exercise 2 – 6:

1. A fly hits the front windscreen of a moving car. Compared to the magnitude of the force the fly exerts on the windscreen, the magnitude of the force the windscreen exerts on the fly during the collision, is:

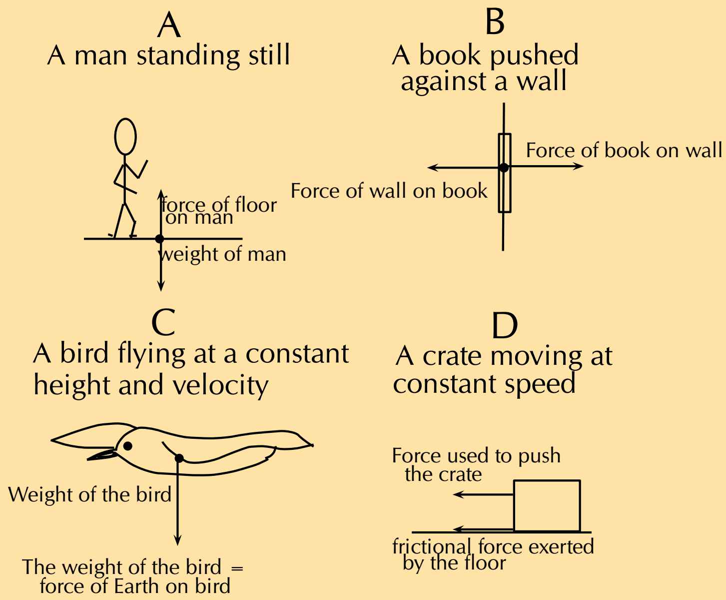
a) zero.

b) smaller, but not zero.

c) bigger.

d) the same.

2. Which of the following pairs of forces correctly illustrates Newton's third law?



Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23JF 2. 23JG

www.everythingscience.co.za m.everythingscience.co.za

Forces in equilibrium ESBKW

At the beginning of this chapter it was mentioned that resultant forces cause objects to accelerate in a straight line. If an object is stationary or moving at constant velocity then either,

- no forces are acting on the object, or
- the forces acting on that object are exactly balanced.

In other words, for stationary objects or objects moving with constant velocity, the resultant force acting on the object is zero.

DEFINITION: *Equilibrium*

An object in equilibrium has both the sum of the forces acting on it equal to zero.

2.4 Forces between masses ESBKX

Gravity is arguably the first force that people really learn about. People don't really think of it as learning about gravity because it is such a big part of our everyday lives. Babies learning to crawl or walk are struggling against gravity, games involving jumping, climbing or balls all give people a sense of the effect of gravity. The saying "everything that goes up, must come down" is all about gravity. Rain falls from the sky because of gravity and so much more. We all know that these things happen but we don't often stop to ask what is gravity, what causes it and how can we describe it more accurately than "everything that goes up, must come down"?

All of the examples mentioned involve objects with mass falling to the ground, they are being attracted to the Earth. Gravity is the name we give to the force that arises between objects because of their mass. It is always an attractive force. It is a non-contact force, therefore it acts at a distance. The Earth is responsible for a gravitational force on the moon which keeps it in its orbit around the Earth and the gravitational force the moon exerts on the Earth is the dominant cause of ocean tides.

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. The gravitational force is relatively simple. It is **always attractive**, and it **depends only on the masses** involved and the **distance between them**. Stated in modern language, Newton's universal law of gravitation states that every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Newton's law of universal gravitation ESBKY

DEFINITION: *Newton's law of universal gravitation*

Every point mass attracts every other point mass by a force directed along the line connecting the two. This force is proportional to the product of the masses and inversely proportional to the square of the distance between them.

The magnitude of the attractive gravitational force between the two point masses, F is given by:

$$F = G\frac{m_1 m_2}{d^2}$$

where: F is in newtons (N), G is the gravitational constant 6,67 $\times$ 10^{-11} N·m^2·kg^{-2}, m_1 is the mass of the first point mass in kilograms (kg), m_2 is the mass of the second point mass in kilograms (kg) and d is the distance between the two point masses in metres (m). For any large objects (not point masses) we use the distance from the **centre of the object(s)** to do the calculation. This is very important when dealing with very large objects like planets. The distance from the centre of the planet and from the surface of the planet differ by a large amount. **Remember** that this is a force of attraction and should be described by a vector. We use Newton's law of universal gravitation to determine the magnitude of the force and then analyse the problem to determine the direction.

For example, consider a man of mass 80 kg standing 10 m from a woman with a mass

of 65 kg. The attractive gravitational force between them would be:

$$\begin{aligned} F &= G\frac{m_1 m_2}{d^2} \\ &= \left(6{,}67 \times 10^{-11}\right)\left(\frac{(80)(65)}{(10)^2}\right) \\ &= 3{,}47 \times 10^{-9}\ \text{N} \end{aligned}$$

If the man and woman move to 1 m apart, then the force is:

$$\begin{aligned} F &= G\frac{m_1 m_2}{d^2} \\ &= \left(6{,}67 \times 10^{-11}\right)\left(\frac{(80)(65)}{(1)^2}\right) \\ &= 3{,}47 \times 10^{-7}\ \text{N} \end{aligned}$$

As you can see, these forces are very small.

Now consider the gravitational force between the Earth and the Moon. The mass of the Earth is 5,98 $\times$ 10^{24} kg, the mass of the Moon is 7,35 $\times$ 10^{22} kg and the Earth and Moon are 3,8 $\times$ 10^{8} m apart. The gravitational force between the Earth and Moon is:

$$\begin{aligned} F &= G\frac{m_1 m_2}{d^2} \\ &= \left(6{,}67 \times 10^{-11}\right)\left(\frac{(5{,}98 \times 10^{24})(7{,}35 \times 10^{22})}{(0{,}38 \times 10^{9})^2}\right) \\ &= 2{,}03 \times 10^{20}\ \text{N} \end{aligned}$$

From this example you can see that the force is very large.

These two examples demonstrate that the greater the masses, the greater the force between them. The $1/d^2$ factor tells us that the distance between the two bodies plays a role as well. The closer two bodies are, the stronger the gravitational force between them is. We feel the gravitational attraction of the Earth most at the surface since that is the closest we can get to it, but if we were in outer-space, we would barely feel the effect of the Earth's gravity!

Remember that $\vec{F} = m\vec{a}$ which means that every object on Earth feels the same gravitational acceleration! That means whether you drop a pen or a book (from the same height), they will both take the same length of time to hit the ground... in fact they will be head to head for the entire fall if you drop them at the same time. We can show this easily by using Newton's second law and the equation for the gravitational force. The force between the Earth (which has the mass M_{Earth}) and an object of mass m_o is $F = \dfrac{G m_o M_{\text{Earth}}}{d^2}$ and the acceleration of an object of mass m_o (in terms of the force acting on it) is $a_o = \dfrac{F}{m_o}$.

So we equate them and we find that:

$$a_o = G\frac{M_{Earth}}{d_{Earth}^2}$$

Since it doesn't depend on the mass of the object, m_o, the acceleration on a body (due to the Earth's gravity) does not depend on the mass of the body. Thus all objects

experience the same gravitational acceleration. The force on different bodies will be different but the acceleration will be the same. Due to the fact that this acceleration caused by gravity is the same on all objects we label it differently, instead of using a we use g_{Earth} which we call the gravitational acceleration and it has a magnitude of approximately 9,8 m·s^{-2}.

The fact that gravitational acceleration is independent of the mass of the object holds for any planet, not just Earth, but each planet will have a different magnitude of gravitational acceleration.

Exercise 2 – 7:

1. When the planet Jupiter is closest to Earth it is $6{,}28 \times 10^8$ km away. If Jupiter has a mass of $1{,}9 \times 10^{27}$ kg, what is the magnitude of the gravitational force between Jupiter and the Earth?
2. When the planet Jupiter is furthest from the Earth it is $9{,}28 \times 10^8$ km away. If Jupiter has a mass of $1{,}9 \times 10^{27}$ kg, what is the magnitude of the gravitational force between Jupiter and the Earth?
3. What distance must a satellite with a mass of 80 kg be away from the Earth to feel a force of 1000 N? How far from Jupiter to feel the same force?
4. The radius of Jupiter is $71{,}5 \times 10^3$ km and the radius of the moon is $1{,}7 \times 10^3$ km, if the moon has a mass of $7{,}35 \times 10^{22}$ kg work out the gravitational acceleration on Jupiter and on the moon.
5. Astrology, NOT astronomy, makes much of the position of the planets at the moment of one's birth. The only known force a planet exerts on Earth is gravitational. Calculate:
   a) the gravitational force exerted on a 4,20 kg baby by a 100 kg father 0,200 m away at birth
   b) the force on the baby due to Jupiter if it is at its closest distance to Earth, some $6{,}29 \times 10^{11}$ m away.
   c) How does the force of Jupiter on the baby compare to the force of the father on the baby?
6. The existence of the dwarf planet Pluto was proposed based on irregularities in Neptune's orbit. Pluto was subsequently discovered near its predicted position. But it now appears that the discovery was fortuitous, because Pluto is small and the irregularities in Neptune's orbit were not well known. To illustrate that Pluto has a minor effect on the orbit of Neptune compared with the closest planet to Neptune:
   a) Calculate the acceleration due to gravity at Neptune due to Pluto when they are $4{,}50 \times 10^{12}$ m apart, as they are at present. The mass of Pluto is $1{,}4 \times 10^{22}$ kg and the mass of Neptune is: $1{,}02 \times 10^{26}$ kg.
   b) Calculate the acceleration due to gravity at Neptune due to Uranus, presently about $2{,}50 \times 10^{12}$ m apart, and compare it with that due to Pluto. The mass of Uranus is $8{,}62 \times 10^{25}$ kg.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23JH 2. 23JJ 3. 23JK 4. 23JM 5. 23JN 6. 23JP

www.everythingscience.co.za m.everythingscience.co.za

Weight and mass ESBKZ

In everyday discussion many people use weight and mass to mean the same thing which is not true.

Mass is a scalar and weight is a vector. Mass is a measurement of how much matter is in an object; weight is a measurement of how hard gravity is pulling on that object. Your mass is the same wherever you are, on Earth; on the moon; floating in space, because the amount of stuff you're made of doesn't change. Your **weight** depends on how strong a gravitational force is acting on you at the moment; you'd weigh less on the moon than on Earth, and in space you'd weigh almost nothing at all. Mass is measured in kilograms, kg, and weight is a force and measured in newtons, N.

When you stand on a scale you are trying measure how much of you there is. People who are trying to reduce their mass hope to see the reading on the scale get smaller but they talk about losing weight. Their weight will decrease but it is because their mass is decreasing. A scale uses the persons weight to determine their mass.

You can use $\vec{F}_g = m\vec{g}$ to calculate weight.

Worked example 21: Newton's second law: lifts and g

QUESTION

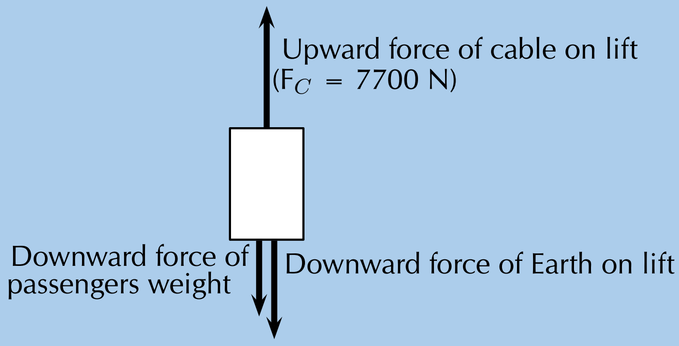
A lift, with a mass of 250 kg, is initially at rest on the ground floor of a tall building. Passengers with an unknown total mass, m, climb into the lift. The lift accelerates upwards at 1,6 m·s^{-2}. The cable supporting the lift exerts a constant upward force of 7700 N.

1. Draw a labelled force diagram indicating all the forces acting on the lift while it accelerates upwards.
2. What is the maximum mass, m, of the passengers the lift can carry in order to achieve a constant upward acceleration of 1,6 m·s^{-2}.

SOLUTION

Step 1: Draw a force diagram.

We choose upwards as the positive direction.



Step 2: Gravitational force

We know that the gravitational acceleration on any object on Earth, due to the Earth, is $\vec{g} = 9{,}8$ m·s^{-2} towards the centre of the Earth (downwards). We know that the force due to gravity on a lift or the passengers in the lift will be $\vec{F}_g = m\vec{g}$.

Step 3: Find the mass, m.

Let us look at the lift with its passengers as a unit. The mass of this unit will be (250 kg $+ m$) and the force of the Earth pulling downwards (F_g) will be $(250 + \text{m}) \times$ 9,8 m·s^{-2}. If we apply Newton's second law to the situation we get:

$$\begin{aligned}
F_{net} &= ma \\
F_C - F_g &= ma \\
7700 - (250 + m)\,(9{,}8) &= (250 + m)\,(1{,}6) \\
7700 - 2500 - 9{,}8\ m &= 400 + 1{,}6\ m \\
4800 &= 11{,}4\ m \\
m &= 421{,}05 \text{ kg}
\end{aligned}$$

Step 4: Quote your final answer

The mass of the passengers is 421,05 kg. If the mass were larger then the total downward force would be greater and the cable would need to exert a larger force in the positive direction to maintain the same acceleration.

In everyday use we often talk about weighing things. We also refer to how much something weighs. It is important to remember that when someone asks how much you weigh or how much an apple weighs they are actually wanting to know your mass or the apples mass, not the force due to gravity acting on you or the apple.

Weightlessness is not because there is no gravitational force or that there is no weight anymore. Weightless is an extreme case of apparent weight. Think about the lift accelerating downwards when you feel a little lighter. If the lift accelerated downwards with the same magnitude as gravitational acceleration there would be no normal force acting on you, the lift and you would be falling with exactly the same acceleration and you would feel weightless. Eventually the lift has to come to a stop.

In a space shuttle in space it is almost exactly the same case. The astronauts and space shuttle feel exactly the same gravitational acceleration so their apparent weight is

zero. The one difference is that they are not falling downwards, they have a very large velocity perpendicular to the direction of the gravitational force that is pulling them towards the earth. They are falling but in a circle around the earth. The gravitational force and their velocity are perfectly balanced that they orbit the earth.

In a weightless environment, defining up and down doesn't make as much sense as in our every day life. In space this affects all sorts of things, for example, when a candle burns the hot gas can't go up because the usual up is defined by which way gravity acts. This has actually been tested.

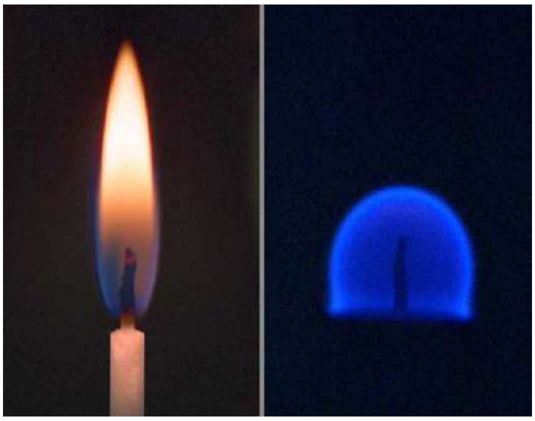


Figure 2.10: A candle burning on earth (left) and one burning in space (right).

Exercise 2 – 8:

1. Jojo has a mass of 87,5 kg, what is his weight on the following planets:
    a) Mercury (radius of $2{,}440 \times 10^3$ km and mass of $3{,}3 \times 10^{23}$ kg)
    b) Mars (radius of $3{,}39 \times 10^3$ km and mass of $6{,}42 \times 10^{23}$ kg)
    c) Neptune (radius of $24{,}76 \times 10^3$ km and mass of $1{,}03 \times 10^{26}$ kg)?
2. If object 1 has a weight of $1{,}78 \times 10^3$ N on Neptune and object 2 has a weight of $3{,}63 \times 10^5$ N on Mars, which has the greater mass?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23JQ 2. 23JR

www.everythingscience.co.za **m.everythingscience.co.za**

Comparative problems ESBM2

Comparative problems involve calculation of something in terms of something else that we know. For example, if you weigh 490 N on Earth and the gravitational acceleration

on Venus is 0,903 that of the gravitational acceleration on the Earth, then you would weigh $0,903 \times 490\text{N} = 442{,}5\text{N}$ on Venus.

Method for answering comparative problems

- Write out equations and calculate all quantities for the given situation
- Write out all relationships between variable from first and second case
- Write out second case
- Substitute all first case variables into second case
- Write second case in terms of first case

Worked example 22: Comparative problem

QUESTION

A man has a mass of 70 kg. The planet Zirgon is the same size as the Earth but has twice the mass of the Earth. What would the man weigh on Zirgon, if the gravitational acceleration on Earth is 9,8 m·s^{-2}?

SOLUTION

Step 1: Determine what information has been given

The following has been provided:

- the mass of the man, m
- the mass of the planet Zirgon (m_Z) in terms of the mass of the Earth (M_{Earth}), $m_Z = 2M_{\text{Earth}}$
- the radius of the planet Zirgon (r_Z) in terms of the radius of the Earth (r_E), $r_Z = r_{\text{Earth}}$

Step 2: Determine how to approach the problem

We are required to determine the man's weight on Zirgon (w_Z). We can do this by using:

$$F_g = mg = G\frac{m_1 \cdot m_2}{d^2}$$

to calculate the weight of the man on Earth and then use this value to determine the weight of the man on Zirgon.

Step 3: Situation on Earth

$$
\begin{aligned}
F_{\text{Earth}} = mg_E &= G\frac{M_{\text{Earth}} \cdot m}{d_E^2} \\
&= (70\text{ kg})\left(9{,}8\text{ m}\cdot\text{s}^{-2}\right) \\
&= 686\text{ N}
\end{aligned}
$$

Step 4: Situation on Zirgon in terms of situation on Earth

Write the equation for the gravitational force on Zirgon and then substitute the values for m_Z and r_Z, in terms of the values for the Earth.

$$
\begin{aligned}
w_Z = mg_Z &= G\frac{m_Z \cdot m}{r_Z^2} \\
&= G\frac{2M_{\text{Earth}} \cdot m}{r_{\text{Earth}}^2} \\
&= 2\left(G\frac{M_{\text{Earth}} \cdot m}{r_E^2}\right) \\
&= 2F_{\text{Earth}} \\
&= 2\,(686\text{ N}) \\
&= 1372\text{ N}
\end{aligned}
$$

Step 5: Quote the final answer

The man weighs 1372 N on Zirgon.

Worked example 23: Comparative problem

QUESTION

A man has a mass of 70 kg. On the planet Beeble how much will he weigh if Beeble has a mass half of that of the Earth and a radius one quarter that of the Earth. Gravitational acceleration on Earth is 9,8 m·s^{-2}.

SOLUTION

Step 1: Determine what information has been given

The following has been provided:

- the mass of the man on Earth, m
- the mass of the planet Beeble (m_B) in terms of the mass of the Earth (M_{Earth}),$m_B = \frac{1}{2}M_{Earth}$
- the radius of the planet Beeble (r_B) in terms of the radius of the Earth (r_E), $r_B = \frac{1}{4}r_E$

Step 2: Determine how to approach the problem

We are required to determine the man's weight on Beeble (w_B). We can do this by using:

$$F_g = mg = G\frac{m_1 m_2}{d^2}$$

to calculate the weight of the man on Earth and then use this value to determine the weight of the man on Beeble.

Step 3: Situation on Earth

$$\begin{aligned} F_{Earth} = mg_{Earth} &= G\frac{M_{Earth}}{r_E^2} \\ &= (70)(9{,}8) \\ &= 686 \text{ N} \end{aligned}$$

Step 4: Situation on Beeble in terms of situation on Earth

Write the equation for the gravitational force on Beeble and then substitute the values for m_B and r_B, in terms of the values for the Earth.

$$\begin{aligned} F_{Beeble} = mg_{Beeble} &= G\frac{M_{Beeble}}{r_B^2} \\ &= G\frac{\frac{1}{2}M_{Earth}}{\frac{1}{4}r_E^2} \\ &= 8\left(G\frac{M_{Earth}}{r_E^2}\right) \\ &= 8(686) \\ &= 5488 \text{ N} \end{aligned}$$

Step 5: Quote the final answer

The man weighs 5488 N on Beeble.

Exercise 2 – 9:

1. Two objects of mass $2X$ and $3X$ respectively, where X is an unknown quantity, exert a force F on each other when they are a certain distance apart. What will be the force between two objects situated the same distance apart but having a mass of $5X$ and $6X$ respectively?

a) 0,2 F

b) 1,2 F

c) 2,2 F

d) 5 F

2. As the distance of an object above the surface of the Earth is greatly increased, the weight of the object would

   a) increase

   b) decrease

   c) increase and then suddenly decrease

   d) remain the same

3. A satellite circles around the Earth at a height where the gravitational force is a factor 4 less than at the surface of the Earth. If the Earth's radius is R, then the height of the satellite above the surface is:

   a) R

   b) 2 R

   c) 4 R

   d) 16 R

4. A satellite experiences a force F when at the surface of the Earth. What will be the force on the satellite if it orbits at a height equal to the diameter of the Earth:

   a) $\frac{1}{F}$

   b) $\frac{1}{2}$ F

   c) $\frac{1}{3}$ F

   d) $\frac{1}{9}$ F

5. The weight of a rock lying on surface of the Moon is W. The radius of the Moon is R. On planet Alpha, the same rock has weight 8W. If the radius of planet Alpha is half that of the Moon, and the mass of the Moon is M, then the mass, in kg, of planet Alpha is:

   a) $\frac{M}{2}$

   b) $\frac{M}{4}$

   c) 2 M

   d) 4 M

6. Consider the symbols of the two physical quantities g and G used in Physics.

   a) Name the physical quantities represented by g and G.

   b) Derive a formula for calculating g near the Earth's surface using Newton's law of universal gravitation. M and R represent the mass and radius of the Earth respectively.

7. Two spheres of mass 800 g and 500 g respectively are situated so that their centres are 200 cm apart. Calculate the gravitational force between them.

8. Two spheres of mass 2 kg and 3 kg respectively are situated so that the gravitational force between them is $2{,}5 \times 10^{-8}$ N. Calculate the distance between them.

9. Two identical spheres are placed 10 cm apart. A force of $1{,}6675 \times 10^{-9}$ N exists between them. Find the masses of the spheres.

10. Halley's comet, of approximate mass 1×10^{15} kg was $1{,}3 \times 10^{8}$ km from the Earth, at its point of closest approach during its last sighting in 1986.

    a) Name the force through which the Earth and the comet interact.
    b) Is the magnitude of the force experienced by the comet the same, greater than or less than the force experienced by the Earth? Explain.
    c) Does the acceleration of the comet increase, decrease or remain the same as it moves closer to the Earth? Explain.
    d) If the mass of the Earth is 6×10^{24} kg, calculate the magnitude of the force exerted by the Earth on Halley's comet at its point of closest approach.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23JS 2. 23JT 3. 23JV 4. 23JW 5. 23JX 6. 23JY
7. 23JZ 8. 23K2 9. 23K3 10. 23K4

www.everythingscience.co.za m.everythingscience.co.za

2.5 Chapter summary ESBM3

See presentation: 23K5 at www.everythingscience.co.za

- The normal force, $\vec{N}$, is the force exerted by a surface on an object in contact with it. The normal force is perpendicular to the surface.
- Frictional force is the force that opposes the motion of an object in contact with a surface and it acts parallel to the surface the object is in contact with. The magnitude of friction is proportional to the normal force.
- For every surface we can determine a constant factor, the coefficient of friction, that allows us to calculate what the frictional force would be if we know the magnitude of the normal force. We know that static friction and kinetic friction have different magnitudes so we have different coefficients for the two types of friction:
  - μ_s is the coefficient of **static** friction
  - μ_k is the coefficient of **kinetic** friction
- The components of the force due to gravity, $\vec{F}_g$, parallel (x-direction) and perpendicular (y-direction) to a slope are given by:

$$F_{gx} = F_g \sin(\theta)$$
$$F_{gy} = F_g \cos(\theta)$$

- **Newton's first law:** An object continues in a state of rest or uniform motion (motion with a constant velocity) unless it is acted on by an unbalanced (net or resultant) force.
- **Newton's second law:** If a resultant force acts on a body, it will cause the body to accelerate in the direction of the resultant force. The acceleration of the body will be directly proportional to the resultant force and inversely proportional to the mass of the body. The mathematical representation is:

$$\vec{F}_{net} = m\vec{a}$$

- **Newton's third law:** If body A exerts a force on body B, then body B exerts a force of equal magnitude on body A, but in the opposite direction.
- **Newton's law of universal gravitation:** Every point mass attracts every other point mass by a force directed along the line connecting the two. This force is proportional to the product of the masses and inversely proportional to the square of the distance between them.

$$F = G\frac{m_1 m_2}{d^2}$$

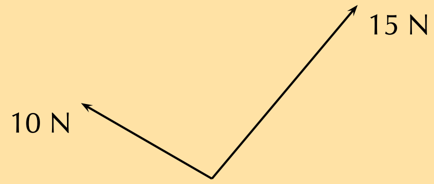
Physical Quantities		
Quantity	**Unit name**	**Unit symbol**
Acceleration (a)	metres per second squared	$m{\cdot}s^{-1}$
Force (F)	newton	N
Mass (m)	kilogram	kg
Tension (T)	newton	N
Weight (N)	newton	N

Exercise 2 – 10: Forces and Newton's Laws

1. A force acts on an object. Name three effects that the force can have on the object.
2. Identify each of the following forces as contact or non-contact forces.
   a) The force between the north pole of a magnet and a paper clip.
   b) The force required to open the door of a taxi.
   c) The force required to stop a soccer ball.
   d) The force causing a ball, dropped from a height of 2 m, to fall to the floor.
3. A book of mass 2 kg is lying on a table. Draw a labelled force diagram indicating all the forces on the book.
4. A constant, resultant force acts on a body which can move freely in a straight line. Which physical quantity will remain constant?
   a) acceleration
   b) velocity
   c) momentum
   d) kinetic energy

[SC 2003/11]

5. Two forces, 10 N and 15 N, act at an angle at the same point.



Which of the following **cannot** be the resultant of these two forces?

a) 2 N
b) 5 N
c) 8 N
d) 20 N

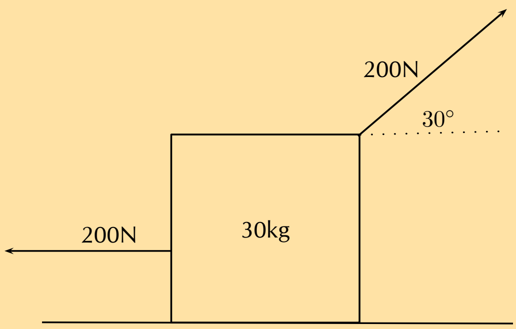
[SC 2005/11 SG1]

6. A concrete block weighing 250 N is at rest on an inclined surface at an angle of 20°. The magnitude of the normal force, in newtons, is

a) 250
b) $250 \cos 20°$
c) $250 \sin 20°$
d) $2500 \cos 20°$

7. A 30 kg box sits on a flat frictionless surface. Two forces of 200 N each are applied to the box as shown in the diagram. Which statement best describes the motion of the box?

a) The box is lifted off the surface.
b) The box moves to the right.
c) The box does not move.
d) The box moves to the left.



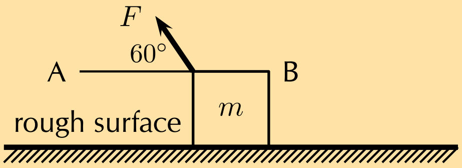
8. A concrete block weighing 200 N is at rest on an inclined surface at an angle of 20°. The normal force, in newtons, is

a) 200
b) $200 \cos 20°$

c) $200 \sin 20°$

d) $2000 \cos 20°$

9. A box, mass m, is at rest on a rough horizontal surface. A force of constant magnitude F is then applied on the box at an angle of 60° to the horizontal, as shown.



If the box has a uniform horizontal acceleration of magnitude, a, the frictional force acting on the box is...

a) $F cos 60° - ma$ in the direction of A

b) $F cos 60° - ma$ in the direction of B

c) $F sin 60° - ma$ in the direction of A

d) $F sin 60° - ma$ in the direction of B

[SC 2003/11]

10. Thabo stands in a train carriage which is moving eastwards. The train suddenly brakes. Thabo continues to move eastwards due to the effect of:

a) his inertia.

b) the inertia of the train.

c) the braking force on him.

d) a resultant force acting on him.

[SC 2002/11 SG]

11. A 100 kg crate is placed on a slope that makes an angle of 45° with the horizontal. The gravitational force on the box is 98 N. The box does not slide down the slope. Calculate the magnitude and direction of the frictional force and the normal force present in this situation.

12. A body moving at a *CONSTANT VELOCITY* on a horizontal plane, has a number of unequal forces acting on it. Which one of the following statements is TRUE?

a) At least two of the forces must be acting in the same direction.

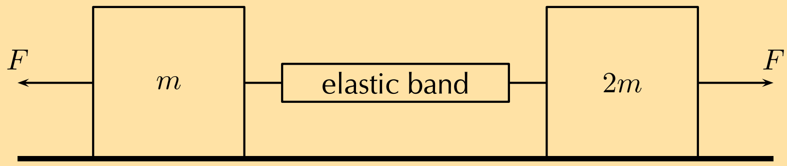
b) The resultant of the forces is zero.

c) Friction between the body and the plane causes a resultant force.

d) The vector sum of the forces causes a resultant force which acts in the direction of motion.

[SC 2002/11 HG1]

13. Two masses of m and $2m$ respectively are connected by an elastic band on a frictionless surface. The masses are pulled in opposite directions by two forces each of magnitude F, stretching the elastic band and holding the masses stationary.

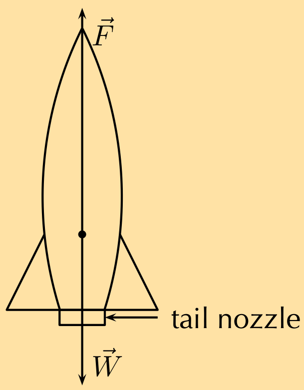


Which of the following gives the magnitude of the tension in the elastic band?

a) zero
b) $\frac{1}{2}F$
c) F
d) $2F$

[IEB 2005/11 HG]

14. A rocket takes off from its launching pad, accelerating up into the air.

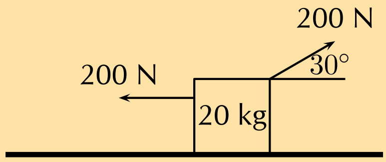


The rocket accelerates because the magnitude of the upward force, F is greater than the magnitude of the rocket's weight, W. Which of the following statements **best** describes how force F arises?

a) F is the force of the air acting on the base of the rocket.
b) F is the force of the rocket's gas jet *pushing down* on the air.
c) F is the force of the rocket's gas jet *pushing down* on the ground.
d) F is the reaction to the force that the rocket exerts on the gases which escape out through the tail nozzle.

[IEB 2005/11 HG]

15. A box of mass 20 kg rests on a smooth horizontal surface. What will happen to the box if two forces each of magnitude 200 N are applied simultaneously to the box as shown in the diagram.



The box will:

a) be lifted off the surface.

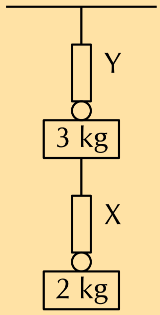
b) move to the left.

c) move to the right.

d) remain at rest.

[SC 2001/11 HG1]

16. A 2 kg mass is suspended from spring balance X, while a 3 kg mass is suspended from spring balance Y. Balance X is in turn suspended from the 3 kg mass. Ignore the weights of the two spring balances.

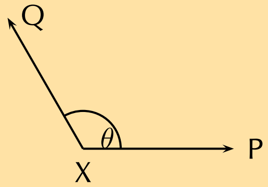


The readings (in N) on balances X and Y are as follows:

	X	**Y**
a)	19,6	29,4
b)	19,6	49
c)	24,5	24,5
d)	49	49

[SC 2001/11 HG1]

17. P and Q are two forces of equal magnitude applied simultaneously to a body at X.



As the angle θ between the forces is **decreased** from 180° to 0°, the magnitude of the resultant of the two forces will

a) initially increase and then decrease.

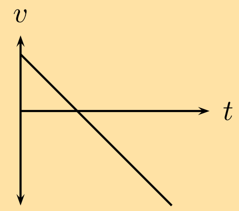
b) initially decrease and then increase.

c) increase only.

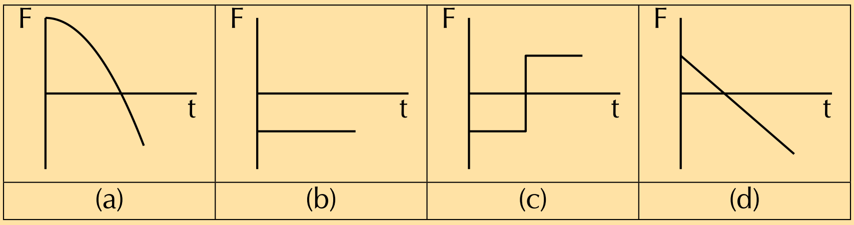
d) decrease only.

[SC 2002/03 HG1]

18. The graph below shows the velocity-time graph for a moving object:

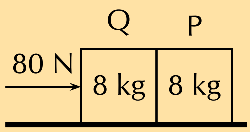


Which of the following graphs could best represent the relationship between the resultant force applied to the object and time?



[SC 2002/03 HG1]

19. Two blocks each of mass 8 kg are in contact with each other and are accelerated along a frictionless surface by a force of 80 N as shown in the diagram. The force which block Q will exert on block P is equal to ...

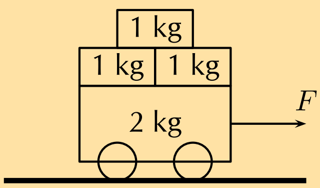


a) 0 N
b) 40 N
c) 60 N
d) 80 N

[SC 2002/03 HG1]

20. A 12 kg box is placed on a rough surface. A force of 20 N applied at an angle of 30° to the horizontal cannot move the box. Calculate the magnitude and direction of the normal and friction forces.

21. Three 1 kg mass pieces are placed on top of a 2 kg trolley. When a force of magnitude F is applied to the trolley, it experiences an acceleration a.



If one of the 1 kg mass pieces falls off while F is still being applied, the trolley will accelerate at ...

a) $\frac{1}{5}a$
b) $\frac{4}{5}a$

c) $\frac{5}{4}a$

d) $5a$

[SC 2002/03 HG1]

22. A car moves along a horizontal road at constant velocity. Which of the following statements is true?

    a) The car is not in equilibrium.
    b) There are no forces acting on the car.
    c) There is zero resultant force.
    d) There is no frictional force.

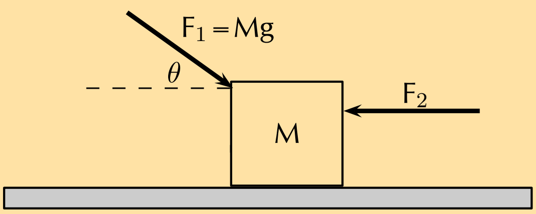
    [IEB 2004/11 HG1]

23. A crane lifts a load vertically upwards at constant speed. The upward force exerted on the load is F. Which of the following statements is correct?

    a) The acceleration of the load is 9,8 m·s^{-1} downwards.
    b) The resultant force on the load is F.
    c) The load has a weight equal in magnitude to F.
    d) The forces of the crane on the load, and the weight of the load, are an example of a Newton's third law 'action-reaction' pair.

    [IEB 2004/11 HG1]

24. A body of mass M is at rest on a smooth horizontal surface with two forces applied to it as in the diagram below. Force F_1 is equal to Mg. The force F_1 is applied to the right at an angle θ to the horizontal, and a force of F_2 is applied horizontally to the left.



    How is the body affected when the angle θ is increased?

    a) It remains at rest.
    b) It lifts up off the surface, and accelerates towards the right.
    c) It lifts up off the surface, and accelerates towards the left.
    d) It accelerates to the left, moving along the smooth horizontal surface.

    [IEB 2004/11 HG1]

25. Which of the following statements correctly explains why a passenger in a car, who is not restrained by the seat belt, continues to move forward when the brakes are applied suddenly?

    a) The braking force applied to the car exerts an equal and opposite force on the passenger.

b) A forward force (called inertia) acts on the passenger.

c) A resultant forward force acts on the passenger.

d) A zero resultant force acts on the passenger.

[IEB 2003/11 HG1]

26. A rocket (mass 20 000 kg) accelerates from rest to 40 $m \cdot s^{-1}$ in the first 1,6 seconds of its journey upwards into space.

The rocket's propulsion system consists of exhaust gases, which are pushed out of an outlet at its base.

a) Explain, with reference to the appropriate law of Newton, how the escaping exhaust gases exert an upwards force (thrust) on the rocket.

b) What is the magnitude of the total thrust exerted on the rocket during the first 1,6 s?

c) An astronaut of mass 80 kg is carried in the space capsule. Determine the resultant force acting on him during the first 1,6 s.

d) Explain why the astronaut, seated in his chair, feels "heavier" while the rocket is launched.

[IEB 2004/11 HG1]

27. a) State Newton's second law of Motion.

b) A sports car (mass 1000 kg) is able to accelerate uniformly from rest to 30 $m \cdot s^{-1}$ in a minimum time of 6 s.

i. Calculate the magnitude of the acceleration of the car.

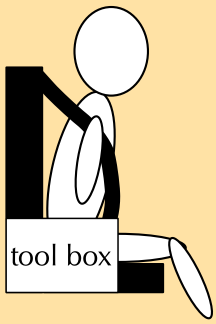
ii. What is the magnitude of the resultant force acting on the car during these 6 s?

c) The magnitude of the force that the wheels of the vehicle exert on the road surface as it accelerates is 7500 N. What is the magnitude of the retarding forces acting on this car?

d) By reference to a suitable Law of Motion, explain why a headrest is important in a car with such a rapid acceleration.

[IEB 2003/11 HG1 - Sports Car]

28. A child (mass 18 kg) is strapped in his car seat as the car moves to the right at constant velocity along a straight level road. A tool box rests on the seat beside him.



The driver brakes suddenly, bringing the car rapidly to a halt. There is negligible friction between the car seat and the box.

a) Draw a labelled free-body diagram of **the forces acting on the child** during the time that the car is being braked.

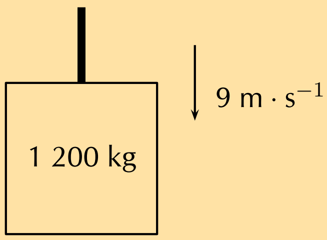
b) Draw a labelled free-body diagram of **the forces acting on the box** during the time that the car is being braked.

c) *Modern cars are designed with safety features (besides seat belts) to protect drivers and passengers during collisions e.g. the crumple zones on the car's body. Rather than remaining rigid during a collision, the crumple zones allow the car's body to collapse steadily.*

State Newton's second law of motion.

[IEB 2005/11 HG1]

29. The total mass of a lift together with its load is 1200 kg. It is moving downwards at a constant velocity of 9 $m \cdot s^{-1}$.



a) What will be the magnitude of the force exerted by the cable on the lift while it is moving downwards at constant velocity? Give an explanation for your answer.

b) *The lift is now uniformly brought to rest over a distance of 18 m.*

Calculate the magnitude of the acceleration of the lift.

c) Calculate the magnitude of the force exerted by the cable while the lift is being brought to rest.

[SC 2003/11 HG1]

30. A driving force of 800 N acts on a car of mass 600 kg.

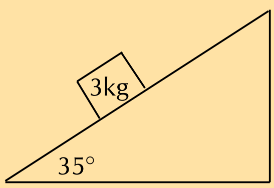
a) Calculate the car's acceleration.

b) Calculate the car's speed after 20 s.

c) Calculate the new acceleration if a frictional force of 50 N starts to act on the car after 20 s.

d) Calculate the speed of the car after another 20 s (i.e. a total of 40 s after the start).

31. A stationary block of mass 3 kg is on top of a plane inclined at 35° to the horizontal.



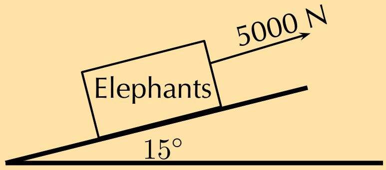
a) Draw a force diagram (not to scale). Include the weight of the block as well as the components of the weight that are perpendicular and parallel to the inclined plane.

b) Determine the values of the weight's perpendicular and parallel components.

32. **A crate on an inclined plane**

Elephants are being moved from the Kruger National Park to the Eastern Cape. They are loaded into crates that are pulled up a ramp (an inclined plane) on frictionless rollers.

The diagram shows a crate being held stationary on the ramp by means of a rope parallel to the ramp. The tension in the rope is 5000 N.



a) Explain how one can deduce the following: "The forces acting on the crate are in equilibrium".

b) Draw a labelled free-body diagram of the forces acting on the elephant. (Regard the crate and elephant as one object, and represent them as a dot. Also show the relevant angles between the forces.)

c) The crate has a mass of 800 kg. Determine the mass of the elephant.

d) The crate is now pulled up the ramp at a constant speed. How does the crate being pulled up the ramp at a constant speed affect the forces acting on the crate and elephant? Justify your answer, mentioning any law or principle that applies to this situation.

[IEB 2002/11 HG1]

33. **Car in Tow**

Car A is towing Car B with a light tow rope. The cars move along a straight, horizontal road.

a) Write down a statement of Newton's second law of Motion (in words).

b) As they start off, Car A exerts a forwards force of 600 N at its end of the tow rope. The force of friction on Car B when it starts to move is 200 N. The mass of Car B is 1200 kg. Calculate the acceleration of Car B.

c) After a while, the cars travel at constant velocity. The force exerted on the tow rope is now 300 N while the force of friction on Car B increases. What is the magnitude and direction of the force of friction on Car B now?

d) Towing with a rope is very dangerous. A solid bar should be used in preference to a tow rope. This is especially true should Car A suddenly apply brakes. What would be the advantage of the solid bar over the tow rope in such a situation?

e) The mass of Car A is also 1200 kg. Car A and Car B are now joined by a solid tow bar and the total braking force is 9600 N. Over what distance could the cars stop from a velocity of 20 $m{\cdot}s^{-1}$?

[IEB 2002/11 HG1]

34. **Testing the Brakes of a Car**

A braking test is carried out on a car travelling at 20 $m{\cdot}s^{-1}$. A braking distance of 30 m is measured when a braking force of 6000 N is applied to stop the car.

a) Calculate the acceleration of the car when a braking force of 6000 N is applied.

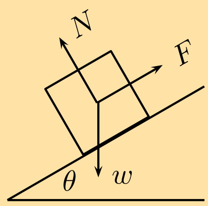
b) **Show** that the mass of this car is 900 kg.

c) How long (in s) does it take for this car to stop from 20 m·s^{-1} under the braking action described above?

d) A trailer of mass 600 kg is attached to the car and the braking test is repeated from 20 m·s^{-1} using the same braking force of 6000 N. How much longer will it take to stop the car with the trailer in tow?

[IEB 2001/11 HG1]

35. A box is held stationary on a smooth plane that is inclined at angle θ to the horizontal.



F is the force exerted by a rope on the box. w is the weight of the box and N is the normal force of the plane on the box. Which of the following statements is correct?

a) $\tan\theta = \frac{F}{w}$

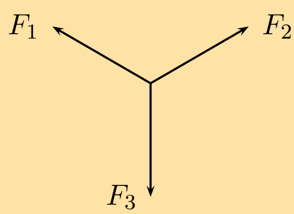
b) $\tan\theta = \frac{F}{N}$

c) $\cos\theta = \frac{F}{w}$

d) $\sin\theta = \frac{N}{w}$

[IEB 2005/11 HG]

36. As a result of three forces F_1, F_2 and F_3 acting on it, an object at point P is in equilibrium.



Which of the following statements is **not true** with reference to the three forces?

a) The resultant of forces F_1, F_2 and F_3 is zero.

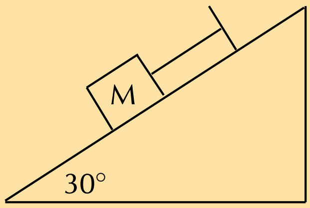
b) Force F_1, F_2 and F_3 lie in the same plane.

c) Force F_3 is the resultant of forces F_1 and F_2.

d) The sum of the components of all the forces in any chosen direction is zero.

[SC 2001/11 HG1]

37. A block of mass M is held stationary by a rope of negligible mass. The block rests on a frictionless plane which is inclined at 30° to the horizontal.

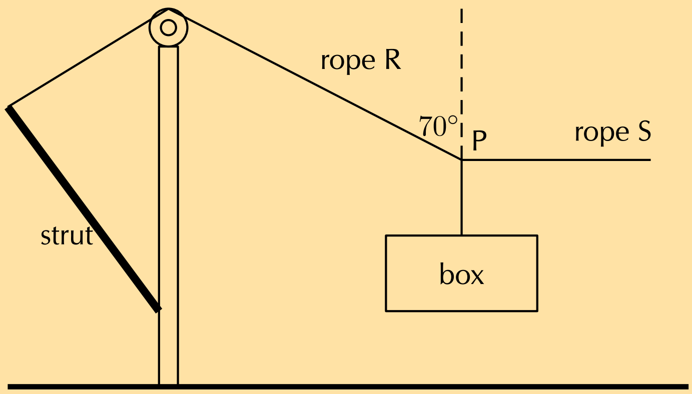


a) Draw a labelled force diagram which shows all the forces acting on the block.

b) Resolve the force due to gravity into components that are parallel and perpendicular to the plane.

c) Calculate the weight of the block when the force in the rope is 8 N.

38. A heavy box, mass m, is lifted by means of a rope R which passes over a pulley fixed to a pole. A second rope S, tied to rope R at point P, exerts a horizontal force and pulls the box to the right. After lifting the box to a certain height, the box is held stationary as shown in the sketch below. Ignore the masses of the ropes. The tension in rope R is 5850 N.



a) Draw a diagram (with labels) of all the forces acting at the point P, when P is in equilibrium.

b) By resolving the force exerted by rope R into components, calculate the...

   i. magnitude of the force exerted by rope S.

   ii. mass, m, of the box.

[SC 2003/11]

39. A tow truck attempts to tow a broken down car of mass 400 kg. The coefficient of static friction is 0,60 and the coefficient of kinetic (dynamic) friction is 0,4. A rope connects the tow truck to the car. Calculate the force required:

a) to just move the car if the rope is parallel to the road.

b) to keep the car moving at constant speed if the rope is parallel to the road.

c) to just move the car if the rope makes an angle of 30° to the road.

d) to keep the car moving at constant speed if the rope makes an angle of 30° to the road.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23K6	2a. 23K7	2b. 23K8	2c. 23K9	2d. 23KB	3. 23KC
4. 23KD	5. 23KF	6. 23KG	7. 23KH	8. 23KJ	9. 23KK
10. 23KM	11. 23KN	12. 23KP	13. 23KQ	14. 23KR	15. 23KS
16. 23KT	17. 23KV	18. 23KW	19. 23KX	20. 23KY	21. 23KZ
22. 23M2	23. 23M3	24. 23M4	25. 23M5	26. 23M6	27. 23M7
28. 23M8	29. 23M9	30. 23MB	31. 23MC	32. 23MD	33. 23MF
34. 23MG	35. 23MJ	36. 23MK	37. 23MM	38. 23MN	39. 23MP

www.everythingscience.co.za m.everythingscience.co.za

Exercise 2 – 11: Gravitation

1. An object attracts another with a gravitational force F. If the distance between the centres of the two objects is now decreased to a third ($\frac{1}{3}$) of the original distance, the force of attraction that the one object would exert on the other would become...

   a) $\frac{1}{9}F$
   b) $\frac{1}{3}F$
   c) $3F$
   d) $9F$

   [SC 2003/11]

2. An object is dropped from a height of 1 km above the Earth. If air resistance is ignored, the acceleration of the object is dependent on the...

   a) mass of the object
   b) radius of the earth
   c) mass of the earth
   d) weight of the object

   [SC 2003/11]

3. A man has a mass of 70 kg on Earth. He is walking on a new planet that has a mass four times that of the Earth and the radius is the same as that of the Earth ($M_E = 6 \times 10^{24}$ kg, $r_E = 6 \times 10^{6}$ m)

   a) Calculate the force between the man and the Earth.
   b) What is the man's weight on the new planet?
   c) Would his weight be bigger or smaller on the new planet? Explain how you arrived at your answer.

4. Calculate the distance between two objects, 5000 kg and 6×10^{12} kg respectively, if the magnitude of the force between them is 3×10^{8} N

5. An astronaut in a satellite 1600 km above the Earth experiences a gravitational force of magnitude 700 N on Earth. The Earth's radius is 6400 km. Calculate:

a) The magnitude of the gravitational force which the astronaut experiences in the satellite.

b) The magnitude of the gravitational force on an object in the satellite which weighs 300 N on Earth.

6. An astronaut of mass 70 kg on Earth lands on a planet which has half the Earth's radius and twice its mass. Calculate the magnitude of the force of gravity which is exerted on him on the planet.

7. Calculate the magnitude of the gravitational force of attraction between two spheres of lead with a mass of 10 kg and 6 kg respectively if they are placed 50 mm apart.

8. The gravitational force between two objects is 1200 N. What is the gravitational force between the objects if the mass of each is doubled and the distance between them halved?

9. Calculate the gravitational force between the Sun with a mass of 2×10^{30} kg and the Earth with a mass of 6×10^{24} kg if the distance between them is $1,4 \times 10^{8}$ km.

10. How does the gravitational force of attraction between two objects change when

    a) the mass of each object is doubled.

    b) the distance between the centres of the objects is doubled.

    c) the mass of one object is halved, and the distance between the centres of the objects is halved.

11. Read each of the following statements and say whether you agree or not. Give reasons for your answer and rewrite the statement if necessary:

    a) The gravitational acceleration g is constant.

    b) The weight of an object is independent of its mass.

    c) G is dependent on the mass of the object that is being accelerated.

12. An astronaut weighs 750 N on the surface of the Earth.

    a) What will his weight be on the surface of Saturn, which has a mass 100 times greater than the Earth, and a radius 5 times greater than the Earth?

    b) What is his mass on Saturn?

13. Your mass is 60 kg in Paris at ground level. How much less would you weigh after taking a lift to the top of the Eiffel Tower, which is 405 m high? Assume the Earth's mass is $6,0 \times 10^{24}$ kg and the Earth's radius is 6400 km.

14. a) State Newton's law of universal gravitation.

    b) Use Newton's law of universal gravitation to determine the magnitude of the acceleration due to gravity on the Moon.

    The mass of the Moon is $7,4 \times 10^{22}$ kg.

    The radius of the Moon is $1,74 \times 10^{6}$ m.

    c) Will an astronaut, kitted out in his space suit, jump higher on the Moon or on the Earth? Give a reason for your answer.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23MS	2. 23MT	3. 23MV	4. 23MW	5. 23MY	6. 23MZ
7. 23N2	8. 23N3	9. 23N4	10. 23N5	11a. 23N6	11b. 23N7
11c. 23N8	12. 23N9	13. 23NC	14. 23ND		

www.everythingscience.co.za **m.everythingscience.co.za**

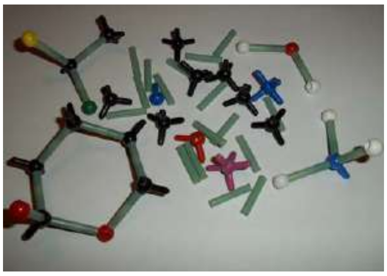
CHAPTER 3

Atomic combinations

We live in a world that is made up of many complex compounds. All around us we see evidence of chemical bonding from the chair you are sitting on, to the book you are holding, to the air you are breathing. Imagine if all the elements on the periodic table did not form bonds but rather remained on their own. Our world would be pretty boring with only 100 or so elements to use.

Imagine you were painting a picture and wanted to show the colours around you. The only paints you have are red, green, yellow, blue, white and black. Yet you are able to make pink, purple, orange and many other colours by mixing these paints. In the same way, the elements can be thought of as natures paint box. The elements can be joined together in many different ways to make new compounds and so create the world around you.

In Grade 10 we started exploring chemical bonding. In this chapter we will go on to explain more about chemical bonding and why chemical bonding occurs. We looked at the three types of bonding: covalent, ionic and metallic. In this chapter we will focus mainly on covalent bonding and on the molecules that form as a result of covalent bonding.



NOTE:
In this chapter we will use the term molecule to mean a covalent molecular structure. This is a covalent compound that interacts and exists as a single entity.

3.1 Chemical bonds ESBM4

Why do atoms bond? ESBM5

As we begin this section, it's important to remember that what we will go on to discuss is a *model* of bonding, that is based on a particular *model* of the atom. You will remember from the discussion on atoms (in Grade 10) that a model is a *representation* of what is happening in reality. In the model of the atom that you are learnt in Grade 10, the atom is made up of a central nucleus, surrounded by electrons that are arranged in fixed energy levels (sometimes called *shells*). Within each energy level, electrons move in *orbitals* of different shapes. The electrons in the outermost energy level of an atom are called the **valence electrons**. This model of the atom is useful in trying to understand how different types of bonding take place between atoms.

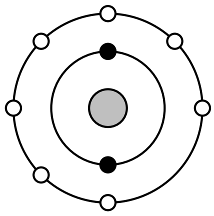


Figure 3.1: Electron arrangement of a fluorine atom. The black electrons (small circles on the inner ring) are the core electrons and the white electrons (small circles on the outer ring) are the valence electrons.

The following points were made in these earlier discussions on electrons and energy levels:

- Electrons always try to occupy the lowest possible energy level.
- The noble gases have a full valence electron orbital. For example neon has the following electronic configuration: $1s^2 2s^2 2p^6$. The second energy level is the outermost (valence) shell and is full.
- Atoms form bonds to try to achieve the same electron configuration as the noble gases.
- Atoms with a full valence electron orbital are less reactive.

TIP
A model takes what we see in the world around us and uses that to make certain predictions about what we cannot see.

Energy and bonding ESBM6

There are two cases that we need to consider when two atoms come close together. The first case is where the two atoms come close together and form a bond. The second case is where the two atoms come close together but do not form a bond. We will use hydrogen as an example of the first case and helium as an example of the second case.

Case 1: A bond forms

Let's start by imagining that there are two hydrogen atoms approaching one another. As they move closer together, there are three forces that act on the atoms at the same time. These forces are described below:

1. **repulsive force** between the electrons of the atoms, since like charges repel

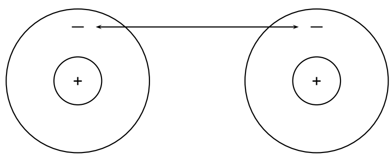


Figure 3.2: Repulsion between electrons

2. **attractive force** between the nucleus of one atom and the electrons of another

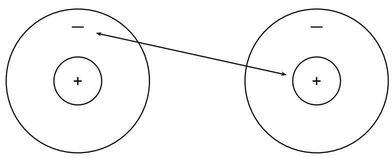


Figure 3.3: Attraction between electrons and protons.

3. **repulsive force** between the two positively-charged nuclei

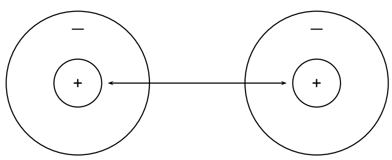


Figure 3.4: Repulsion between protons

These three forces work together when two atoms come close together. As the total force experienced by the atoms changes, the amount of energy in the system also changes.

Now look at Figure 3.5 to understand the energy changes that take place when the two atoms move towards each other.

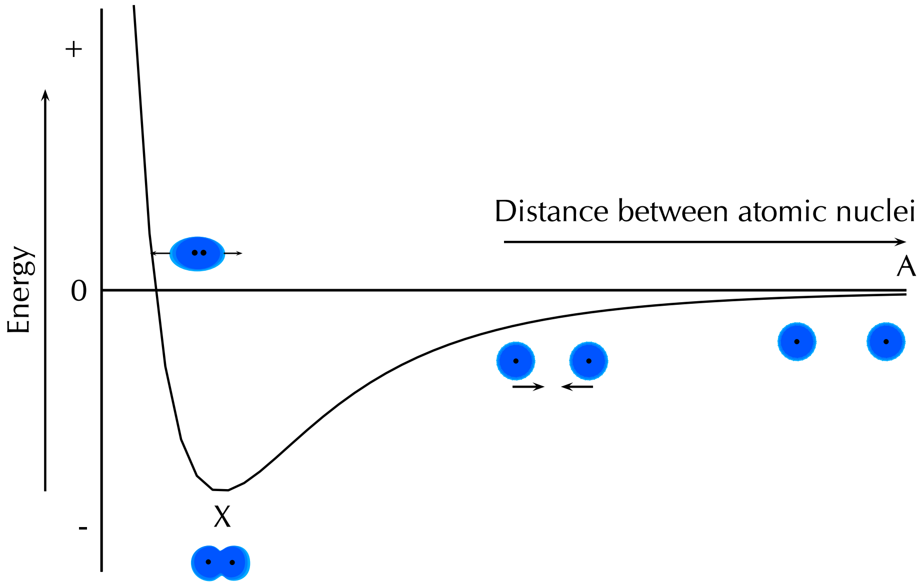


Figure 3.5: Graph showing the change in energy that takes place as two hydrogen atoms move closer together.

Let us imagine that we have fixed the one atom and we will move the other atom closer to the first atom. As we move the second hydrogen atom closer to the first (from point A to point X) the energy of the system decreases. Attractive forces dominate this part of the interaction. As the second atom approaches the first one and gets closer to point X, more energy is needed to pull the atoms apart. This gives a negative potential energy.

At point X, the attractive and repulsive forces acting on the two hydrogen atoms are balanced. The energy of the system is at a minimum.

Further to the left of point X, the repulsive forces are stronger than the attractive forces and the energy of the system increases.

For hydrogen the energy at point X is low enough that the two atoms stay together and do not break apart again. This is why when we draw the Lewis diagram for a hydrogen molecule we draw two hydrogen atoms next to each other with an electron pair between them.

$$\mathbf{H} \overset{\times}{\bullet} \mathbf{H}$$

We also note that this arrangement gives both hydrogen atoms a full outermost energy level (through the sharing of electrons or covalent bonding).

Case 2: A bond does not form

Now if we look at helium we see that each helium atom has a filled outer energy level. Looking at Figure 3.6 we find that the energy minimum for two helium atoms is very close to zero. This means that the two atoms can come together and move apart very easily and never actually stick together.

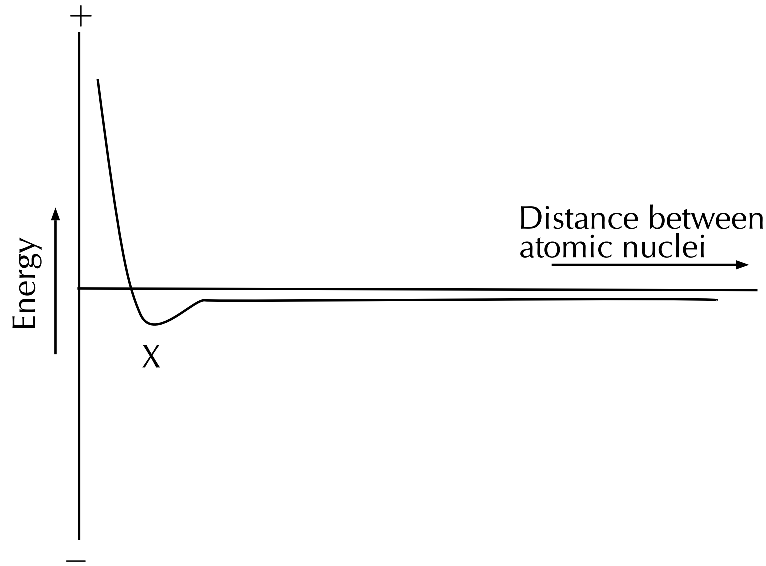


Figure 3.6: Graph showing the change in energy that takes place as two helium atoms move closer together.

For helium the energy minimum at point X is not low enough that the two atoms stay together and so they move apart again. This is why when we draw the Lewis diagram for helium we draw one helium atom on its own. There is no bond.

We also see that helium already has a full outermost energy level and so no compound forms.

He:

See simulation: 23NF at www.everythingscience.co.za

Valence electrons and Lewis diagrams ESBM7

Now that we understand a bit more about bonding we need to refresh the concept of Lewis diagrams that you learnt about in Grade 10. With the knowledge of why atoms bond and the knowledge of how to draw Lewis diagrams we will have all the tools that we need to try to predict which atoms will bond and what shape the molecule will be.

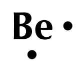
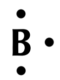
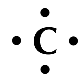
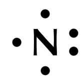
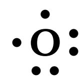
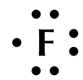
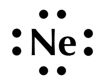
In grade 10 we learnt how to write the electronic structure for any element. For drawing Lewis diagrams the one that you should be familiar with is the spectroscopic notation. For example the electron configuration of chlorine in spectroscopic notation is: $1s^2 2s^2 2p^5$. Or if we use the condensed form: $[\text{He}]2s^2 2p^5$. The condensed spectroscopic notation quickly shows you the valence electrons for the element.

TIP
A **Lewis diagram** uses dots or crosses to represent the **valence electrons** on different atoms. The chemical symbol of the element is used to represent the nucleus and the core electrons of the atom.

TIP
You can place the unpaired electrons anywhere (top, bottom, left or right). The exact ordering in a Lewis diagram does not matter.

Using the number of valence electrons we can easily draw Lewis diagrams for any element. In Grade 10 you learnt how to draw Lewis diagrams. We will refresh the concepts here as they will aid us in our discussion of bonding.

Lewis diagrams for the elements in period 2 are shown below:

Element	Group number	Valence electrons	Spectroscopic notation	Lewis diagram
Lithium	1	1	$[He]2s^1$	
Beryllium	2	2	$[He]2s^2$	
Boron	13	3	$[He]2s^22p^1$	
Carbon	14	4	$[He]2s^22p^2$	
Nitrogen	15	5	$[He]2s^22p^3$	
Oxygen	16	6	$[He]2s^22p^4$	
Fluorine	17	7	$[He]2s^22p^5$	
Neon	18	8	$[He]2s^22p^6$	

Exercise 3 – 1: Lewis diagrams

Give the spectroscopic notation and draw the Lewis diagram for:

1. magnesium
2. sodium
3. chlorine
4. aluminium
5. argon

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23NG 2. 23NH 3. 23NJ 4. 23NK 5. 23NM

www.everythingscience.co.za **m.everythingscience.co.za**

Covalent bonds and bond formation ESBM8

Covalent bonding involves the sharing of electrons to form a chemical bond. The outermost orbitals of the atoms overlap so that unpaired electrons in each of the bonding atoms can be shared. By overlapping orbitals, the outer energy shells of all the bonding atoms are filled. The shared electrons move in the orbitals around *both* atoms. As

they move, there is an attraction between these negatively charged electrons and the positively charged nuclei. This attractive force holds the atoms together in a covalent bond.

TIP
Covalent bonds are examples of interatomic forces.

TIP
Remember that it is only the *valence electrons* that are involved in bonding, and so when diagrams are drawn to show what is happening during bonding, it is only these electrons that are shown. Dots or crosses represent electrons in different atoms.

DEFINITION: *Covalent bond*

A form of chemical bond where pairs of electrons are shared between atoms.

We will look at a few simple cases to deduce some rules about covalent bonds.

Case 1: Two atoms that each have an unpaired electron

For this case we will look at hydrogen chloride and methane.

Worked example 1: Lewis diagrams: Simple molecules

QUESTION

Represent hydrogen chloride (HCl) using a Lewis diagram.

SOLUTION

Step 1: For each atom, determine the number of valence electrons in the atom, and represent these using dots and crosses.

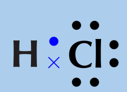
The electron configuration of hydrogen is $1s^1$ and the electron configuration for chlorine is $[He]2s^22p^5$. The hydrogen atom has 1 valence electron and the chlorine atom has 7 valence electrons.

The Lewis diagrams for hydrogen and chlorine are:

Notice the single unpaired electron (highlighted in blue) on each atom. This does not mean this electron is different, we use highlighting here to help you see the unpaired electron.

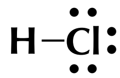
Step 2: Arrange the electrons so that the outermost energy level of each atom is full.

Hydrogen chloride is represented below.



Notice how the two unpaired electrons (one from each atom) form the covalent bond.

The dot and cross in between the two atoms, represent the pair of electrons that are shared in the covalent bond. We can also show this bond using a single line:



Note how we still show the other electron pairs around chlorine.

From this we can conclude that any electron on its own will try to pair up with another electron. So in practise atoms that have at least one unpaired electron can form bonds with any other atom that also has an unpaired electron. This is not restricted to just two atoms.

Worked example 2: Lewis diagrams: Simple molecules

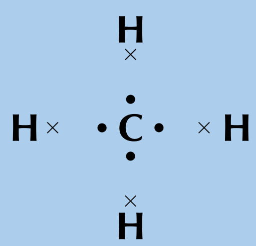
QUESTION

Represent methane (CH_4) using a Lewis diagram

SOLUTION

Step 1: For each atom, determine the number of valence electrons in the atom, and represent these using dots and crosses.

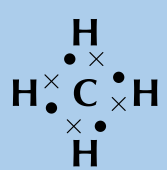
The electron configuration of hydrogen is $1s^1$ and the electron configuration for carbon is $[He]2s^22p^2$. Each hydrogen atom has 1 valence electron and the carbon atom has 4 valence electrons.



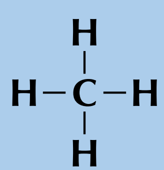
Remember that we said we can place unpaired electrons at any position (top, bottom, left, right) around the elements symbol.

Step 2: Arrange the electrons so that the outermost energy level of each atom is full.

The methane molecule is represented below.



or



Exercise 3 – 2:

Represent the following molecules using Lewis diagram:

1. chlorine (Cl_2)
2. boron trifluoride (BF_3)

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23NN 2. 23NP

www.everythingscience.co.za m.everythingscience.co.za

TIP
Notice how in this example we wrote a 2 in front of the hydrogen? Instead of writing the Lewis diagram for hydrogen twice, we simply write it once and use the 2 in front of it to indicate that two hydrogens are needed for each oxygen.

Case 2: Atoms with lone pairs

We will use water as an example. Water is made up of one oxygen and two hydrogen atoms. Hydrogen has one unpaired electron. Oxygen has two unpaired electrons and two electron pairs. From what we learnt in the first examples we see that the unpaired electrons can pair up. But what happens to the two pairs? Can these form bonds?

Worked example 3: Lewis diagrams: Simple molecules

QUESTION

Represent water (H_2O) using a Lewis diagram

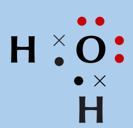
SOLUTION

Step 1: For each atom, determine the number of valence electrons in the atom, and represent these using dots and crosses.

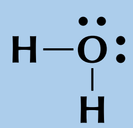
The electron configuration of hydrogen is $1s^1$ and the electron configuration for oxygen is $[He]2s^22p^4$. Each hydrogen atom has 1 valence electron and the oxygen atom has 6 valence electrons.

Step 2: Arrange the electrons so that the outermost energy level of each atom is full.

The water molecule is represented below.



or



TIP
A lone pair can be used to form a dative covalent bond.

And now we can answer the questions that we asked before the worked example. We see that oxygen forms two bonds, one with each hydrogen atom. Oxygen however keeps its electron pairs and does not share them. We can generalise this to any atom. If an atom has an electron pair it will normally not share that electron pair.

A **lone pair** is an unshared electron pair. A lone pair stays on the atom that it belongs to.

In the example above the lone pairs on oxygen are highlighted in red. When we draw the bonding pairs using lines it is much easier to see the lone pairs on oxygen.

Exercise 3 – 3:

Represent the following molecules using Lewis diagrams:

1. ammonia (NH_3)
2. oxygen difluoride (OF_2)

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23NQ 2. 23NR

www.everythingscience.co.za m.everythingscience.co.za

Case 3: Atoms with multiple bonds

We will use oxygen and hydrogen cyanide as examples.

Worked example 4: Lewis diagrams: Molecules with multiple bonds

QUESTION

Represent oxygen (O_2) using a Lewis diagram

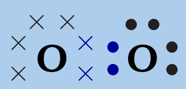
SOLUTION

Step 1: For each atom, determine the number of valence electrons that the atom has from its electron configuration.

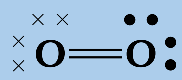
The electron configuration of oxygen is $[He]2s^22p^4$. Oxygen has 6 valence electrons.

Step 2: Arrange the electrons in the O_2 molecule so that the outermost energy level in each atom is full.

The O_2 molecule is represented below. Notice the two electron pairs between the two oxygen atoms (highlighted in blue). Because these two covalent bonds are between the same two atoms, this is a *double* bond.



or



Each oxygen atom uses its two unpaired electrons to form two bonds. This forms a double covalent bond (which is shown by a double line between the two oxygen atoms).

Worked example 5: Lewis diagrams: Molecules with multiple bonds

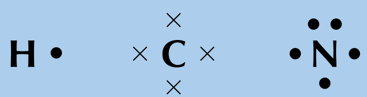
QUESTION

Represent hydrogen cyanide (HCN) using a Lewis diagram

SOLUTION

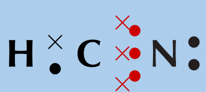
Step 1: For each atom, determine the number of valence electrons that the atom has from its electron configuration.

The electron configuration of hydrogen is $1s^1$, the electron configuration of nitrogen is $[He]2s^22p^3$ and for carbon is $[He]2s^22p^2$. Hydrogen has 1 valence electron, carbon has 4 valence electrons and nitrogen has 5 valence electrons.

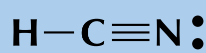


Step 2: Arrange the electrons in the HCN molecule so that the outermost energy level in each atom is full.

The HCN molecule is represented below. Notice the three electron pairs (highlighted in red) between the nitrogen and carbon atom. Because these three covalent bonds are between the same two atoms, this is a *triple* bond.



or



As we have just seen carbon shares one electron with hydrogen and three with nitrogen. Nitrogen keeps its electron pair and shares its three unpaired electrons with carbon.

Exercise 3 – 4:

Represent the following molecules using Lewis diagrams:

1. acetylene (C_2H_2)
2. formaldehyde (CH_2O)

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23NS 2. 23NT

www.everythingscience.co.za m.everythingscience.co.za

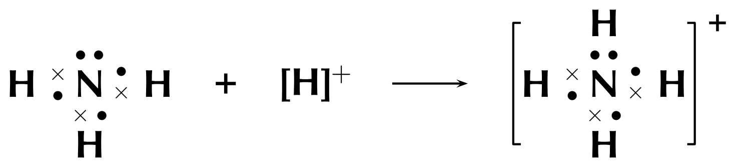
Case 4: Co-ordinate or dative covalent bonds

DEFINITION: *Dative covalent bond*

This type of bond is a description of covalent bonding that occurs between two atoms in which both electrons shared in the bond come from the same atom.

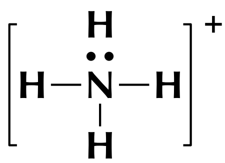
A dative covalent bond is also known as a coordinate covalent bond. Earlier we said that atoms with a pair of electrons will normally not share that pair to form a bond. But now we will see how an electron pair can be used by atoms to form a covalent bond.

One example of a molecule that contains a dative covalent bond is the ammonium ion (NH_4^+) shown in the figure below. The hydrogen ion H^+ does not contain any electrons, and therefore the electrons that are in the bond that forms between this ion and the nitrogen atom, come only from the nitrogen.



Notice that the hydrogen ion is charged and that this charge is shown on the ammonium ion using square brackets and a plus sign outside the square brackets.

We can also show this as:



Note that we do not use a line for the dative covalent bond.

Another example of this is the hydronium ion (H_3O^+).

To summarise what we have learnt:

- Any electron on its own will try to pair up with another electron. So in theory atoms that have at least one unpaired electron can form bonds with any other atom that also has an unpaired electron. This is not restricted to just two atoms.
- If an atom has an electron pair it will normally not share that pair to form a bond. This electron pair is known as a lone pair.
- If an atom has more than one unpaired electron it can form multiple bonds to another atom. In this way double and triple bonds are formed.
- A dative covalent bond can be formed between an atom with no electrons and an atom with a lone pair.

See simulation: 23NV at www.everythingscience.co.za

Exercise 3 – 5: Atomic bonding and Lewis diagrams

1. Represent each of the following *atoms* using Lewis diagrams:

   a) calcium
   b) lithium
   c) phosphorous
   d) potassium
   e) silicon
   f) sulfur

2. Represent each of the following *molecules* using Lewis diagrams:

   a) bromine (Br_2)
   b) carbon dioxide (CO_2)
   c) nitrogen (N_2)
   d) hydronium ion (H_3O^+)
   e) sulfur dioxide (SO_2)

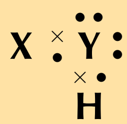
3. Two chemical reactions are described below.

   - nitrogen and hydrogen react to form NH_3
   - carbon and hydrogen bond to form CH_4

   For each reaction, give:

   a) the number of valence electrons for each of the atoms involved in the reaction
   b) the Lewis diagram of the product that is formed
   c) the name of the product

4. A chemical compound has the following Lewis diagram:



   a) How many valence electrons does element Y have?
   b) How many valence electrons does element X have?
   c) How many covalent bonds are in the molecule?
   d) Suggest a name for the elements X and Y.

5. Complete the following table:

Compound	CO_2	CF_4	HI	C_2H_2
Lewis diagram				
Total number of bonding pairs				
Total number of non-bonding pairs				
Single, double or triple bonds				

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 23NW 1b. 23NX 1c. 23NY 1d. 23NZ 1e. 23P2 1f. 23P3
2a. 23P4 2b. 23P5 2c. 23P6 2d. 23P7 2e. 23P8 3. 23P9
4. 23PB 5. 23PC

www.everythingscience.co.za m.everythingscience.co.za

3.2 Molecular shape ESBM9

Molecular shape (the shape that a single molecule has) is important in determining how the molecule interacts and reacts with other molecules. Molecular shape also influences the boiling point and melting point of molecules. If all molecules were linear then life as we know it would not exist. Many of the properties of molecules come from the particular shape that a molecule has. For example if the water molecule was linear, it would be non-polar and so would not have all the special properties it has.

Valence shell electron pair repulsion (VSEPR) theory ESBMB

The shape of a covalent molecule can be predicted using the Valence Shell Electron Pair Repulsion (VSEPR) theory. Very simply, VSEPR theory says that the valence electron pairs in a molecule will arrange themselves around the central atom(s) of the molecule so that the repulsion between their negative charges is as small as possible.

In other words, the valence electron pairs arrange themselves so that they are as **far apart** as they can be.

DEFINITION: *Valence Shell Electron Pair Repulsion Theory*

Valence shell electron pair repulsion (VSEPR) theory is a model in chemistry, which is used to predict the shape of individual molecules. VSEPR is based upon minimising the extent of the electron-pair repulsion around the central atom being considered.

VSEPR theory is based on the idea that the geometry (shape) of a molecule is mostly determined by repulsion among the pairs of electrons around a central atom. The pairs of electrons may be bonding or non-bonding (also called lone pairs). Only valence electrons of the central atom influence the molecular shape in a meaningful way.

Determining molecular shape ESBMC

To predict the shape of a covalent molecule, follow these steps:

1. Draw the molecule using a Lewis diagram. Make sure that you draw *all* the valence electrons around the molecule's central atom.
2. Count the number of electron pairs around the central atom.
3. Determine the basic geometry of the molecule using the table below. For example, a molecule with two electron pairs (and no lone pairs) around the central atom has a *linear* shape, and one with four electron pairs (and no lone pairs) around the central atom would have a *tetrahedral* shape.

TIP
The central atom is the atom around which the other atoms are arranged. So in a molecule of water, the central atom is oxygen. In a molecule of ammonia, the central atom is nitrogen.

The table below gives the common molecular shapes. In this table we use **A** to represent the central atom, **X** to represent the terminal atoms (i.e. the atoms around the central atom) and **E** to represent any lone pairs.

Number of bonding electron pairs	Number of lone pairs	Geometry	General formula
1 or 2	0	linear	AX or AX_2
2	2	*bent or angular*	AX_2E_2
3	0	trigonal planar	AX_3
3	1	*trigonal pyramidal*	AX_3E
4	0	tetrahedral	AX_4
5	0	trigonal bipyramidal	AX_5
6	0	octahedral	AX_6

Table 3.1: The effect of electron pairs in determining the shape of molecules. Note that in the general example A is the central atom and X represents the terminal atoms.

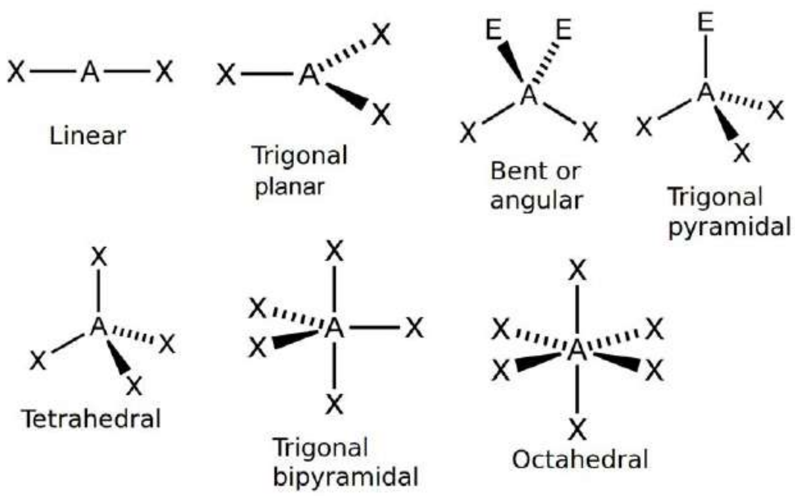


Figure 3.7: The common molecular shapes.

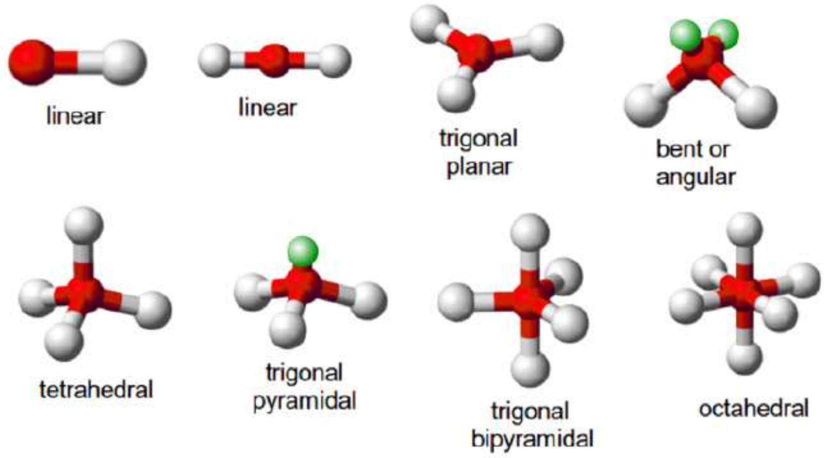


Figure 3.8: The common molecular shapes in 3-D.

In Figure 3.8 the green balls represent the lone pairs (E), the white balls (X) are the terminal atoms and the red balls (A) are the center atoms.

Of these shapes, the ones with no lone pairs are called the **ideal shapes**. The five ideal shapes are: linear, trigonal planar, tetrahedral, trigonal bypramidal and octahedral.

One important point to note about molecular shape is that all diatomic (compounds with two atoms) compounds are **linear**. So H_2, HCl and Cl_2 are all linear.

See simulation: 23PD at www.everythingscience.co.za

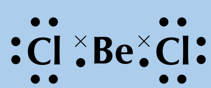
Worked example 6: Molecular shape

QUESTION

Determine the shape of a molecule of $BeCl_2$

SOLUTION

Step 1: Draw the molecule using a Lewis diagram



The central atom is beryllium.

Step 2: Count the number of electron pairs around the central atom

There are two electron pairs.

Step 3: Determine the basic geometry of the molecule

There are two electron pairs and no lone pairs around the central atom. $BeCl_2$ has the general formula: AX_2. Using this information and Table 3.1 we find that the molecular shape is linear.

Step 4: Write the final answer

The molecular shape of $BeCl_2$ is linear.

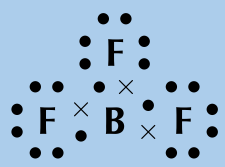
Worked example 7: Molecular shape

QUESTION

Determine the shape of a molecule of BF_3

SOLUTION

Step 1: Draw the molecule using a Lewis diagram



The central atom is boron.

Step 2: Count the number of electron pairs around the central atom

There are three electron pairs.

Step 3: Determine the basic geometry of the molecule

There are three electron pairs and no lone pairs around the central atom. The molecule has the general formula AX_3. Using this information and Table 3.1 we find that the molecular shape is trigonal planar.

Step 4: Write the final answer

The molecular shape of BF_3 is trigonal planar.

Worked example 8: Molecular shape

QUESTION

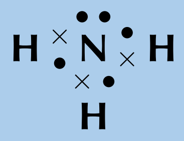
Determine the shape of a molecule of NH_3

SOLUTION

Step 1: Draw the molecule using a Lewis diagram

TIP

We can also work out the shape of a molecule with double or triple bonds. To do this, we count the double or triple bond as one pair of electrons.



The central atom is nitrogen.

Step 2: Count the number of electron pairs around the central atom

There are four electron pairs.

Step 3: Determine the basic geometry of the molecule

There are three bonding electron pairs and one lone pair. The molecule has the general formula AX_3E. Using this information and Table 3.1 we find that the molecular shape is trigonal pyramidal.

Step 4: Write the final answer

The molecular shape of NH_3 is trigonal pyramidal.

Group discussion: Building molecular models

In groups, you are going to build a number of molecules using jellytots to represent the atoms in the molecule, and toothpicks to represent the bonds between the atoms. In other words, the toothpicks will hold the atoms (jellytots) in the molecule together. Try to use different coloured jellytots to represent different elements.

You will need jellytots, toothpicks, labels or pieces of paper.

On each piece of paper, write the words: "lone pair".

You will build models of the following molecules:

HCl, CH_4, H_2O, BF_3, PCl_5, SF_6 and NH_3.

For each molecule, you need to:

- Determine the molecular geometry of the molecule
- Build your model so that the atoms are as far apart from each other as possible (remember that the electrons around the central atom will try to avoid the repulsions between them).
- Decide whether this shape is accurate for that molecule or whether there are any lone pairs that may influence it. If there are lone pairs then add a toothpick to the central jellytot. Stick a label (i.e. the piece of paper with "lone pair" on it) onto this toothpick.

- Adjust the position of the atoms so that the bonding pairs are further away from the lone pairs.
- How has the shape of the molecule changed?
- Draw a simple diagram to show the shape of the molecule. It doesn't matter if it is not 100% accurate. This exercise is only to help you to visualise the 3-dimensional shapes of molecules. (See Figure 3.8 to help you).

Do the models help you to have a clearer picture of what the molecules look like? Try to build some more models for other molecules you can think of.

TIP
Depending on which source you use for electronegativities you may see slightly different values.

Exercise 3 – 6: Molecular shape

Determine the shape of the following molecules.

1. $BeCl_2$
2. F_2
3. PCl_5
4. SF_6
5. CO_2
6. CH_4
7. H_2O
8. COH_2

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23PF 2. 23PG 3. 23PH 4. 23PJ 5. 23PK 6. 23PM
7. 23PN 8. 23PP

www.everythingscience.co.za **m.everythingscience.co.za**

3.3 Electronegativity ESBMD

So far we have looked at covalent molecules. But how do we know that they are covalent? The answer comes from electronegativity. Each element (except for the noble gases) has an electronegativity value.

Electronegativity is a measure of how strongly an atom pulls a shared electron pair towards it. The table below shows the electronegativities of the some of the elements.

For a full list of electronegativities see the periodic table at the front of the book. On this periodic table the electronegativity values are given in the top right corner. Do not confuse these values with the other numbers shown for the elements. Electronegativities will always be between 0 and 4 for any element. If you use a number greater than 4 then you are not using the electronegativity.

FACT
The concept of electronegativity was introduced by *Linus Pauling* in 1932, and this became very useful in explaining the nature of bonds between atoms in molecules. For this work, Pauling was awarded the Nobel Prize in Chemistry in 1954. He also received the Nobel Peace Prize in 1962 for his campaign against above-ground nuclear testing.

Element	Electronegativity	Element	Electronegativity
Hydrogen (H)	2,1	Lithium (Li)	1,0
Beryllium (Be)	1,5	Boron (B)	2,0
Carbon (C)	2,5	Nitrogen (N)	3,0
Oxygen (O)	3,5	Fluorine (F)	4,0
Sodium (Na)	0,9	Magnesium (Mg)	1,2
Aluminium (Al)	1,5	Silicon (Si)	1,8
Phosphorous (P)	2,1	Sulfur (S)	2,5
Chlorine (Cl)	3,0	Potassium (K)	0,8
Calcium (Ca)	1,0	Bromine (Br)	2,8

Table 3.2: Table of electronegativities for selected elements.

DEFINITION: *Electronegativity*

Electronegativity is a chemical property which describes the power of an atom to attract electrons towards itself.

The greater the electronegativity of an atom of an element, the stronger its attractive pull on electrons. For example, in a molecule of hydrogen bromide (HBr), the electronegativity of bromine (2,8) is higher than that of hydrogen (2,1), and so the shared electrons will spend more of their time closer to the bromine atom. Bromine will have a slightly negative charge, and hydrogen will have a slightly positive charge. In a molecule like hydrogen (H_2) where the electronegativities of the atoms in the molecule are the same, both atoms have a neutral charge.

Worked example 9: Calculating electronegativity differences

QUESTION

Calculate the electronegativity difference between hydrogen and oxygen.

SOLUTION

Step 1: Read the electronegativity of each element off the periodic table.

From the periodic table we find that hydrogen has an electronegativity of 2,1 and oxygen has an electronegativity of 3,5.

Step 2: Calculate the electronegativity difference

$3,5 - 2,1 = 1,4$

Exercise 3 – 7:

1. Calculate the electronegativity difference between:
   Be and C; H and C; Li and F; Al and Na; C and O.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23PQ

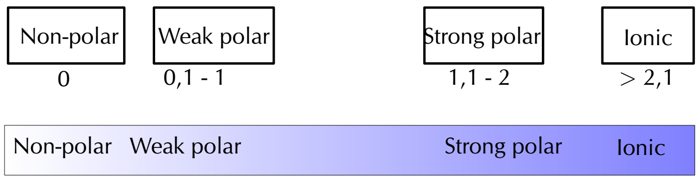
www.everythingscience.co.za   m.everythingscience.co.za

TIP
Note that metallic bonding is not given here. Metals have low electronegativities and so the valence electrons are not drawn strongly to any one atom. Instead, the valence electrons are loosely shared by all the atoms in the metallic network.

TIP
The symbol δ is read as delta.

Electronegativity and bonding ESBMF

The electronegativity difference between two atoms can be used to determine what type of bonding exists between the atoms. The table below lists the approximate values. Although we have given ranges here bonding is more like a spectrum than a set of boxes.



Electronegativity difference	Type of bond
0	Non-polar covalent
0 - 1	Weak polar covalent
1,1 - 2	Strong polar covalent
$> 2,1$	Ionic

Non-polar and polar covalent bonds ESBMG

It is important to be able to determine if a molecule is polar or non-polar since the *polarity* of molecules affects properties such as *solubility*, *melting points* and *boiling points*.

Electronegativity can be used to explain the difference between two types of covalent bonds. **Non-polar covalent bonds** occur between two identical non-metal atoms, e.g. H_2, Cl_2 and O_2. Because the two atoms have the same electronegativity, the electron pair in the covalent bond is shared equally between them. However, if two different non-metal atoms bond then the shared electron pair will be pulled more strongly by the atom with the higher electronegativity. As a result, a **polar covalent bond** is formed where one atom will have a slightly negative charge and the other a slightly positive charge.

This slightly positive or slightly negative charge is known as a partial charge. These partial charges are represented using the symbols δ^+ (slightly positive) and δ^- (slightly negative). So, in a molecule such as hydrogen chloride (HCl), hydrogen is H^{δ^+} and chlorine is Cl^{δ^-}.

TIP
To determine if a molecule is symmetrical look first at the atoms around the central atom. If they are different then the molecule is not symmetrical. If they are the same then the molecule may be symmetrical and we need to look at the shape of the molecule.

Polar molecules

ESBMH

Some molecules with polar covalent bonds are **polar molecules**, e.g. H_2O. But not *all* molecules with polar covalent bonds are polar. An example is CO_2. Although CO_2 has two polar covalent bonds (between $C^{\delta+}$ atom and the two $O^{\delta-}$ atoms), the molecule itself is not polar. The reason is that CO_2 is a linear molecule, with both terminal atoms the same, and is therefore symmetrical. So there is no difference in charge between the two ends of the molecule.

DEFINITION: *Polar molecules*

A **polar molecule** is one that has one end with a slightly positive charge, and one end with a slightly negative charge. Examples include water, ammonia and hydrogen chloride.

DEFINITION: *Non-polar molecules*

A **non-polar molecule** is one where the charge is equally spread across the molecule or a symmetrical molecule with polar bonds. Examples include carbon dioxide and oxygen.

We can easily predict which molecules are likely to be polar and which are likely to be non-polar by looking at the molecular shape. The following activity will help you determine this and will help you understand more about symmetry.

Activity: Polar and non-polar molecules

The following table lists the molecular shapes. Build the molecule given for each case using jellytots and toothpicks. Determine if the shape is symmetrical. (Does it look the same whichever way you look at it?) Now decide if the molecule is polar or non-polar.

Geometry	Molecule	Symmetrical	Polar or non-polar
Linear	HCl		
Linear	CO_2		
Linear	HCN		
Bent or angular	H_2O		
Trigonal planar	BF_3		
Trigonal planar	BF_2Cl		
Trigonal pyramidal	NH_3		
Tetrahedral	CH_4		
Tetrahedral	CH_3Cl		
Trigonal bipyramidal	PCl_5		
Trigonal bipyramidal	PCl_4F		
Octahedral	SF_6		
Octahedral	SF_5Cl		

See simulation: 23PR at www.everythingscience.co.za

Worked example 10: Polar and non-polar molecules

QUESTION

State whether hydrogen (H_2) is polar or non-polar.

SOLUTION

Step 1: Determine the shape of the molecule

The molecule is linear. There is one bonding pair of electrons and no lone pairs.

Step 2: Write down the electronegativities of each atom

Hydrogen: 2,1

Step 3: Determine the electronegativity difference for each bond

There is only one bond and the difference is 0.

Step 4: Determine the polarity of each bond

The bond is non-polar.

Step 5: Determine the polarity of the molecule

The molecule is non-polar.

Worked example 11: Polar and non-polar molecules

QUESTION

State whether methane (CH_4) is polar or non-polar.

SOLUTION

Step 1: Determine the shape of each molecule

The molecule is tetrahedral. There are four bonding pairs of electrons and no lone pairs.

Step 2: Determine the electronegativity difference for each bond

There are four bonds. Since each bond is between carbon and hydrogen, we only need to calculate one electronegativity difference. This is: $2,5 - 2,1 = 0,4$

Step 3: Determine the polarity of each bond

Each bond is polar.

Step 4: Determine the polarity of the molecule

The molecule is symmetrical and so is non-polar.

Worked example 12: Polar and non-polar molecules

QUESTION

State whether hydrogen cyanide (HCN) is polar or non-polar.

SOLUTION

Step 1: Determine the shape of the molecule

The molecule is linear. There are four bonding pairs, three of which form a triple bond and so are counted as 1. There is one lone pair on the nitrogen atom.

Step 2: Determine the electronegativity difference and polarity for each bond

There are two bonds. One between hydrogen and carbon and the other between carbon and nitrogen. The electronegativity difference between carbon and hydrogen is 0,4 and the electronegativity difference between carbon and nitrogen is 0,5. Both of the bonds are polar.

Step 3: Determine the polarity of the molecule

The molecule is not symmetrical and so is polar.

Exercise 3 – 8: Electronegativity

1. In a molecule of beryllium chloride ($BeCl_2$):
   a) What is the electronegativity of beryllium?
   b) What is the electronegativity of chlorine?
   c) Which atom will have a slightly positive charge and which will have a slightly negative charge in the molecule? Represent this on a sketch of the molecule using partial charges.
   d) Is the bond a non-polar or polar covalent bond?
   e) Is the molecule polar or non-polar?

2. Complete the table below:

Molecule	Difference in electronegativity between atoms	Non-polar/polar covalent bond	Polar/non-polar molecule
H_2O			
HBr			
F_2			
CH_4			
PF_5			
$BeCl_2$			
CO			
C_2H_2			
SO_2			
BF_3			

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23PS 2. 23PT

www.everythingscience.co.za m.everythingscience.co.za

3.4 Energy and bonding ESBMJ

As we saw earlier in the chapter we can show the energy changes that occur as atoms come together (Figure 3.5). Shown below is the same image but this time with two extra pieces of information: the bond energy and the bond length.

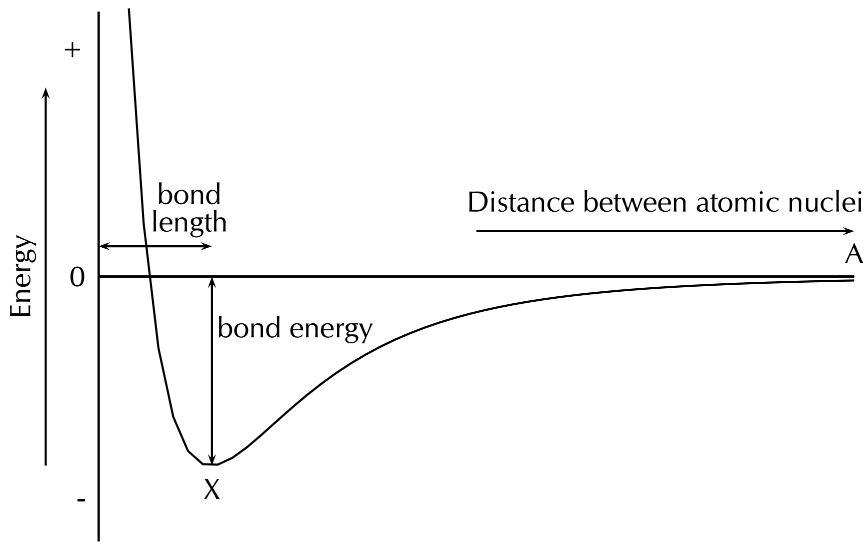


Figure 3.9: Graph showing the change in energy that takes place as atoms move closer together.

DEFINITION: *Bond length*

The distance between the nuclei of two adjacent atoms when they bond.

DEFINITION: *Bond energy*

The amount of energy that must be added to the system to break the bond that has formed.

It is important to remember that bond length is measured between two atoms that are bonded to each other. The following diagrams show the bond length for CO and for CO_2. The grey circle represents carbon and the white circle represents oxygen.

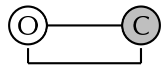


Figure 3.10:

The bond length for carbon monoxide (CO).

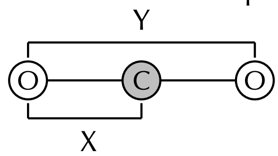


Figure 3.11:

The bond length for each C – O bond in carbon dioxide (CO_2) is indicated by X. Y **is not** the bond length.

A third property of bonds is the bond strength. **Bond strength** means how strongly one atom attracts and is held to another. The strength of a bond is related to the bond length, the size of the bonded atoms and the number of bonds between the atoms. In general:

- the shorter the bond length, the stronger the bond between the atoms.
- the smaller the atoms involved, the stronger the bond.
- the more bonds that exist between the same atoms, the stronger the bond.

3.5 Chapter summary ESBMK

See presentation: 23PV at www.everythingscience.co.za

- A **chemical bond** is the physical process that causes atoms to be attracted together and to be bound in new compounds.
- The noble gases have a full valence shell. Atoms bond to try fill their outer valence shell.
- There are three **forces** that act between atoms: attractive forces between the positive nucleus of one atom and the negative electrons of another; repulsive forces between like-charged electrons, and repulsion between like-charged nuclei.
- The **energy** of a system of two atoms is at a minimum when the attractive and repulsive forces are balanced.
- **Lewis** diagrams are one way of representing molecular structure. In a Lewis diagram, dots or crosses are used to represent the valence electrons around the central atom.

- A **covalent bond** is a form of chemical bond where pairs of electrons are shared between two atoms.
- A **single** bond occurs if there is one electron pair that is shared between the same two atoms.
- A **double** bond occurs if there are two electron pairs that are shared between the same two atoms.
- A **triple** bond occurs if there are three electron pairs that are shared between the same two atoms.
- A **dative covalent bond** is a description of covalent bonding that occurs between two atoms in which both electrons shared in the bond come from the same atom.
- Dative covalent bonds occur between atoms of elements with a lone pair and atoms of elements with no electrons. Examples include the hydronium ion (H_3O^+) and the ammonium ion (NH_4^+).
- The **shape of molecules** can be predicted using the VSEPR theory.
- Valence shell electron pair repulsion (VSEPR) theory is a model in chemistry, which is used to predict the shape of individual molecules. VSEPR is based upon minimising the extent of the electron-pair repulsion around the central atom being considered.
- **Electronegativity** is a chemical property which describes the power of an atom to attract electrons towards itself in a chemical.
- Electronegativity can be used to explain the difference between two types of covalent bonds: **polar covalent bonds** (between non-identical atoms) and **non-polar covalent bonds** (between identical atoms or atoms with the same electronegativity).
- A **polar** molecule is one that has one end with a slightly positive charge, and one end with a slightly negative charge. Examples include water, ammonia and hydrogen chloride.
- A **non-polar** molecule is one where the charge is equally spread across the molecule or a symmetrical molecule with polar bonds.
- **Bond length** is the distance between the nuclei of two atoms when they bond.
- **Bond energy** is the amount of energy that must be added to the system to break the bond that has formed.
- **Bond strength** means how strongly one atom attracts and is held to another atom. Bond strength depends on the length of the bond, the size of the atoms and the number of bonds between the two atoms.

Exercise 3 – 9:

1. Give **one word/term** for each of the following descriptions.
   a) The distance between two adjacent atoms in a molecule.
   b) A type of chemical bond that involves the sharing of electrons between two atoms.

c) A measure of an atom's ability to attract electrons to itself in a chemical bond.

2. Which ONE of the following best describes the bond formed between an H^+ ion and the NH_3 molecule?

   a) Covalent bond
   b) Dative covalent (co-ordinate covalent) bond
   c) Ionic Bond
   d) Hydrogen Bond

3. Explain the meaning of each of the following terms:

   a) valence electrons
   b) bond energy
   c) covalent bond

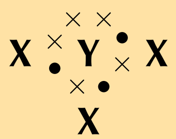
4. Which of the following reactions will *not* take place? Explain your answer.

   a) $H + H \rightarrow H_2$
   b) $Ne + Ne \rightarrow Ne_2$
   c) $Cl + Cl \rightarrow Cl_2$

5. Draw the Lewis diagrams for each of the following:

   a) An atom of strontium (Sr). (Hint: Which group is it in? It will have an identical Lewis diagram to other elements in that group).
   b) An atom of iodine.
   c) A molecule of hydrogen bromide (HBr).
   d) A molecule of nitrogen dioxide (NO_2). (Hint: There will be a single unpaired electron).

6. Given the following Lewis diagram, where X and Y each represent a different element:



   a) How many valence electrons does X have?
   b) How many valence electrons does Y have?
   c) Which elements could X and Y represent?

7. Determine the shape of the following molecules:

a) O_2	c) BCl_3	e) CCl_4	g) Br_2
b) MgI_2	d) CS_2	f) CH_3Cl	h) SCl_5F

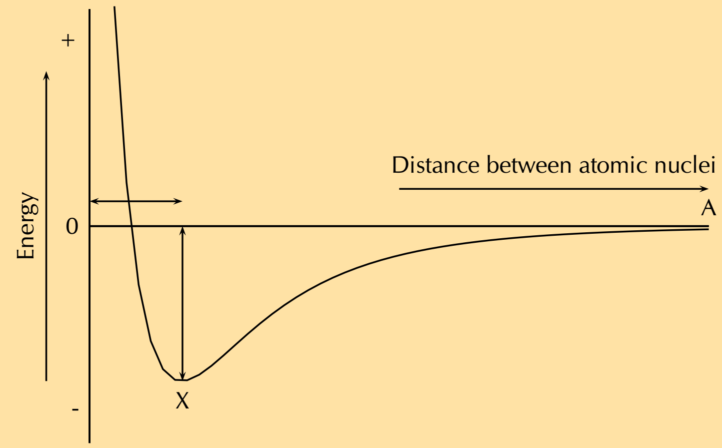
8. Complete the following table:

Element pair	Electronegativity difference	Type of bond that could form
Hydrogen and lithium		
Hydrogen and boron		
Hydrogen and oxygen		
Hydrogen and sulfur		
Magnesium and nitrogen		
Magnesium and chlorine		
Boron and fluorine		
Sodium and fluorine		
Oxygen and nitrogen		
Oxygen and carbon		

9. Are the following molecules polar or non-polar?

   a) O_2 b) $MgBr_2$ c) BF_3 d) CH_2O

10. Given the following graph for hydrogen:



   a) The bond length for hydrogen is 74 pm. Indicate this value on the graph. (Remember that pm is a picometer and means 74×10^{-12} m).
   The bond energy for hydrogen is 436 $kJ \cdot mol^{-1}$. Indicate this value on the graph.
   b) What is important about point X?

11. Hydrogen chloride has a bond length of 127 pm and a bond energy of 432 $kJ \cdot mol^{-1}$. Draw a graph of energy versus distance and indicate these values on your graph. The graph does not have to be accurate, a rough sketch graph will do.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 23PW 1b. 23PX 1c. 23PY 2. 23PZ 3a. 23Q2 3b. 23Q3
3c. 23Q4 4. 23Q5 5a. 23Q6 5b. 23Q7 5c. 23Q8 5d. 23Q9
6. 23QB 7a. 23QC 7b. 23QD 7c. 23QF 7d. 23QG 7e. 23QH
7f. 23QJ 7g. 23QK 7h. 23QM 8. 23QN 9a. 23QP 9b. 23QQ
9c. 23QR 9d. 23QS 10. 23QT 11. 23QV

www.everythingscience.co.za m.everythingscience.co.za

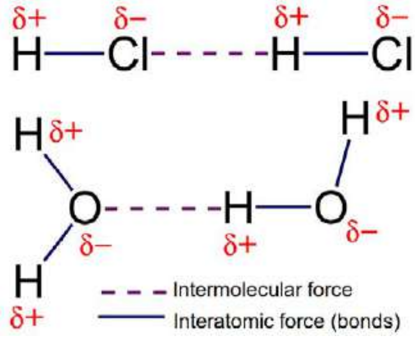
CHAPTER 4

Intermolecular forces

All around us we see matter in different phases. The air we breathe is a gas, while the water you drink is a liquid and the chair you are sitting on is a solid. In this chapter we are going to look at one of the reasons that matter exists as solids and liquids.

In the previous chapter, we discussed the different forces that exist *between atoms* (interatomic forces). When atoms are joined to one another they form molecules, and these molecules in turn have forces that bind them together. These forces are known as **intermolecular forces**.

Intermolecular forces allow us to determine which substances are likely to dissolve in which other substances and what the melting and boiling points of substances are. Without intermolecular forces holding molecules together we would not exist. Note that we will use the term molecule throughout this chapter as the compounds we are looking at are all covalently bonded and do not exist as giant networks (recall from grade 10 that there are three types of bonding: metallic, ionic and covalent). Sometimes you will see the term simple molecule. This is a covalent molecular structure.



NOTE:
Interatomic (between atoms) forces are also known as intramolecular (within molecules) forces. You can remember this by thinking of **inter**national which means between nations.

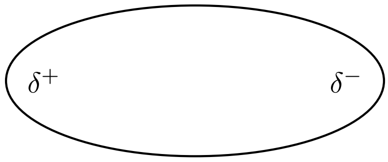
4.1 Intermolecular and interatomic forces ESBMM

DEFINITION: *Intermolecular forces*

Intermolecular forces are forces that act between molecules.

You will also recall from the previous chapter, that we can describe molecules as being either **polar** or **non-polar**. A polar molecule is one in which there is a difference in electronegativity between the atoms in the molecule, such that the shared electron pair spends more time close to the atom that attracts it more strongly. The result is that one end of the molecule will have a slightly positive charge (δ^{+}), and the other end will have a slightly negative charge (δ^{-}). The molecule is said to be a **dipole**.

A dipole molecule is a molecule that has two (di) poles. One end of the molecule is slightly positive and the other is slightly negative. We can depict this very simply as an oval with one positive side and one negative. In reality however, the molecules do not look like this, they look more like the images in Figure 4.1.



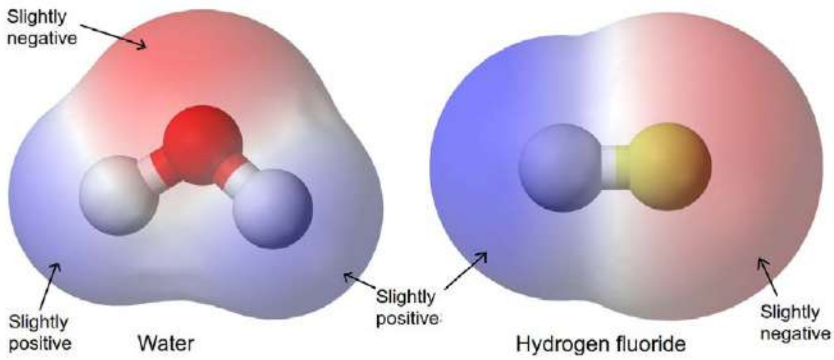


Figure 4.1: A different representation of dipole molecules. The red region is slightly negative, and the blue region is slightly positive.

It is important to remember that just because the bonds within a molecule are polar, the molecule itself may not necessarily be polar. The shape of the molecule may also affect its polarity. A few examples are shown in Table 4.1 to refresh your memory. Note that we have shown tetrahedral molecules with all the terminal atoms at 90° to each other (i.e. flat or 2-dimensional), but the shape is really 3-dimensional.

Molecule	Chemical formula	Bond between atoms	Shape of molecule	Polarity of molecule
Hydrogen	H_2	Non-polar covalent	Linear molecule **H** —— **H**	Non-polar
Hydrogen chloride	HCl	Polar covalent	Linear molecule $\mathbf{H}^{\delta+}$ —— $\mathbf{Cl}^{\delta-}$	Polar
Carbon tetrafluoride	CF_4	Polar covalent	Tetrahedral molecule $\mathbf{F}^{\delta-}$ | $\mathbf{F}^{\delta-}$ — $\mathbf{C}^{\delta+}$ — $\mathbf{F}^{\delta-}$ | $\mathbf{F}^{\delta-}$	Non-polar
Trifluoro-methane	CHF_3	Polar covalent	Tetrahedral molecule **F** | **H** — **C** — **F** | **F**	Polar

Table 4.1: Polarity in molecules with different atomic bonds and molecular shapes.

It is important to be able to recognise whether the molecules in a substance are polar or non-polar because this will determine what type of intermolecular forces there are. This is important in explaining the properties of the substance.

1. **Ion-dipole forces**

   As the name suggests, this type of intermolecular force exists between an ion and a dipole (polar) molecule. You will remember that an *ion* is a charged atom, and this will be attracted to one of the charged ends of the polar molecule. A positive ion will be attracted to the negative pole of the polar molecule, while a negative ion will be attracted to the positive pole of the polar molecule. This can be seen when sodium chloride (NaCl) dissolves in water. The positive sodium ion (Na^+) will be attracted to the slightly negative oxygen atoms in the water molecule, while the negative chloride ion (Cl^-) is attracted to the slightly positive hydrogen atoms. These intermolecular forces weaken the ionic bonds between the sodium and chloride ions so that the sodium chloride dissolves in the water (Figure 4.2).

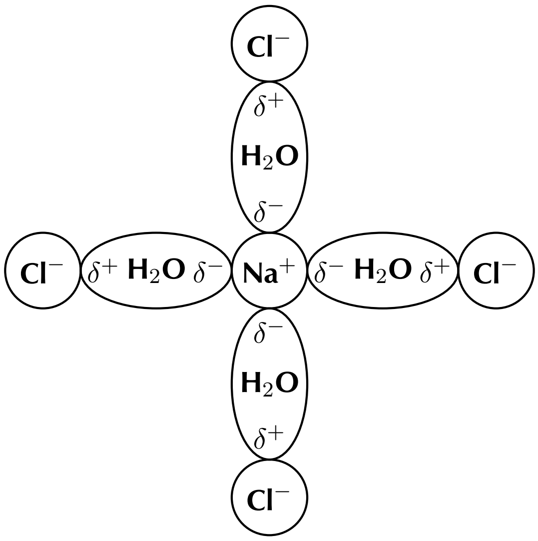
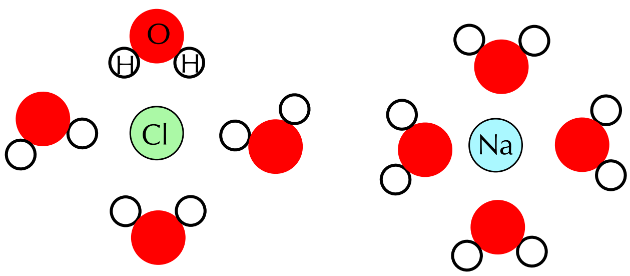


Figure 4.2: Ion-dipole forces in a sodium chloride solution.

   This is a simplified diagram to highlight the regions of positive and negative charge. When sodium chloride dissolves in water it can more accurately be shown as:



2. **Ion-induced-dipole forces**

   Similar to ion-dipole forces these forces exist between ions and non-polar molecules. The ion induces a dipole in the non-polar molecule leading to a weak, short lived force which holds the compounds together.

These forces are found in haemoglobin (the molecule that carries oxygen around your body). Haemoglobin has Fe^{2+} ions. Oxygen (O_2) is attracted to these ions by ion-induced dipole forces.

3. **Dipole-dipole forces**

   When one dipole molecule comes into contact with another dipole molecule, the positive pole of the one molecule will be attracted to the negative pole of the other, and the molecules will be held together in this way (Figure 4.3). Examples of materials/substances that are held together by dipole-dipole forces are HCl, SO_2 and CH_3Cl.

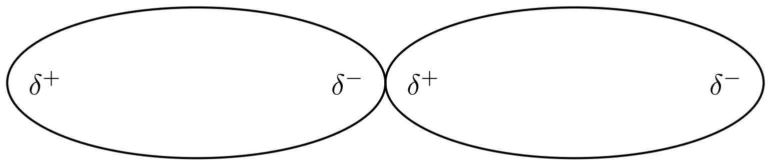


Figure 4.3: Two dipole molecules are held together by the attractive force between their oppositely charged poles.

   One special case of this is hydrogen bonding.

4. **Induced dipole forces**

   We know that while carbon dioxide is a non-polar molecule, we can still freeze it (and we can also freeze all other non-polar substances). This tells us that there must be some kind of attractive force in these kinds of molecules (molecules can only be solids or liquids if there are attractive forces pulling them together). This force is known as an induced dipole force.

   In non-polar molecules the electronic charge is usually evenly distributed but it is possible that at a particular moment in time, the electrons might not be evenly distributed (remember that the electrons are always moving in their orbitals). The molecule will have a *temporary dipole*. In other words, each end of the molecules has a slight charge, either positive or negative. When this happens, molecules that are next to each other attract each other very weakly. These forces are found in the halogens (e.g. F_2 and I_2) and in other non-polar molecules such as carbon dioxide and carbon tetrachloride.

   All covalent molecules have induced dipole forces. For non-polar covalent molecules these forces are the only intermolecular forces. For polar covalent molecules, dipole-dipole forces are found in addition to the induced dipole forces.

5. **Dipole-induced-dipole forces**

   This type of force occurs when a molecule with a dipole induces a dipole in a non-polar molecule. It is similar to an ion-induced dipole force. An example of this type of force is chloroform ($CHCl_3$) in carbon tetrachloride (CCl_4).

The following image shows the types of intermolecular forces and the kinds of compounds that lead to those forces.

FACT

These intermolecular forces are also sometimes called "London forces" or "momentary dipole" forces or "dispersion" forces.

TIP

When the noble gases condense, the intermolecular forces that hold the liquid together are induced dipole forces.

TIP
Do not confuse hydrogen bonds with actual chemical bonds. Hydrogen bonding is an example of a case where a scientist named something believing it to be one thing when in fact it was another. In this case the strength of the hydrogen bonds misled scientists into thinking this was actually a chemical bond, when it is really just an intermolecular force.

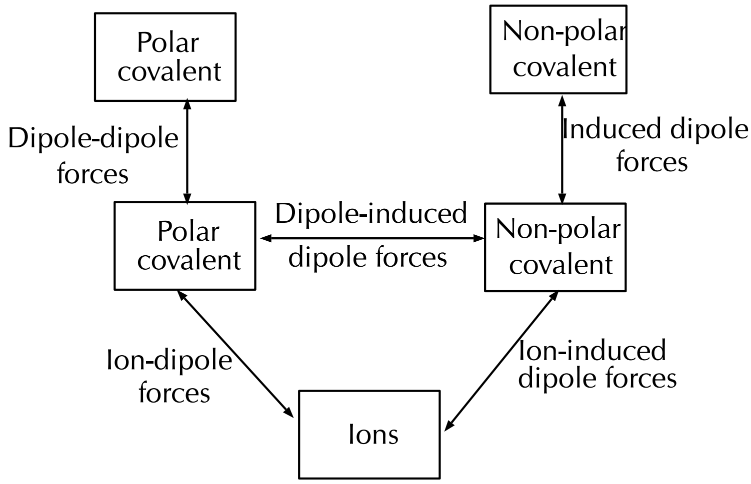


Figure 4.4: The types of intermolecular forces. The boxes represent the type of compound while the lines represent the type of force.

The last three forces (dipole-dipole forces, dipole-induced dipole forces and induced dipole forces) are sometimes collectively known as van der Waals' forces. We will now look at a special case of dipole-dipole forces in more detail.

Hydrogen bonds

As the name implies, this type of intermolecular bond involves a hydrogen atom. When a molecule contains a hydrogen atom covalently bonded to a highly electronegative atom (O, N or F) this type of intermolecular force can occur. The highly electronegative atom on one molecule attracts the hydrogen atom on a nearby molecule.

Water molecules for example, are held together by hydrogen bonds between the hydrogen atom of one molecule and the oxygen atom of another (Figure 4.5). Hydrogen bonds are a relatively strong intermolecular force and are stronger than other dipole-dipole forces. It is important to note however, that hydrogen bonds are weaker than the covalent and ionic bonds that exist between *atoms*.

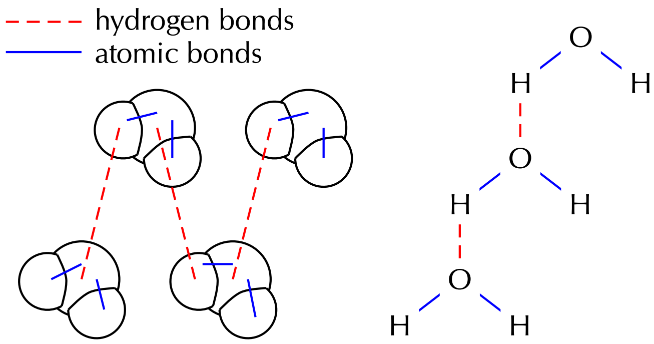


Figure 4.5: Two representations showing the hydrogen bonds between water molecules: space-filling model and structural formula.

The difference between intermolecular and interatomic forces ESBMP

It is important to realise that there is a difference between the types of interactions that occur in molecules and the types that occur between molecules. In the previous chapter we focused on the interactions between atoms. These are known as interatomic forces or chemical bonds. We also studied covalent molecules in more detail.

Remember that a covalent bond has an electronegativity difference of less than 2,1. Covalent molecules have covalent bonds between their atoms. Van der Waals' forces only occur in covalent molecules. We can show the interatomic and intermolecular forces between covalent compounds diagrammatically or in words. Intermolecular forces occur between molecules and do not involve individual atoms. Interatomic forces are the forces that hold the the atoms in molecules together. Figure 4.5 shows this.

	Interatomic forces	**Intermolecular forces**
Atoms or molecules	Forces between atoms	Forces between molecules
Strength of forces	Strong forces	Relatively weak forces
Distance between atoms or molecules	Very short distances	Larger distances than bonds

Table 4.2: The differences between interatomic and intermolecular forces.

Worked example 1: Intermolecular forces

QUESTION

Which intermolecular forces are found in carbon tetrachloride (CCl_4)?

SOLUTION

Step 1: Think about what you know about the molecule.

Carbon has an electronegativity of 2,5. Chlorine has an electronegativity of 3,0. The electronegativity difference between carbon and chlorine is 1,0 (recall the section on electronegativity in the previous chapter). We also know that the bond between carbon and chlorine is polar.

Also from the previous chapter we know that carbon tetrachloride is a tetrahedral molecule (recall molecular shape). Carbon tetrachloride is symmetrical and so is non-polar overall.

Step 2: Now decide which case it is

Carbon tetrachloride is non-polar and so the only kind of force that can exist is **induced dipole**.

Worked example 2: Intermolecular forces

QUESTION

Which intermolecular forces are found in the following solution: sodium chloride in water?

SOLUTION

Step 1: Think about what you know about the molecules

Sodium chloride is ionic. (the electronegativity difference is 2,1). Water has polar bonds (the electronegativity difference is 1,4). Water is a polar molecule (its molecular shape is bent or angular).

Step 2: Now decide which case it is

This is an ionic substance interacting with a polar substance. This interaction is an **ion-dipole** force.

Exercise 4 – 1:

Which intermolecular forces are found in:

1. hydrogen fluoride (HF)
2. methane (CH_4)
3. potassium chloride in ammonia (KCl in NH_3)
4. krypton (Kr)

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23QW 2. 23QX 3. 23QY 4. 23QZ

www.everythingscience.co.za m.everythingscience.co.za

Understanding intermolecular forces ESBMQ

The types of intermolecular forces that occur in a substance will affect its properties, such as its **phase**, **melting point** and **boiling point**. You should remember from the kinetic theory of matter (see grade 10), that the *phase* of a substance is determined by how strong the forces are between its particles. The weaker the forces, the more likely the substance is to exist as a gas. This is because the particles are able to move far apart since they are not held together very strongly. If the forces are very strong, the particles are held closely together in a solid structure. Remember also that the *temperature* of a material affects the energy of its particles. The more energy the particles have, the more likely they are to be able to overcome the forces that are holding them together. This can cause a change in phase.

Shown below are the three phases of water. Note that we are showing two dimensional figures when in reality these are three dimensional.

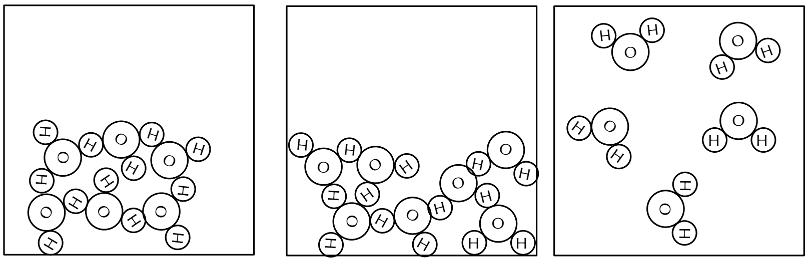


Figure 4.6: The three phases of water.

The effects of intermolecular forces

The following five experiments investigate the effect of various physical properties (evaporation, surface tension, solubility, boiling point and capillarity) of substances and determine how these properties relate to intermolecular forces. Each experiment will look at a different property.

Formal experiment: The effects of intermolecular forces: Part 1

Aim:

To investigate evaporation and to determine the relation between evaporation and intermolecular forces.

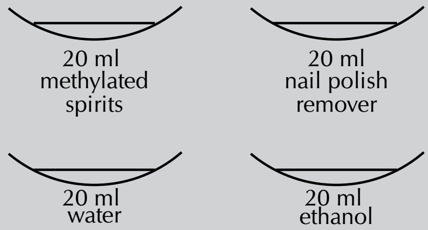
Apparatus:

You will need the following items for this experiment:

- ethanol, water, nail polish remover (acetone), methylated spirits
- evaporating dishes (or shallow basins)

Method:

1. Place 20 ml of each substance given in separate evaporating dishes.
2. Carefully move each dish to a warm (sunny) spot.
3. Mark the level of liquid in each dish using a permanent marker. Make several marks at different positions around the dish. If the permanent marker is leaving a smudge rather than a noticeable mark, carefully wipe the side of the dish and try again.
4. Observe each dish every minute and note which liquid evaporates fastest.



Results:

Record your results in the table below. You do not need to measure the level of the liquid, but rather just write how much the level had dropped (e.g. for water you might write did not notice any decrease in the level or for ethanol you might write almost all the liquid had evaporated).

Substance	1 min	2 min	3 min	4 min	5 min
Ethanol					
Water					
Nail polish remover					
Methylated spirits					

Discussion and conclusion:

You should find that water takes the longest time to evaporate. Water has strong intermolecular forces (hydrogen bonds). Ethanol (CH_3CH_2OH) and methylated spirits (mainly ethanol (CH_3CH_2OH) with some methanol (CH_3OH)) both have hydrogen bonds but these are slightly weaker than the hydrogen bonds in water. Nail polish remover (acetone (CH_3COCH_3)) has dipole-dipole forces only and so evaporates quickly.

Substances with weaker intermolecular forces evaporate faster than substances with stronger intermolecular forces.

Formal experiment: The effects of intermolecular forces: Part 2

Aim:

To investigate surface tension and to determine the relation between surface tension and intermolecular forces.

Apparatus:

You will need the following items for this experiment:

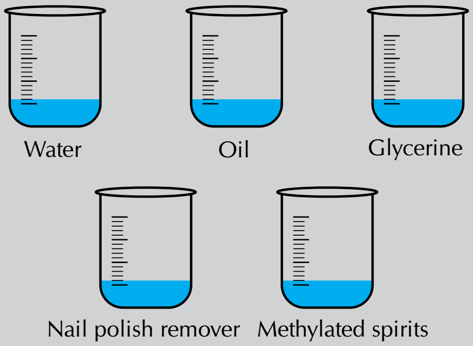
- water, cooking oil (sunflower oil), glycerin, nail polish remover (acetone), methylated spirits
- small glass beakers or glass measuring cylinders
- small piece of glass or clear plastic (about 5 cm by 5 cm.)

Method:

1. Place about 50 ml of each substance given in separate small beakers or measuring cylinders.

2. Observe the shape of the meniscus. (This is the level of the liquid). Note what happens at the edges where the liquid touches the glass. (You can place a few drops of food colouring in each substance to help you see the meniscus.)

3. Now place a drop of the substance on a small piece of glass. Observe the shape of the drop.



Results:

Record your results in the table below. You just need to give a qualitative result (in other words what you see in the experiment).

Substance	Shape of meniscus	Shape of droplet
Water		
Oil		
Glycerine		
Nail polish remover		
Methylated spirits		

Discussion and conclusion:

The meniscus for all these substances should be concave (i.e. higher at the edges than in the middle). This is because the forces holding the molecules in the substance together are weaker than the attraction between the substance and the glass of the tube.

You should also have noticed that water, oil and Glycerine tend to form a drop, while nail polish remover and methylated spirits do not. Strong intermolecular forces help hold the substance together, while weaker ones do not hold the molecules in the substance together as much.

Water has the strongest intermolecular forces (hydrogen bonds) of all the substances used. Glycerine and methylated spirits also have hydrogen bonds, but these intermolecular forces are slightly weaker than in water. Sunflower oil is mostly non-polar but has very long molecules which help account for the higher surface tension.

Substances with strong intermolecular forces will generally have a greater surface tension than substances with weaker intermolecular forces.

Formal experiment: The effects of intermolecular forces: Part 3

Aim:

To investigate solubility and to determine the relation between solubility and intermolecular forces.

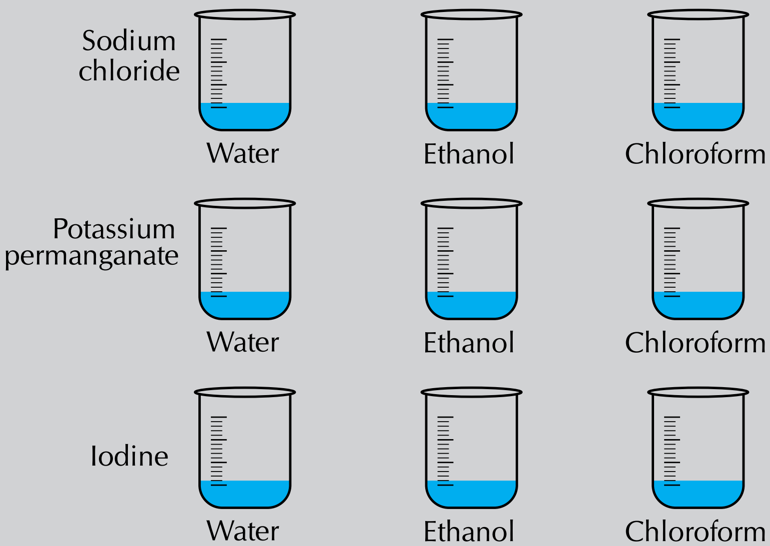
Apparatus:

You will need the following items for this experiment:

- Solids: sodium chloride (table salt), iodine, potassium permanganate
- Solvents: water, ethanol, chloroform
- 9 beakers or test-tubes
- 3 A4 sheets of paper

Method:

1. Place about 20 ml of each solvent given in separate beakers. Place this set on a piece of paper labelled "sodium chloride".
2. Repeat this step twice. The second set is for potassium permanganate (so your piece of paper will say "potassium permanganate") and the third set is for iodine (so your piece of paper will say "iodine"). You should now have nine beakers in total.
3. Into the first set, add about 2 g of sodium chloride.
4. Into the second set, add about 2 g of potassium permanganate.
5. Into the third set, add about 2 g of iodine.
6. Observe how much of each substance dissolves in the solvent.



Results:

Record your results in the table below. If you observe only a small amount of the solid dissolving then write that very little solid dissolved. If all the solid dissolves then write that all the solid dissolved.

Substance	Water	Chloroform	Ethanol
Sodium chloride			
Potassium permanganate			
Iodine			

Discussion and conclusion:

You should find that the sodium chloride and potassium permanganate dissolved (at least a bit) in all the substances. The iodine did not dissolve in any of the substances. The three solvents (water, chloroform and ethanol) are all polar and have dipole-dipole forces. Sodium chloride and potassium permanganate are both ionic substances, while iodine is non-polar.

Substances will dissolve in solvents that have similar intermolecular forces or in solvents where the ionic bonds can be disrupted by the formation of ion-dipole forces.

Formal experiment: The effects of intermolecular forces: Part 4

Aim:

To investigate boiling point and to determine the relation between boiling point and intermolecular forces.

Apparatus:

You will need the following items for this experiment:

- water, cooking oil (sunflower oil), Glycerine, nail polish remover, methylated spirits
- test-tubes and a beaker
- hot plate

Method:

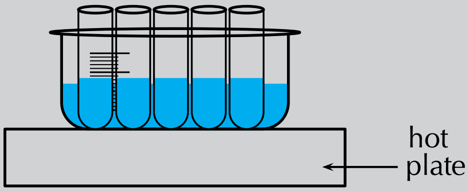
WARNING!

Methylated spirits and nail polish remover are highly flammable. They will easily catch fire if left near an open flame. For this reason they must be heated in a water bath. This experiment MUST be performed in a well ventilated room.

---

1. Place about 20 ml of each substance given in separate test-tubes.
2. Half-fill the beaker with water and place on the hot plate.
3. Place the test-tubes in the beaker.

4. Observe how long each substance takes to boil. As soon as a substance boils, remove it from the water bath.



Results:

Write down the order in which the substances boiled, starting with the substance that boiled first and ending with the substance that boiled last.

Discussion and conclusion:

You should have found that the nail polish remover and the methylated spirits boil before the water, oil and Glycerine.

Glycerine, water and methylated spirits have hydrogen bonds between the molecules. However, in water and Glycerine these intermolecular forces are very strong while in the methylated spirits they are slightly weaker. This leads to the higher boiling point for water and Glycerine. Nail polish remover has weaker dipole-dipole forces.

Although cooking oil is non-polar and has induced dipole forces the molecules are very large and so these increase the strength of the intermolecular forces.

Substances with strong intermolecular forces will have a higher boiling point than substances with weaker intermolecular forces.

Formal experiment: The effects of intermolecular forces: Part 5

Aim:

To investigate capillarity (how far up a tube a liquid rises or how far down a liquid falls) and to determine the relation between capillarity and intermolecular forces.

Apparatus:

You will need the following items for this experiment:

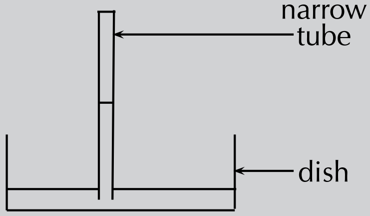
- water, cooking oil (sunflower oil), nail polish remover, methylated spirits
- large shallow dish, narrow glass tube (with one end closed)

Method:

1. Place about 20 ml of water in the shallow dish.
2. Hold the narrow tube just above the level of the water in the dish.

3. Observe how far up the tube the water travels.

4. Repeat for the other three substances, remembering to wash and dry the dish and tube well between each one.



Results:

Record your results in the table below. You do not need to measure how far up the tube the substance travels but rather say if it only travelled a short distance or a long distance.

Substance	Distance travelled up tube
Water	
Oil	
Nail polish remover	
Methylated spirits	

Discussion and conclusion:

Water travels the greatest distance up the tube. Nail polish remover travels the least distance.

Capillarity is related to surface tension. If the attractive force between the glass walls of the tube and the substance are stronger than the intermolecular forces in the substance, than the edges of the liquid will be pulled above the surface of the liquid. This in turn helps pull the liquid up the tube.

Substances with strong intermolecular forces will travel further up a narrow tube (have a greater capillarity) than substances with weaker intermolecular forces.

From these experiments we can see how intermolecular forces (a microscopic property) affect the macroscopic behaviour of substances. If a substance has weak intermolecular forces then it will evaporate easily. Substances with weak intermolecular forces also have low surface tension and do not rise as far up in narrow tubes as substances with strong intermolecular forces. Boiling points are lower for substances with weak intermolecular forces. Substances are more likely to be soluble in liquids with similar intermolecular forces.

We will now look at some more properties (molecular size, viscosity, density, melting and boiling points, thermal expansion, thermal conductivity) in detail.

FACT
It is partly the stronger intermolecular forces that explain why petrol (mainly octane (C_8H_{18})) is a liquid, while candle wax ($C_{23}H_{48}$) is a solid. If these intermolecular forces did not increase with increasing molecular size we would not be able to put liquid fuel into our cars or use solid candles.

Molecular size

The alkanes are a group of organic compounds that contain carbon and hydrogen bonded together. The carbon atoms link together to form chains of varying lengths.

The boiling point and melting point of these molecules is determined by their molecular structure, and their surface area. The more carbon atoms there are in an alkane, the greater the surface area and therefore the higher the boiling point. The melting point also increases as the number of carbon atoms in the molecule increases. This can be seen in the table below.

Formula	CH_4	C_2H_6	C_5H_{12}	C_6H_{14}	$C_{20}H_{42}$
Name	methane	ethane	pentane	hexane	icosane
Molecular mass (g·mol^{-1})	16	30	72	86	282
Melting point (°C)	-183	-183	-130	-95	37
Boiling point (°C)	-164	-89	36	69	343
Phase at room temperature	gas	gas	liquid	liquid	solid

You will also notice that, when the molecular mass of the alkanes is low (i.e. there are few carbon atoms), the organic compounds are *gases* because the intermolecular forces are weak. As the number of carbon atoms and the molecular mass increases, the compounds are more likely to be liquids or solids because the intermolecular forces are stronger.

You should see that the larger a molecule is the stronger the intermolecular forces are between its molecules. This is one of the reasons why methane (CH_4) is a gas at room temperature while pentane (C_5H_{12}) is a liquid and icosane ($C_{20}H_{42}$) is a solid.

Viscosity

Viscosity is the resistance to flow of a liquid. Compare how easy it is to pour water and syrup or honey. The water flows much faster than the syrup or honey.

You can see this if you take a cylinder filled with water and a cylinder filled with glycerin. Drop a small metal ball into each cylinder and note how easy it is for the ball to fall to the bottom. In the glycerin the ball falls slowly, while in the water it falls faster.

Substances with stronger intermolecular forces are more viscous than substances with weaker intermolecular forces.

Activity: Machine and motor oils

You are given the following information about engine oils.

Oil	Use	Other info
SAE 30 monograde	Engines	Low viscosity
SAE 50 monograde	Engines	High viscosity
SAE 15W-40 multigrade	Engines	Medium viscosity
SAE 0W-40 multigrade	Engines	Medium viscosity

Multigrade oils can be used even in cold weather since they remain fluid (The first number is the rating for winter weather and the W shows that this is the rating in winter. The second number is the viscosity rating in summer). Monograde oils are given their viscosity rating at 100°C. The viscosity is an indication of how well the oil flows. The more viscous an oil the larger the molecules that are in the oil.

- Which oil has the longest molecules?
- Which oil has the shortest molecules?
- Which oil has the strongest overall intermolecular forces?
- Which oil has the weakest overall intermolecular forces?
- What can you conclude about the link between the magnitude of the intermolecular force and viscosity?

Density

DEFINITION: *Density*

Density is a measure of the mass in a unit volume.

The solid phase is often the most dense phase (water is one noteworthy exception to this). This can be explained by the strong intermolecular forces found in a solid. These forces pull the molecules together which results in more molecules in one unit volume than in the liquid or gas phases. The more molecules in a unit volume the denser that substance will be.

Melting and boiling points

Intermolecular forces affect the boiling and melting points of substances. Substances with weak intermolecular forces will have low melting and boiling points while those with strong intermolecular forces will have high melting and boiling points. In the experiment on intermolecular forces you investigated the boiling points of several substances, and should have seen that molecules with weaker intermolecular forces have

a lower boiling point than molecules with stronger intermolecular forces.

One further point to note is that covalent network structures (recall from grade 10 that these are covalent compounds that form large networks and an example is diamond) will have high melting and boiling points due to the fact that some bonds (i.e. the strong forces between atoms) have to break before the substance can melt. Covalent molecular substances (e.g. water, sugar) often have lower melting and boiling points, because of the presence of the weaker intermolecular forces holding these molecules together.

Thermal expansion

As substances are heated their molecules start moving more vigorously (their kinetic energy increases). This causes the liquid to expand on heating. You can observe this in a thermometer. As the alcohol (or mercury) is heated it expands and rises up the tube.

This is why when you tile a floor you have to leave gaps between the tiles to allow for expansion. It is also why power lines sag slightly and bridges have slight gaps for expansion.

Thermal conductivity

Different materials conduct heat differently. The following activity will highlight this.

Investigation: Thermal conductivity

Take a long thin piece of graphite and a long thin piece of copper (or other metal). Attach a bit of wax to the one end of each rod (you will need to melt the wax a bit first to make it stick). While the wax is still soft, press a toothpick into the blob of wax.

Now suspend the graphite and copper rods from a desk or chair using a piece of string and heat the other end. Observe which toothpick falls off first. Try to explain why.

Heat is transferred through a substance from the point being heated to the other end. This is why the bottom of a pot gets hot first (assuming you are heating the pot on a stove plate). In metals there are some free, delocalised electrons which can help transfer the heat energy through the metal. In covalent molecular compounds there are no free, delocalised electrons and the heat does not travel as easily through the material.

Worked example 3: Understanding intermolecular forces

QUESTION

Explain why the melting point of oxygen (O_2) is much lower than the melting point of hydrogen chloride HCl.

SOLUTION

Step 1: Write down what you know about melting points and forces

The stronger the intermolecular force, the higher the melting point. So if a substance has strong intermolecular forces, then that substance will have a high melting point.

Step 2: Write down which forces occur in the two given compounds

Oxygen is non-polar and has induced dipole forces. Hydrogen chloride is polar and has dipole-dipole forces.

Step 3: Combine all the facts to get the answer

We know that stronger intermolecular forces lead to higher melting points. We also know that oxygen has weaker intermolecular forces than hydrogen chloride (induced dipole versus dipole-dipole forces). Therefore oxygen will have a lower melting point than hydrogen chloride since oxygen has weaker intermolecular forces.

Exercise 4 – 2: Types of intermolecular forces

1. Given the following diagram:

   H——Cl- - -H——Cl

   a) Name the molecule and circle it on the diagram
   b) Label the interatomic forces (covalent bonds)
   c) Label the intermolecular forces

2. Given the following molecules and solutions:

   HCl, CO_2, I_2, H_2O, $KI(aq)$, NH_3, $NaCl(aq)$, HF, $MgCl_2$ in CCl_4, NO, Ar, SiO_2

   Complete the table below by placing each molecule next to the correct type of intermolecular force.

Type of force	Molecules
Ion-dipole	
Ion-induced-dipole	
Dipole-dipole (no hydrogen bonding)	
Dipole-dipole (hydrogen bonding)	
Induced dipole	
Dipole-induced-dipole	

   In which one of the substances listed above are the intermolecular forces:

   a) strongest
   b) weakest

3. Use your knowledge of different types of intermolecular forces to explain the following statements:

FACT
There are about 55,5 mol of water in 1 L. This is equivalent to $3,34 \times 10^{25}$ molecules of water. That's a lot of water molecules!

a) The boiling point of F_2 is much lower than the boiling point of NH_3
b) Water evaporates slower than carbon tetrachloride (CCl_4).
c) Sodium chloride is likely to dissolve in methanol (CH_3OH) .

4. Tumi and Jason are helping their dad tile the bathroom floor. Their dad tells them to leave small gaps between the tiles. Why do they need to leave these small gaps?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23R2 2. 23R3 3a. 23R4 3b. 23R5 3c. 23R6 4. 23R7

www.everythingscience.co.za **m.everythingscience.co.za**

4.2 The chemistry of water ESBMR

See video: 23R8 at www.everythingscience.co.za

We will now look at how intermolecular forces apply to a very special liquid. This section shows how the knowledge of intermolecular forces can be applied to the case of water.

The microscopic structure of water ESBMS

In many ways, water behaves very differently from other compounds. These properties are directly related to the microscopic structure of water, and more specifically to the *shape* of the molecule and its *polar nature*, and to the *intermolecular forces* that hold water molecules together.

In the previous chapter you learnt about molecular shape and polarity. Water has two hydrogen atoms around a central oxygen atom. The central oxygen atom also has two lone pairs of electrons. This gives water a bent (or angular) shape. It also means that water is polar since the two hydrogen atoms are not parallel to each other and so do not cancel out the bond polarity (refer back to the previous chapter on molecular shape). We can see this in the following image:

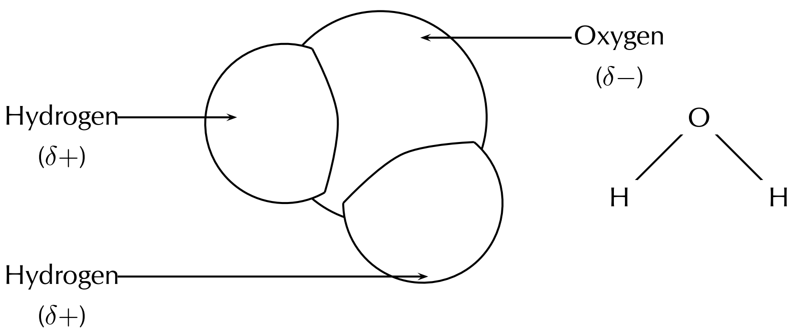


Figure 4.7: Diagrams showing the structure of a water molecule. Each molecule is made up of two hydrogen atoms that are attached to one oxygen atom.

Water molecules are held together by **hydrogen bonds**. Hydrogen bonds are a much stronger type of intermolecular force than those found in many other substances, and this affects the properties of water.

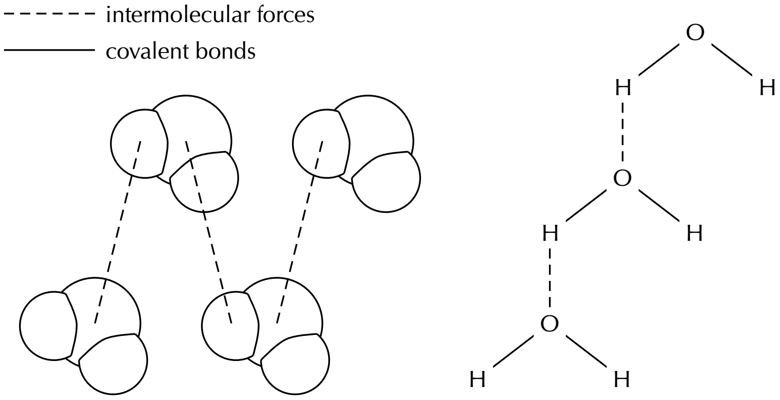


Figure 4.8: Intermolecular and covalent bonds (interatomic forces) in water. Note that the diagram on the left only shows *intermolecular* forces. The covalent bonds (interatomic forces) are between the atoms of each water molecule.

FACT

It is the high specific heat of water and its ability to absorb infra-red radiation that allows water to regulate the climate. Towns close to the sea often have less extreme temperatures than inland towns due to the oceans ability to absorb the heat.

The unique properties of water ESBMT

We will now look at a few of the properties of water.

1. **Specific heat**

   DEFINITION: *Specific heat*

   Specific heat is the amount of heat energy that is needed to increase the temperature of a unit mass of a substance by one degree.

   Water has a high specific heat, meaning that a lot of energy must be absorbed by water before its temperature changes.

   You have probably observed this phenomenon if you have boiled water in a pot on the stove. The metal of the pot heats up very quickly, and can burn your fingers if you touch it, while the water may take several minutes before its temperature increases even slightly. How can we explain this in terms of hydrogen bonding? Remember that increasing the temperature of a substance means that its particles will move more quickly. However, before they can move faster, the intermolecular forces between them must be disrupted. In the case of water, these forces are strong hydrogen bonds, and so a lot of energy is needed just to break these, before the particles can start moving further apart.

2. **Absorption of infra-red radiation**

   Water is able to absorb infra-red radiation (heat) from the sun. As a result of this, the oceans and other water bodies act as heat reservoirs, and are able to help moderate the Earth's climate.

3. **Melting point and boiling point**

   The melting point of water is 0 °C and its boiling point is 100 °C (at standard pressure or 0,987 atm). This large difference between the melting and boiling point is very important because it means that water can exist as a liquid over a large range of temperatures. (This temperature range is only large in the world

FACT

When the boiling point of water is measured at sea level (e.g. towns like Cape Town and Durban), it is often very close to 100 °C since the atmospheric pressure is almost the same as the standard pressure. If you measure the boiling point of water in a town at a higher altitudes (e.g. Johannesburg or Polokwane) it will have a slightly lower boiling point.

FACT

Antarctica, the "frozen continent", has one of the world's largest and deepest freshwater lakes. And this lake is hidden beneath 4 km of ice! Lake Vostok is 200 km long and 50 km wide. The thick, glacial blanket of ice acts as an insulator, preventing the water from freezing.

around us, if we look at space and the universe then this is a very narrow temperature range.)

In grade 10 you studied the heating and cooling curve of water. This is given below.

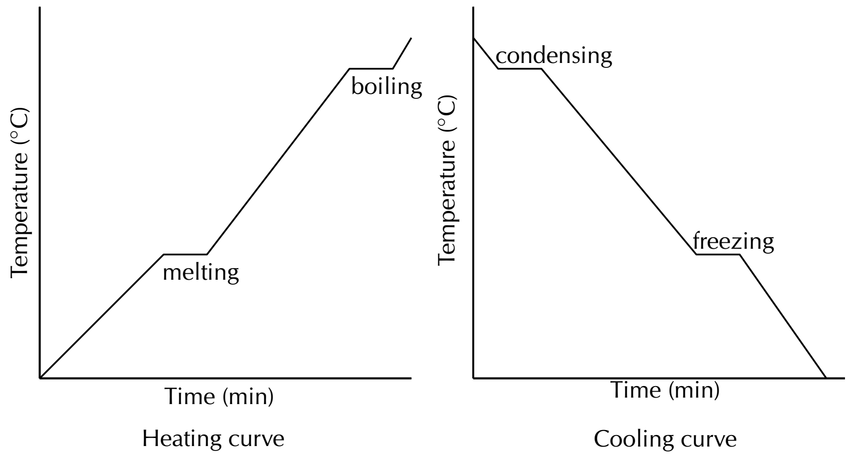


Figure 4.9: Heating and cooling curves for water.

4. **High heat of vaporisation**

DEFINITION: *Heat of vaporisation*

Heat of vaporisation is the energy that is needed to change a given quantity of a substance into a gas.

The strength of the hydrogen bonds between water molecules also means that it has a high heat of vaporisation. "Heat of vaporisation" is the heat energy that is needed to change water from the liquid to the gas phase. Because the forces between molecules are strong, water has to be heated to 100 °C before it changes phase. At this temperature, the molecules have enough energy to break the intermolecular forces that hold the molecules together. The heat of vaporisation for water is 40,65 $kJ{\cdot}mol^{-1}$.

It is very important for life on earth that water does have a high heat of vaporisation. Can you imagine what a problem it would be if water's heat of vaporisation was much lower? All the water that makes up the cells in our bodies would evaporate and most of the water on earth would no longer be able to exist as a liquid!

5. **Less dense solid phase**

Another unusual property of water is that its solid phase (ice) is *less dense* than its liquid phase. You can observe this if you put ice into a glass of water. The ice doesn't sink to the bottom of the glass, but floats on top of the liquid. This phenomenon is also related to the hydrogen bonds between water molecules. While other materials contract when they solidify, water expands. The ability of ice to float as it solidifies is a very important factor in the environment. If ice sank, then eventually all ponds, lakes, and even the oceans would freeze solid as soon as temperatures dropped below freezing, making life as we know it impossible on Earth. During summer, only the upper few metres of the ocean would thaw. Instead, when a deep body of water cools, the floating ice insulates the liquid water below, preventing it from freezing and allowing life to exist under the frozen surface.

It should be clear now, that water is an amazing compound, and that without its unique properties, life on Earth would definitely not be possible.

Worked example 4: Properties of water

QUESTION

Explain why water takes a long time to heat up, but the pot that you are heating it in gets hot quickly.

SOLUTION

Step 1: Decide which property is applied here

We are asked why water takes a long time to heat up compared to the pot you are heating it in. The property that applies here is the high specific heat of water. The other properties of water do not apply here since we are comparing the pot to the water and the pot is not changing phase.

Step 2: Write the final answer

Water has a high specific heat, while the metal that the pot is made of does not. The metal pot needs less energy to heat it up and so it gets hotter faster. Water needs a lot of energy to change its temperature and so it takes longer to get hot.

Informal experiment:The properties of water

Aim:

To investigate the properties of water.

Apparatus:

- water
- ice
- beakers
- carbon tetrachloride, oil, ethanol
- various solids (e.g. sodium chloride, potassium chloride, potassium permanganate, iodine)

Method:

1. Pour about 100 ml of water into a glass beaker.
2. Place the beaker on a stand and heat it over a Bunsen burner for about a minute.
3. After this time, carefully touch the side of the beaker (Make sure you touch the glass very lightly because it will be very hot and may burn you!). Then test the temperature of the water.

4. Note what happens when you place ice into water.
5. Carefully layer water and carbon tetrachloride in a test-tube. Which substances floats on top? Try adding the water first and the carbon tetrachloride second and then the other way around. Repeat with oil and ethanol.
6. Dissolve the different solid substances in water. Observe how much of each solid (if any) dissolves.

Results:

Record all your results in the following table.

Property	Observation
Temperature	
Ice in water	
Carbon tetrachloride and water	
Oil and water	
Ethanol and water	
Solubility	

Conclusion:

You should find that the glass beaker heats up faster than the water. You should also find that water is more dense in the liquid phase than in the solid phase. Water floats on some liquids and other liquids float on water. Water is a good solvent for polar and ionic substances.

Activity: How humans have used the properties of water

Carry out some research into: water bags on cars, clay pots and carafes for water and safe or "cool" rooms to keep food cool. Find out which people groups use these things and how the properties of water help in each case.

Exercise 4 – 3: The properties of water

1. Hope returns home from school on a hot day and pours herself a glass of water. She adds ice cubes to the water and notices that they float on the water.
   a) What property of ice cubes allows them to float in the water?
   b) Briefly describe how this property affects the survival of aquatic life during winter.
2. Which properties of water allow it to remain in its liquid phase over a large temperature range? Explain why this is important for life on earth.

3. Which property of water allows the oceans to act as heat reservoirs? What effect does this have on the Earths climate?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23R9 2. 23RB 3. 23RC

www.everythingscience.co.za m.everythingscience.co.za

4.3 Chapter summary ESBMV

See presentation: 23RD at www.everythingscience.co.za

- **Intermolecular forces** are the forces that act between molecules.
- The **type** of intermolecular force in a substance, will depend on the **nature of the molecules**.
- **Polar molecules** have an unequal distribution of charge, meaning that one part of the molecule is slightly positive and the other part is slightly negative. The molecule is said to be a dipole.
- **Non-polar molecules** have an equal distribution of charge.
- There are five types of intermolecular forces: ion-dipole forces, ion-induced-dipole forces, dipole-dipole forces, dipole-induced dipole forces and induced dipole forces.
- **Ion-dipole** forces exist between **ions and polar (dipole) molecules**. The ion is attracted to the part of the molecule that has an opposite charge to its own.
- **Ion-induced dipole** forces exist between **ions and non-polar molecules**. An ion induces a dipole in the non-polar molecule.
- **Dipole-dipole** forces exist between two **polar (dipole) molecules**.
- **Dipole-induced dipole** forces exist between a **polar molecule** and a **non-polar molecule**.
- **Induced dipole** forces exist between two **non-polar molecules**.
- Dipole-dipole forces, dipole-induced dipole forces and induced dipole forces are collectively called **van der Waals' forces**.
- **Hydrogen bonds** are a type of dipole-dipole force that occurs when a hydrogen atom is attached to a highly electronegative atom (oxygen, fluorine, nitrogen). A hydrogen atom on one molecule is attracted to the electronegative atom on a second molecule.
- Intermolecular forces affect the **properties** of substances.
- Substances with **larger molecules** have **stronger** intermolecular forces than substances with smaller molecules.

- **Viscosity** is the resistance to flow of a liquid. Substances that are very viscous have larger molecules and stronger intermolecular forces than substances with smaller molecules.
- **Density** is a measure of the mass in a unit volume. Solids have strong intermolecular forces and so have more molecules in one unit volume.
- Substances with **weak** intermolecular forces will have **low melting and boiling** points while those with strong intermolecular forces will have high melting and boiling points.
- **Thermal expansion** is the expansion of a liquid on heating.
- **Thermal conductivity** is a measure of how much a material conducts heat.
- Water has **strong hydrogen bonds** which hold the molecules together. It is these intermolecular forces that give water its unique properties.
- Water has the following properties: a high specific heat, absorption of infrared radiation, a large range in which it exists as a liquid, a high heat of vaporisation and has a less dense solid phase.
- **Specific heat** is the amount of heat energy that is needed to increase the temperature of a unit mass of a substance by one degree.
- **Heat of vaporisation** is the energy that is needed to change a given quantity of a substance into a gas.

Exercise 4 – 4:

1. Give one word or term for each of the following descriptions:
   a) The attractive force that exists between molecules.
   b) A molecule that has an unequal distribution of charge.
   c) The amount of heat energy that is needed to increase the temperature of a unit mass of a substance by one degree.
2. Refer to the list of substances below:
   HCl, Cl_2, H_2O, NH_3, N_2, HF
   Select the true statement from the list below:
   a) NH_3 is a non-polar molecule
   b) The melting point of NH_3 will be higher than for Cl_2
   c) Ion-dipole forces exist between molecules of HF
   d) At room temperature N_2 is usually a liquid
3. The following table gives the melting points of various hydrides:

Hydride	**Melting point** (°C)
HI	-34
NH_3	-33
H_2S	-60
CH_4	-164

a) In which of these hydrides does hydrogen bonding occur?

i. HI only

ii. NH_3 only

iii. HI and NH_3 only

iv. HI, NH_3 and H_2S

b) Draw a graph to show the melting points of the hydrides.

c) Explain the shape of the graph.

(IEB Paper 2, 2003)

4. The respective boiling points for four chemical substances are given below:

Substance	Boiling point
Hydrogen sulphide	-60 °C
Ammonia	-33 °C
Hydrogen fluoride	20 °C
Water	100 °C

a) Which one of the substances exhibits the strongest forces of attraction between its molecules in the liquid state?

b) Give the name of the force responsible for the relatively high boiling points of hydrogen fluoride and water and explain how this force originates.

c) The shapes of the molecules of hydrogen sulfide and water are similar, yet their boiling points differ. Explain.

(IEB Paper 2, 2002)

5. Susan states that van der Waals forces include ion-dipole forces, dipole-dipole forces and induced dipole forces.

Simphiwe states that van der Waals forces include ion-dipole forces, ion-induced dipole forces and induced dipole forces.

Thembile states that van der Waals forces include dipole-induced dipole forces, dipole-dipole forces and induced dipole forces.

Who is correct and why?

6. Jason and Bongani are arguing about which molecules have which intermolecular forces. They have drawn up the following table:

Compound	Type of force
Potassium iodide in water (KI(aq))	dipole-induced dipole forces
Hydrogen sulfide (H_2S)	induced dipole forces
Helium (He)	ion-induced dipole forces
Methane (CH_4)	induced dipole forces

a) Jason says that hydrogen sulfide (H_2S) is non-polar and so has induced dipole forces. Bongani says hydrogen sulfide is polar and has dipole-dipole forces. Who is correct and why?

b) Bongani says that helium (He) is an ion and so has ion-induced dipole forces. Jason says helium is non-polar and has induced dipole forces. Who is correct and why?

c) They both agree on the rest of the table. However, they have not got the correct force for potassium iodide in water (KI(aq)). What type of force actually exists in this compound?

7. Khetang is looking at power lines around him for a school project. He notices that they sag slightly between the pylons. Why do power lines need to sag slightly?

8. Briefly describe how the properties of water make it a good liquid for life on Earth.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 23RF 1b. 23RG 1c. 23RH 2. 23RJ 3. 23RK 4. 23RM
5. 23RN 6. 23RP 7. 23RQ 8. 23RR

www.everythingscience.co.za **m.everythingscience.co.za**

CHAPTER 5

Geometrical optics

Your reflection in the mirror, a straw in a glass of water, telescopes, communications, spotlights and car headlights. All these things rely on the way light reflects off surfaces or refracts in different media. If light did not reflect and refract as it does, you would not be able to see how you look before going out or communicate over long distances.

Key Mathematics Concepts

- Equations — Mathematics, Grade 10, Equations and inequalities
- Trigonometry — Mathematics, Grade 10, Trigonometry

5.1 Summary of properties of light ESBMW

Imagine you are indoors on a sunny day. A beam of sunlight through a window lights up a section of the floor. How would you draw this sunbeam? You might draw a series of parallel lines showing the path of the sunlight from the window to the floor. This is not exactly accurate — no matter how hard you look, you will not find unique lines of light in the sunbeam! However, this is a good way to draw light and to model light geometrically, as we will see in this chapter.

We call these narrow, imaginary lines of light **light rays**. Recall that light can behave like a wave and so you can think of a light ray as the path of a point on the crest of a wave.

We can use light rays to model the behaviour of light relative to mirrors, lenses, telescopes, microscopes, and prisms. The study of how light interacts with materials is called **optics**. When dealing with light rays, we are usually interested in the shape of a material and the angles at which light rays hit it. From these angles, we can determine, for example, the distance between an object and its reflection. We call these methods **geometrical optics**.

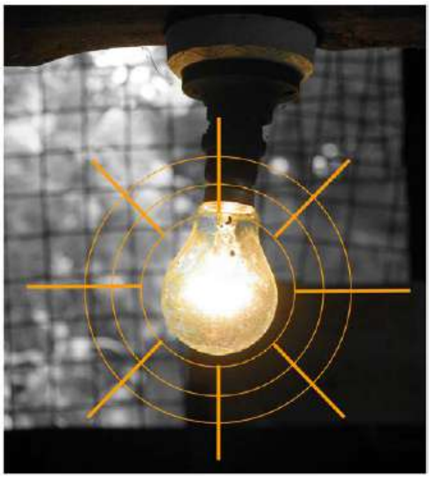
5.2 Light rays ESBMX

In physics we use the idea of a *light ray* to indicate the direction in which light travels. In geometrical optics, we represent light rays with straight arrows to show how light

propagates. Light rays are not an exact description of the light; rather they show the direction in which the light wavefronts are travelling.

DEFINITION: *Light ray*

Light rays are lines which are perpendicular to the light's wavefronts. In geometrical optics we represent light rays with arrows with straight lines.



The lightbulb is a source of light. The light wavefronts are shown by the concentric circles coming from the bulb. We represent the direction in which the wavefronts are moving by drawing light rays (the arrows) perpendicular to the wavefronts.

In Figure 5.1, the light rays from the object enter the eye and the eye *sees* the object. Note that we can *only* see an object when light from the object enters our eyes. The object must be a source of light (e.g. a light bulb) or else it must reflect light from a source (e.g. the moon which reflects light from the sun), and the light must enter our eyes.

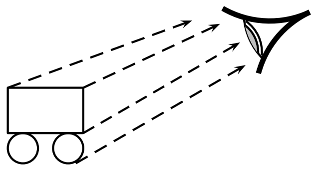


Figure 5.1: We can only see an object when light from that object enters our eyes. When the light travels from the object to the eye, the eye can see the object. Light rays entering the eye from the cart are shown as dashed lines. The second wheel of the cart will be invisible as no straight, unobstructed lines exist between it and the eye.

From the figures you can see that the light rays showing the path of light are straight arrows. In geometrical optics, light travels in straight lines.

Investigation: To demonstrate that light travels in straight lines

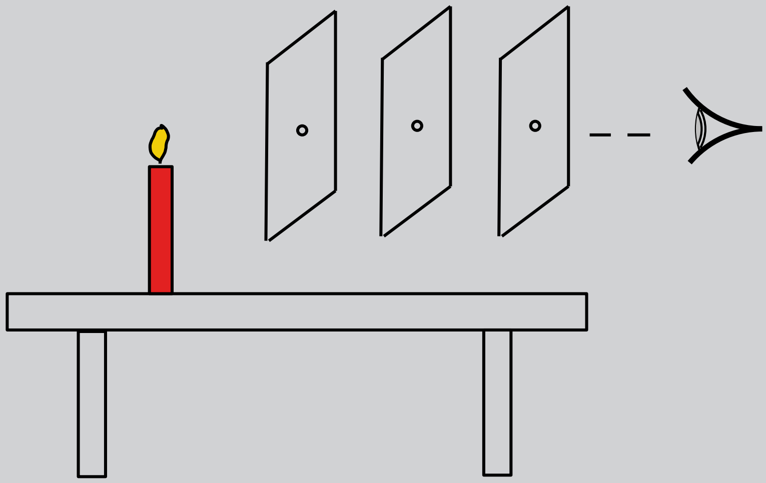
Apparatus:

You will need a candle, matches and three sheets of paper.

Method:

1. Make a small hole in the middle of each of the three sheets of paper.
2. Light the candle.

3. Look at the burning candle through the hole in the first sheet of paper.
4. Place the second sheet of paper between you and the candle so that you can still see the candle through the holes.
5. Now do the same with the third sheet so that you can still see the candle. The sheets of paper must not touch each other.



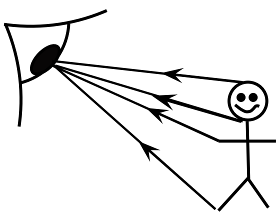
6. What do you notice about the holes in the paper?

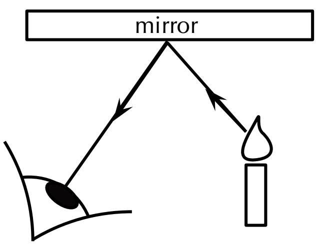
Conclusions:

In the investigation you will notice that the holes in the paper need to be in a straight line. This shows that light travels in a straight line. We cannot see around corners. This also proves that light does not bend around a corner, but travels straight.

Ray diagrams ESBMY

A ray diagram is a drawing that shows the path of light rays. Light rays are drawn using straight lines and arrow heads. The figure below shows some examples of ray diagrams.





5.3 Properties of light: revision ESBMZ

When light interacts with objects or a medium, such as glass or water, it displays certain properties: it can either be **reflected**, **absorbed** or **transmitted**.

When you smile into a mirror, you see your own face smiling back at you. This is caused by the reflection of light rays on the mirror. Reflection occurs when an incoming light ray bounces off a surface. In the following figure, a still lake reflects the landscape surrounding it.

Figure 5.2: A landscape reflection from a still lake.

To describe the reflection of light, we will use the following terminology. The incoming light ray is called the **incident ray**. The light ray moving away from the surface is the **reflected ray**. The most important characteristic of these rays is their angles in relation to the reflecting surface. These angles are measured with respect to the normal of the surface. The **normal** is an imaginary line perpendicular to the surface. The **angle of incidence,** θ_i is measured between the incident ray and the surface normal. The **angle of reflection,** θ_r is measured between the reflected ray and the surface normal. This is shown in Figure 5.3.

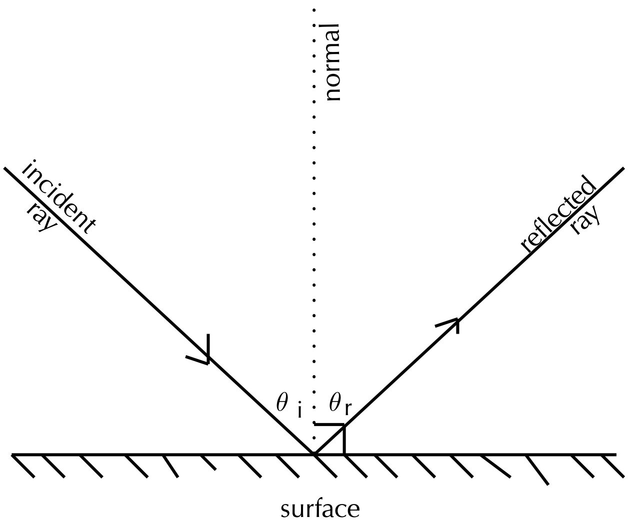


Figure 5.3: The angles of incidence and reflection are measured with respect to the normal to the surface.

When a ray of light is reflected, the reflected ray lies in the same plane as the incident ray and the normal. This plane is called the **plane of incidence** and is shown in Figure 5.4.

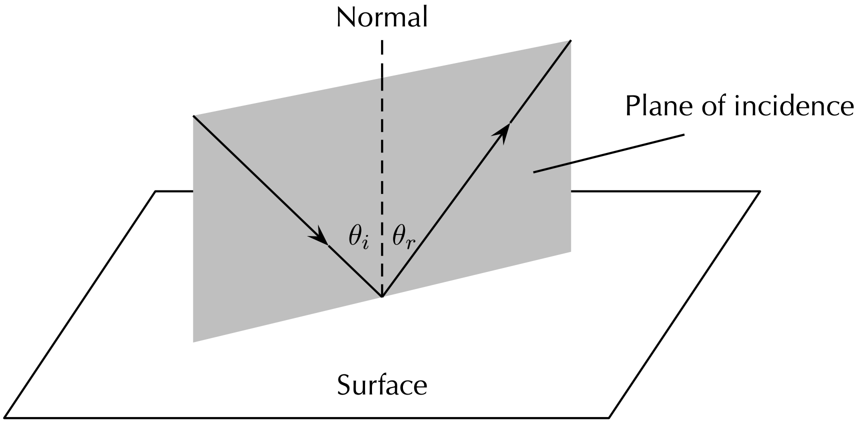


Figure 5.4: The plane of incidence is the plane including the incident ray and the normal to the surface. The reflected ray also lies in the plane of incidence.

DEFINITION: *Law of reflection*

The angle of incidence is equal to the angle of reflection:

$$\theta_i = \theta_r$$

and the incident ray, reflected ray, and the normal, all lie in the same plane.

The simplest example of the law of reflection is if the angle of incidence is 0°. In this case, the angle of reflection is also 0°. You see this when you look straight into a mirror.

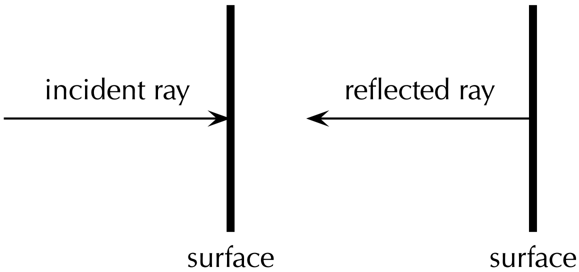


Figure 5.5: When a light ray strikes a surface at right angles to the surface, then the ray is reflected directly back.

Applying what we know from the law of reflection, if a light ray strikes a surface at 60° to the normal to the surface, then the angle that the reflected ray makes with the normal must also be 60° as shown in Figure 5.6.

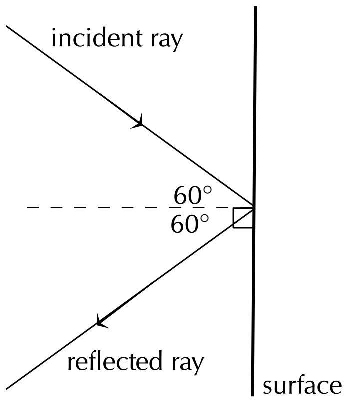


Figure 5.6: Ray diagram showing angle of incidence and angle of reflection. The law of reflection states that when a light ray reflects off a surface, the angle of reflection θ_r is the same as the angle of incidence θ_i.

Real world applications of reflection

A parabolic reflector is a mirror or dish (e.g. a satellite dish) which has a parabolic shape. Some examples of very useful parabolic reflectors are car headlamps, spotlights, telescopes and satellite dishes. In the case of car headlights or spotlights, the outgoing light produced by the bulb is reflected by a parabolic mirror behind the bulb so that it leaves as a collimated beam (i.e. all the reflected rays are parallel). The reverse situation is true for a telescope where the incoming light from distant objects arrives as parallel rays and is focused by the parabolic mirror to a point, called the focus, where an image can be made. The surface of this sort of reflector has to be shaped very carefully so that the rays all arrive at the same focal point.

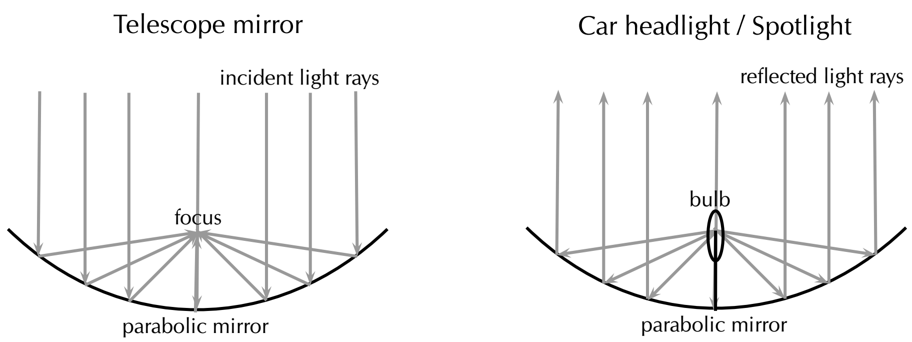
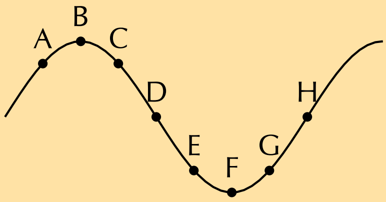


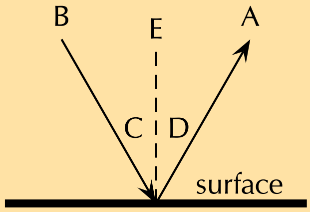
Figure 5.7: On the left is a ray diagram showing how a telescope mirror works to collect incoming incident light (parallel rays) from a distant object such as a star or galaxy and focus the rays to a point where a detector e.g. a camera, can make an image. The diagram on the right shows how the same kind of parabolic reflector can cause light coming from a car headlight or spotlight bulb to be collimated. In this case the reflected rays are parallel.

Exercise 5 – 1: Rays and Reflection

1. Are light rays real? Explain.
2. The diagram shows a curved surface. Draw normals to the surface at the marked points.



3. Which of the labels, A–H, in the diagram, correspond to the following:

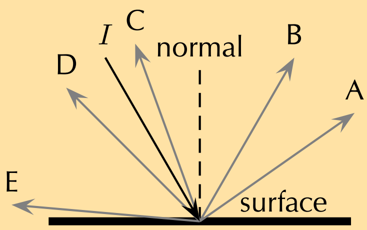


   a) normal
   b) angle of incidence
   c) angle of reflection
   d) incident ray
   e) reflected ray

4. State the Law of Reflection. Draw a diagram, label the appropriate angles and write a mathematical expression for the Law of Reflection.

5. Draw a ray diagram to show the relationship between the angle of incidence and the angle of reflection.

6. The diagram shows an incident ray I. Which of the other 5 rays (A, B, C, D, E) best represents the reflected ray of I?



7. A ray of light strikes a surface at 15° to the normal to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.

8. A ray of light leaves a surface at 45° to the normal to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.

9. A ray of light strikes a surface at 25° to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.

10. A ray of light leaves a surface at 65° to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.

11. A beam of light (for example from a torch) is generally not visible at night, as it travels through air. Try this for yourself. However, if you shine the torch through dust, the beam is visible. Explain why this happens.
12. If a torch beam is shone across a classroom, only students in the direct line of the beam would be able to see that the torch is shining. However, if the beam strikes a wall, the entire class will be able to see the spot made by the beam on the wall. Explain why this happens.
13. A scientist looking into a flat mirror hung perpendicular to the floor cannot see her feet but she can see the hem of her lab coat. Draw a ray diagram to help explain the answers to the following questions:
    a) Will she be able to see her feet if she backs away from the mirror?
    b) What if she moves towards the mirror?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23RS	2. 23RT	3a. 23RV	3b. 23RW	3c. 23RX	3d. 23RY
3e. 23RZ	4. 23S2	5. 23S3	6. 23S4	7. 23S5	8. 23S6
9. 23S7	10. 23S8	11. 23S9	12. 23SB	13. 23SC	

www.everythingscience.co.za **m.everythingscience.co.za**

Absorption

In addition to being reflected, light can also be **absorbed**. Recall from grade 10 that visible light covers a range of wavelengths in the electromagnetic spectrum. The colours we see with our eyes correspond to light waves with different wavelengths or frequencies.

Imagine that you shine a torch or other source of white light onto a white piece of paper. The light is reflected off the paper and into our eyes and we see the colour of the paper as white. Now if we were to shine the same light onto a red apple we will notice that the colour of the apple appears red. This means that the surface of the apple is only reflecting red light into our eyes. All the other wavelengths in the incident white light are absorbed by the apple's skin. If you touch the apple, it will feel warm because it is absorbing the energy from all the light it is absorbing. Because of absorption and reflection, we can perceive colours of different objects. White objects reflect all or most of the wavelengths of light falling on them, coloured objects reflect particular wavelengths of light and absorb the rest. Black objects absorb all the light falling on them. This is why wearing a white t-shirt outside in the sun is cooler than wearing a black t-shirt, since the white t-shirt reflects most of the light falling on it, while the black t-shirt will absorb it and heat up.

Transmission

A further property of light is that it can be **transmitted** through objects or a medium. Objects through which light can be transmitted are called *transparent* and objects

which block out light or that light cannot pass through are called *opaque*.

For example, glass windows allow visible light to pass through them which is why we can see through windows. The light rays from things outside the window can pass through or be transmitted through the glass and into our eyes. Brick walls on the other hand are opaque to visible light. We cannot see through brick walls because the light cannot be transmitted through the wall into our eyes. The transmission of light through an object depends on the wavelength of the light. For example, short wavelength visible light cannot be transmitted through a brick wall whereas long wavelength radio waves can easily pass through walls and be received by a radio or cell phone. In other words, a brick wall is transparent to radio waves!

5.4 The speed of light ESBN3

One of the most exciting discoveries in physics during the last century, and the cornerstone of Einstein's Theory of Relativity, is that light travels at a constant speed in a given medium. Light also has a maximum speed at which it can propagate, and nothing can move faster than the speed of light. The maximum speed at which light can travel is when it propagates through free space (a vacuum) at 299 792 485 $m{\cdot}s^{-1}$. A vacuum is a region with no matter in it, not even air. However, the speed of light in air is very close to that in a vacuum.

We use the symbol c to represent the speed of light in a vacuum and approximate it as $c = 3 \times 10^8$ $m{\cdot}s^{-1}$

DEFINITION: *Speed of light*

The speed of light, c, is constant in a given medium and has a maximum speed in vacuum of

$$c = 3 \times 10^8 \text{ m}{\cdot}\text{s}^{-1}$$

5.5 Refraction ESBN4

See video: 23SD at www.everythingscience.co.za

See video: 23SF at www.everythingscience.co.za

In the previous sections we studied light reflecting off various surfaces. What happens when light passes from one medium into another? The speed of light, like that of all waves, is dependent on the medium through which it is travelling. When light moves from one medium into another (for example, from air to glass), the speed of light changes. If the light ray hits the boundary of the new medium (for example the edge of a glass block) at any angle which is not perpendicular to or parallel with the boundary, the light ray will change its direction through the next medium, or appear to 'bend'. This is called **refraction** of light. It is important to note that while the speed of the light changes when it passes into the new medium, the frequency of the light

remains the same.

DEFINITION: *Refraction*

Refraction occurs at the boundary of two media when light travels from one medium into the other and its speed changes but its frequency remains the same. If the light ray hits the boundary at an angle which is not perpendicular to or parallel to the surface, then it will change direction and appear to 'bend'.

Refraction is nicely demonstrated when you look from above at an angle at a straw in a glass of water. The straw appears bent in the liquid. This is because the light rays leaving the straw change direction when they hit the surface between the liquid and the air. Your eyes trace the light rays backwards as straight lines to the point they would have come from if they had not changed direction and as a result you see the tip of the straw as being shallower in the liquid than it really is.

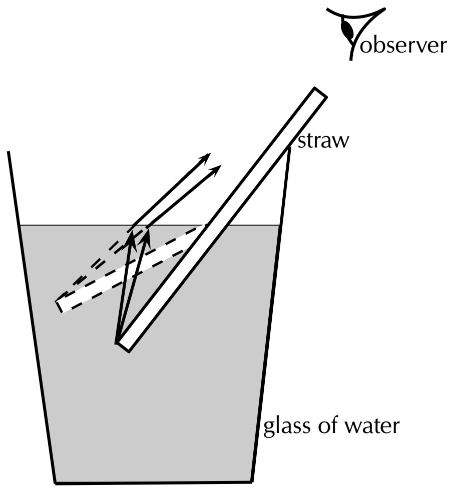


Figure 5.8: Due to refraction, a straw in a glass of water appears bent when an observer looks down at an angle from above the water surface.

Refractive index ESBN5

The speed of light and therefore the degree of bending of the light depends on the *refractive index* of material through which the light passes. The refractive index (symbol n) is the ratio of the speed of light in a vacuum to its speed in the material.

DEFINITION: *Refractive index*

The refractive index (symbol n) of a material is the ratio of the speed of light in a vacuum to its speed in the material and gives an indication of how difficult it is for light to get through the material.

$$n = \frac{c}{v}$$

where

n = refractive index (no unit)
c = speed of light in a vacuum ($3{,}00 \times 10^8$ m·s^{-1})
v = speed of light in a given medium (m·s^{-1})

Using the definition of refractive index

$$n = \frac{c}{v}$$

we can see how the speed of light changes in different media, because the speed of light in a vacuum, c, is constant.

If the refractive index, n, increases, the speed of light in the material, v, must decrease. Therefore light travels slower through materials of high refractive index, n.

Table 5.1 shows refractive indices for various materials. Light travels slower in any material than it does in a vacuum, so all values for n are greater than 1.

Medium	Refractive Index
Vacuum	1
Helium	1,000036
Air*	1,0002926
Carbon dioxide	1,00045
Water: Ice	1,31
Water: Liquid (20°C)	1,333
Acetone	1,36
Ethyl Alcohol (Ethanol)	1,36
Sugar solution (30%)	1,38
Fused quartz	1,46
Glycerine	1,4729
Sugar solution (80%)	1,49
Rock salt	1,516
Crown Glass	1,52
Sodium chloride	1,54
Polystyrene	1,55 to 1,59
Bromine	1,661
Sapphire	1,77
Glass (typical)	1,5 to 1,9
Cubic zirconia	2,15 to 2,18
Diamond	2,419
Silicon	4,01

Table 5.1: Refractive indices of some materials. n_{air} is calculated at standard temperature and pressure (STP).

Worked example 1: Refractive index

QUESTION

Calculate the speed of light through glycerine which has a refractive index of 1,4729.

SOLUTION

Step 1: Determine what is given and what is required

We are given the refractive index, n of glycerine and we need to calculate the speed of light in glycerine.

Step 2: Determine how to approach the problem

We can use the definition of refractive index since the speed of light in vacuum is a constant and we know the value of glycerine's refractive index.

Step 3: Do the calculation

$$n = \frac{c}{v}$$

Rearrange the equation to solve for v and substitute in the known values:

$$\begin{aligned} v &= \frac{c}{n} \\ &= \frac{3 \times 10^8 \text{ m·s}^{-1}}{1{,}4729} \\ &= 2{,}04 \times 10^8 \text{ m·s}^{-1} \end{aligned}$$

Exercise 5 – 2: Refractive index

1. Use the values given in Table 5.1, and the definition of refractive index to calculate the speed of light in water (ice).
2. Calculate the refractive index of an unknown substance where the speed of light through the substance is $1{,}974 \times 10^8$ m·s^{-1}. Round off your answer to 2 decimal places. Using Table 5.1, identify what the unknown substance is.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23SG 2. 23SH

www.everythingscience.co.za **m.everythingscience.co.za**

Optical density ESBN6

Optical density is a measure of the refracting power of a medium. In other words, the higher the optical density, the more the light will be refracted or slowed down as it moves through the medium. Optical density is related to refractive index in that materials with a high refractive index will also have a high optical density. Light will travel slower through a medium with a high refractive index and high optical density and faster through a medium which has a low refractive index and a low

optical density.

DEFINITION: *Optical density*

Optical density is a measure of the refracting power of a medium.

Representing refraction with ray diagrams ESBN7

It is useful to draw ray diagrams to understand how the geometrical optics concepts we have discussed previously work. Before we can draw the diagrams we need to define a few concepts such as the normal to a surface, the angle of incidence and the angle of refraction.

DEFINITION: *Normal*

The normal to a surface is the line which is perpendicular to the plane of the surface.

DEFINITION: *Angle of incidence*

The angle of incidence is the angle defined between the normal to a surface and the incoming (incident) light ray.

DEFINITION: *Angle of refraction*

The angle of refraction is the angle defined between the normal to a surface and the refracted light ray.

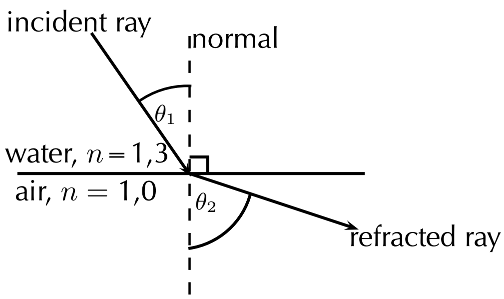


Figure 5.9: The diagram shows the boundary between two media: water and air. An incoming light ray is refracted when it passes through the surface of the water into the air. The angle of incidence is θ_1 and the angle of refraction is θ_2.

When light travels from one medium to another, it is refracted. If the angle of incidence is not equal to zero, the light ray will change direction from its original path as it is refracted.

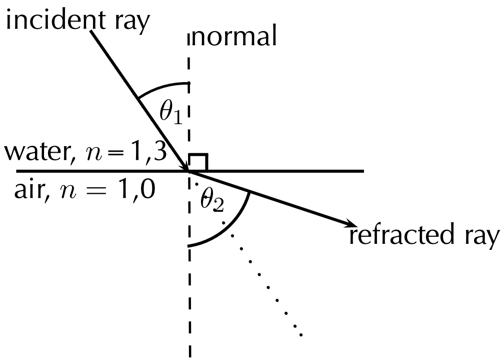


Figure 5.10: Light is moving from an optically dense medium to an optically less dense medium.

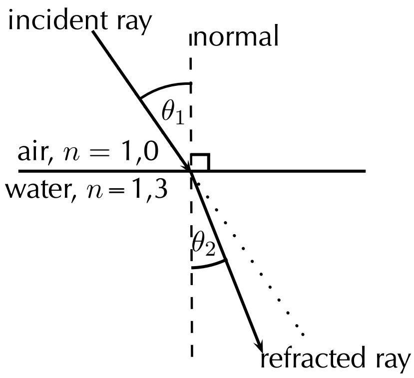


Figure 5.11: Light is moving from an optically less dense medium to an optically denser medium.

A practical demonstration of the propagation of light from air into glass and back into air or light from one medium to another can be done. You will need a glass block, ray box, colour filters, non-rectangular glass blocks, water, paper, ruler, pencil and protractor.

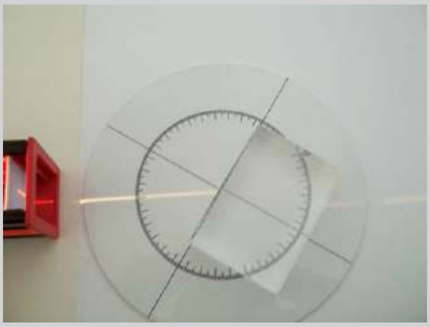
General experiment: Propagation of light from air into glass and back into air

Aim:

To investigate propagation of light from air into glass and back into air

Apparatus:

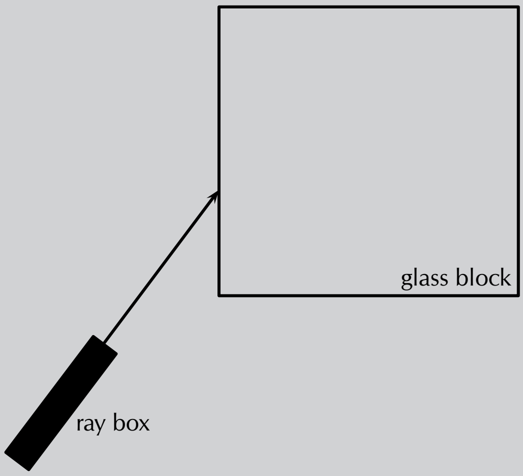
ray box, rectangular glass block, plain paper, pencil, ruler, protractor.



Method:

Follow the steps below:

1. Place the glass block on a plain piece of paper and use your pencil to draw around the block to make its outline on the paper.
2. Now turn on the ray box and aim the light ray through the left side of the glass block as illustrated in the diagram:



3. Now use your pencil to draw a dot somewhere on the incident light ray and another dot at the point where it enters the glass block.
4. Use your pencil to draw a dot at the point where the light exits the glass block and also somewhere else along the exiting ray.
5. Turn off the ray box and remove the glass block from the paper. Use your ruler to join the dots so that you have drawn a picture that looks like the figure above.
6. Now draw the normals to the surfaces where the light ray enters and leaves the block and mark the angle of incidence and angle of refraction on the left surface and the right surface.

Questions for discussion:

1. At the surface where the light enters the glass block, what do you notice about the angle of incidence compared to the angle of refraction?

2. Now look at the surface where the light exits the glass block. Compare the angle of incidence and angle of refraction here.
3. How do the optical densities and indices of refraction for air and glass compare?

Informal experiment:Propagation of light from one medium into another medium

Aim:

To investigate the propagation of light from one medium into another

Apparatus:

ray box, glass blocks of various shapes, a transparent container filled with water, plain paper, pencil, ruler, protractor.

Method:

Starting with the rectangular glass block, repeat the steps below for each of the various differently shaped glass blocks:

1. Place the glass block on a plain piece of paper and use your pencil to draw around the block to make its outline on the paper.
2. Turn on the ray box and aim the light ray through one of the block's surfaces.
3. Draw a dot at the point where the light enters the block and another dot somewhere else along the incident ray. Also draw a dot at the point where the ray exits the surface of the block and another somewhere along the exiting ray.
4. Remove the glass block and turn off the ray box. Use your ruler to join the dots.
5. Now draw the normals to the surfaces where the light ray enters and exits the block.
6. Use your protractor to measure the angles of incidence and refraction at the surfaces where the light ray enters and exits the block.

Now follow the same steps as before but place differently coloured filters at the surface of the blocks where the light enters the block.

Lastly, replace the glass blocks with the container of water and repeat the same steps as above. For this case, try to aim the ray box so that the angle of incidence on the water container is the same as it was for the rectangular glass block you investigated first. This is easier if you place your new piece of paper on top of the drawing you made with the rectangular block. Line up the water container where you drew the edge of your glass block. Then you can aim the ray box so that the light lines up with the incident ray on your paper and follow the steps as before.

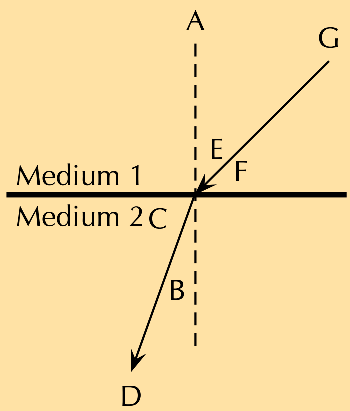
Questions for discussion:

1. For each of your differently shaped glass blocks, how does the angle of incidence going from air to glass compare to the angle of refraction?
2. What happens when you place a coloured filter at the boundary between the air and a glass block?
3. When you compare your two diagrams for the rectangular glass block and the water container, if you did it correctly, the angles of incidence from air to glass/water should be the same. What can you say about the angles of refraction for glass and water respectively? How does this compare with what you already know about the refractive indices or optical densities of these materials?

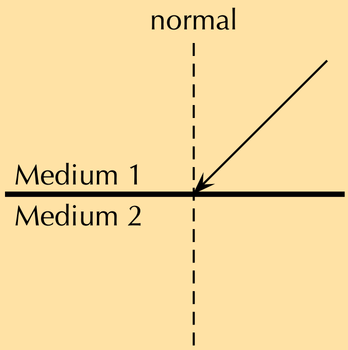
See simulation: 23SJ at www.everythingscience.co.za

Exercise 5 – 3: Refraction

1. Explain refraction in terms of a change of wave speed in different media.
2. In the diagram, label the following:
   a) angle of incidence
   b) angle of refraction
   c) incident ray
   d) refracted ray
   e) normal



3. What is *the angle of refraction*?
4. Describe what is meant by *the refractive index of a medium*.
5. In the diagram, a ray of light strikes the interface between two media.



Draw what the refracted ray would look like if:

a) medium 1 had a higher refractive index than medium 2.

b) medium 1 had a lower refractive index than medium 2.

6. **Challenge question:** What values of n are physically impossible to achieve? Explain your answer. The values provide the limits of possible refractive indices.

7. **Challenge question:** You have been given a glass beaker full of an unknown liquid. How would you identify what the liquid is? You have the following pieces of equipment available for the experiment: a laser or ray box, a protractor, a ruler, a pencil, and a reference guide containing optical properties of various liquids.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23SK 2. 23SM 3. 23SN 4. 23SP 5. 23SQ 6. 23SR
7. 23SS

www.everythingscience.co.za **m.everythingscience.co.za**

5.6 Snell's Law ESBN8

Now that we know that the degree of bending, or the angle of refraction, is dependent on the refractive index of a medium, how do we calculate the angle of refraction? The answer to this question was discovered by a Dutch physicist called Willebrord Snell in 1621 and is now called Snell's Law or the Law of Refraction.

DEFINITION: *Snell's law*

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

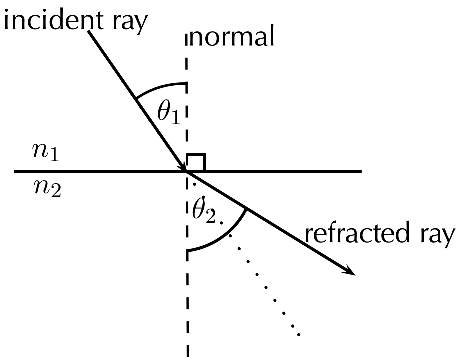
where

n_1 = Refractive index of material 1
n_2 = Refractive index of material 2
θ_1 = Angle of incidence
θ_2 = Angle of refraction

FACT
Snell never published his discovery of the Law of Refraction. His work was actually published by another prominent physicist of the time, Christiaan Huygens, who gave credit to Snell.

Remember that angles of incidence and refraction are measured from the normal, which is an imaginary line perpendicular to the surface.

Suppose we have two media with refractive indices n_1 and n_2. A light ray is incident on the surface between these materials with an **angle of incidence** θ_1. The refracted ray that passes through the second medium will have an **angle of refraction** θ_2.



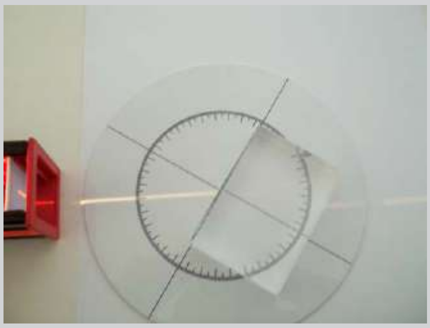
Formal experiment: Verifying Snell's Law

Aim:

To verify Snell's law

Apparatus:

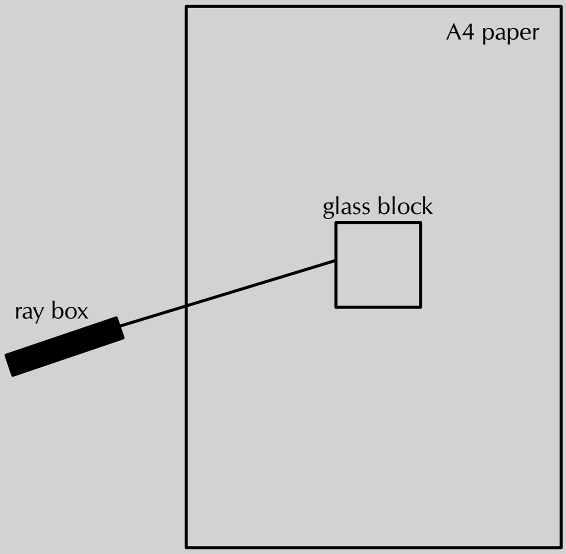
glass block, ray box, 360° protractor, 5 pieces of A4 paper, pencil, ruler



Method:

This experiment will require you to follow the steps below 5 times (once for each piece of A4 paper).

1. Place the glass block in the middle of the A4 piece of paper so that its sides are parallel to each of the sides of the paper and draw around the block with a pencil to make its outline on the piece of paper.
2. Turn on the ray box and aim the light ray towards the glass block so that it makes an angle with the nearest surface of the block as shown in the picture. For each piece of paper, change the angle of the incoming ray.



3. You will now need to mark on the paper, the path of the incoming and outgoing light rays. Do this by first drawing a dot on the paper somewhere along the incoming light ray. Now draw a second dot on the paper at the point where the incoming light ray hits the surface of the block. Do the same thing for the outgoing light ray; mark the point where it leaves the block and some other point along its path.

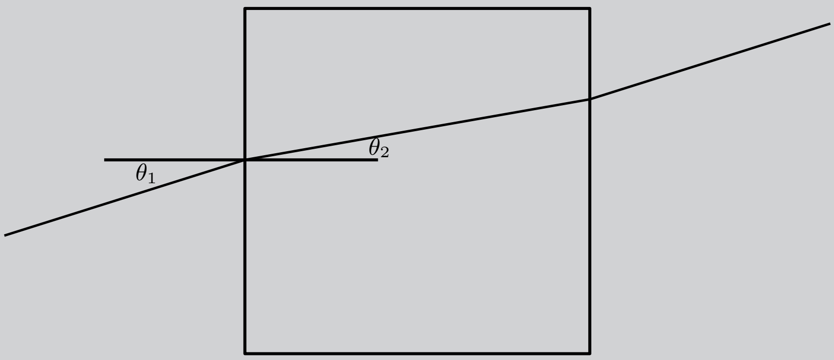
4. Now switch off the ray box and remove the glass block from the paper. Use a ruler to join the dots of the incoming ray. Now join the dots of the outgoing ray. Lastly, draw a line which joins the point where the incoming ray hits the block and where the outgoing ray leaves the block. This is the path of the light ray through the glass.

5. The aim of this experiment is to verify Snell's law. i.e. $n_1 \sin\theta_1 = n_2 \sin\theta_2$. We know the refractive index of our two media:

   For air, $n_1 =$ 1,0

   For glass, $n_2 =$ 1,5

   Now we need to measure the two angles, θ_1 and θ_2. To do this, we need to draw the normal to the surface where the light ray enters the block. Use the protractor to measure an angle of 90° to the entry surface and draw the normal. At this point, the drawing on your piece of paper should look something like the picture:



6. Now measure θ_1 and θ_2 using the protractor. Enter the values you measured into a table which looks like:

θ_1	θ_2	$n_1 \sin\theta_1$	$n_2 \sin\theta_2$

Discussion:

Have a look at your completed table. You should have 5 rows filled in, one for each of your pieces of A4 paper. For each row, what do you notice about the values in the last two columns? Do your values agree with what Snell's law predicts?

Formal experiment: Using Snell's law to determine the refractive index of an unknown material

Aim:

To determine the refractive index of an unknown material

Apparatus:

ray box, 360° protractor, 5 pieces of A4 paper, a block of unknown transparent material, pencil, ruler

Method:

This experiment will require you to follow the steps below at least 5 times. The steps are the same as you followed in the previous experiment.

1. Place the block in the middle of the A4 piece of paper so that its sides are parallel to each of the sides of the paper and draw around the block with a pencil to make its outline on the piece of paper.
2. Turn on the ray box and aim the light ray towards the block so that it makes an angle with the nearest surface of the block. For each piece of paper, change the angle of the incoming ray.
3. You will now need to mark on the paper, the path of the incoming and outgoing light rays. Do this by first drawing a dot on the paper somewhere along the incoming light ray. Now draw a second dot on the paper at the point where the incoming light ray hits the surface of the block. Do the same thing for the outgoing light ray; mark the point where it leaves the block and some other point along its path.
4. Now switch off the ray box and remove the block from the paper. Use a ruler to join the dots of the incoming ray. Now join the dots of the outgoing ray. Lastly, draw a line which joins the point where the incoming ray hits the block and where the outgoing ray leaves the block. This is the path of the light ray through the block.

5. The aim of this experiment is to determine the refractive index n_2 of the unknown material using Snell's law which states $n_1 \sin\theta_1 = n_2 \sin\theta_2$. We know that for air n_1 = 1,0 and we can measure the angle of incidence, θ_1 and the angle of refraction, θ_2. Then we can solve the equation for the unknown, n_2.

   To measure θ_1 and θ_2, we need to draw the normal to the surface where the light ray enters the block. Use the protractor to measure an angle of 90° to the entry surface and draw the normal. Now use the protractor to measure θ_1 and θ_2.

6. Enter the values you measured into a table which looks like the one below and calculate the resulting value for n_2.

θ_1	θ_2	$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2}$

Question:

What do you notice about all your values in the last column of the table?

Discussion:

You should have noticed that the values in the last column of the table are similar but not identical. This is due to measurement errors when you measured the angles of incidence and angles of refraction. These sorts of errors are common in all physics experiments and lead to a measure of uncertainty in the final extracted value. However, since we did the same experiment 5 times, we can average the 5 independent measurements of n_2 to get a good approximation to the real value for our unknown material.

If

$$n_2 > n_1$$

then from Snell's Law,

$$\sin\theta_1 > \sin\theta_2$$

For angles smaller than 90°, $\sin\theta$ increases as θ increases. Therefore,

$$\theta_1 > \theta_2.$$

This means that the angle of incidence is greater than the angle of refraction and the light ray is bent toward the normal.

Similarly, if

$$n_2 < n_1$$

then from Snell's Law,

$$\sin\theta_1 < \sin\theta_2.$$

For angles smaller than 90°, $\sin\theta$ increases as θ increases. Therefore,

$$\theta_1 < \theta_2.$$

This means that the angle of incidence is less than the angle of refraction and the light ray is bent away from the normal.

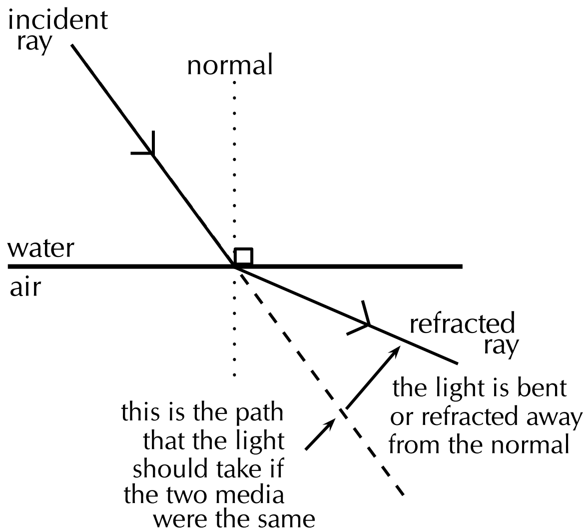


Figure 5.12: Light is moving from a medium with a higher refractive index to one with a lower refractive index. Light is refracted away from the normal.

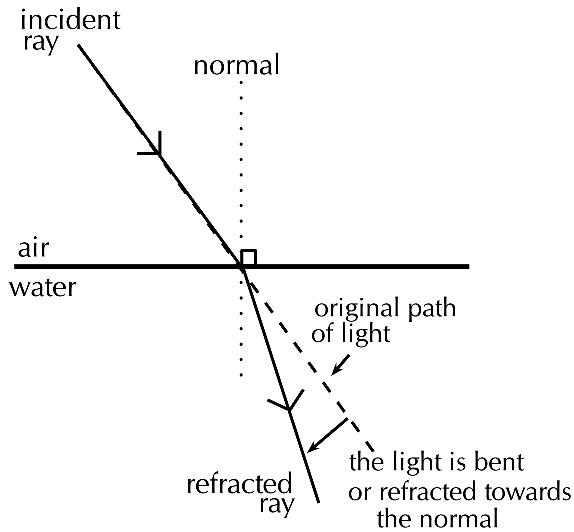


Figure 5.13: Light is moving from a medium with a lower refractive index to a medium with a higher refractive index. Light is refracted towards the normal.

What happens to a ray that lies along the normal line? In this case, the angle of incidence is 0° and

$$\begin{aligned}\sin\theta_2 &= \frac{n_1}{n_2}\sin\theta_1\\ &= 0\\ \therefore \theta_2 &= 0.\end{aligned}$$

This shows that if the light ray is incident at 0°, then the angle of refraction is also 0°. The direction of the light ray is unchanged, however, the speed of the light will change as it moves into the new medium. Therefore refraction still occurs although it will not be easily observed.

Let's use an everyday example that you can more easily imagine. Imagine you are pushing a lawnmower or a cart through short grass. As long as the grass is pretty much the same length everywhere, it is easy to keep the mower or cart going in a straight line. However, if the wheels on one side of the mower or cart enter an area of grass that is longer than the grass on the other side, that side of the mower will move more slowly since it is harder to push the wheels through the longer grass. The result is that the mower will start to turn inwards, towards the longer grass side as you can see in the picture. This is similar to light which passes through a medium and then enters a new medium with a higher refractive index or higher optical density. The light will change direction towards the normal, just like the mower in the longer grass. The opposite happens when the mower moves from an area of longer grass into an area with shorter grass. The side of the mower in the shorter grass will move faster and the mower will turn outwards, just like a light ray moving from a medium with high refractive index into a medium with low refractive index moves away from the normal.

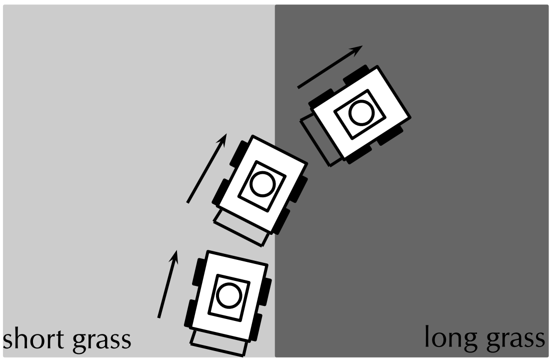


Figure 5.14: The lawnmower is moving from an area of short grass into an area of longer grass. When it reaches the boundary between the areas, the wheels which are in the long grass move slower than the wheels which are still in the short grass, causing the lawnmower to change direction and its path to bend inwards.

Worked example 2: Using Snell's Law

QUESTION

Light is refracted at the boundary between water and an unknown medium. If the angle of incidence is 25°, and the angle of refraction is 20,6°, calculate the refractive index of the unknown medium and use Table 5.1 to identify the material.

SOLUTION

Step 1: Determine what is given and what is being asked

The angle of incidence θ_1 =25°

The angle of refraction θ_2 =20,6°

We can look up the refractive index for water in Table 5.1: n_1 = 1,333

We need to calculate the refractive index for the unknown medium and identify it.

Step 2: Determine how to approach the problem

We can use Snell's law to calculate the unknown refractive index, n_2

According to Snell's Law:

$$\begin{aligned} n_1 \sin\theta_1 &= n_2 \sin\theta_2 \\ n_2 &= \frac{n_1 \sin\theta_1}{\sin\theta_2} \\ n_2 &= \frac{1{,}333 \sin 25°}{\sin 20{,}6°} \\ n_2 &= 1{,}6 \end{aligned}$$

Step 3: Identify the unknown medium

According to Table 5.1, typical glass has a refractive index between 1,5 to 1,9. Therefore the unknown medium is typical glass.

Worked example 3: Using Snell's law

QUESTION

A light ray with an angle of incidence of 35° passes from water to air. Find the angle of refraction using Snell's Law and Table 5.1. Discuss the meaning of your answer.

SOLUTION

Step 1: Determine the refractive indices of water and air

From Table 5.1, the refractive index is 1,333 for water and about 1 for air. We know the angle of incidence, so we are ready to use Snell's Law.

Step 2: Substitute values

According to Snell's Law:

$$\begin{aligned} n_1 \sin\theta_1 &= n_2 \sin\theta_2 \\ 1{,}33 \sin 35° &= 1 \sin\theta_2 \\ \sin\theta_2 &= 0{,}763 \\ \theta_2 &= 49{,}7° \text{ or } 130{,}3° \end{aligned}$$

Since 130,3° is larger than 90°, the solution is:

$$\theta_2 = 49{,}7°$$

Step 3: Discuss the answer

The light ray passes from a medium of high refractive index to one of low refractive index. Therefore, the light ray is bent away from the normal.

Worked example 4: Using Snell's Law

QUESTION

A light ray passes from water to diamond with an angle of incidence of 75°. Calculate the angle of refraction. Discuss the meaning of your answer.

SOLUTION

Step 1: Determine the refractive indices of water and air

From Table 5.1, the refractive index is 1,333 for water and 2,42 for diamond. We know the angle of incidence, so we are ready to use Snell's Law.

Step 2: Substitute values and solve

According to Snell's Law:

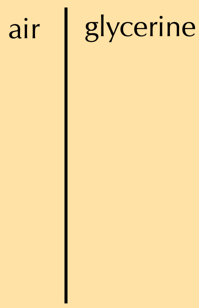
$$\begin{aligned} n_1 \sin\theta_1 &= n_2 \sin\theta_2 \\ 1{,}33 \sin 75^\circ &= 2{,}42 \sin\theta_2 \\ \sin\theta_2 &= 0{,}531 \\ \theta_2 &= 32{,}1^\circ \end{aligned}$$

Step 3: Discuss the answer

The light ray passes from a medium of low refractive index to one of high refractive index. Therefore, the light ray is bent towards the normal.

Exercise 5 – 4: Snell's Law

1. State Snell's Law.
2. Light travels from a region of glass into a region of glycerine, making an angle of incidence of 40°.



   a) Draw the incident and refracted light rays on the diagram and label the angles of incidence and refraction.
   b) Calculate the angle of refraction.

3. A ray of light travels from silicon to water. If the ray of light in the water makes an angle of 69° to the normal to the surface, what is the angle of incidence in the silicon?

4. Light travels from a medium with $n = 1{,}25$ into a medium of $n = 1{,}34$, at an angle of 27° from the normal.

   a) What happens to the speed of the light? Does it increase, decrease, or remain the same?

   b) What happens to the wavelength of the light? Does it increase, decrease, or remain the same?

   c) Does the light bend towards the normal, away from the normal, or not at all?

5. Light travels from a medium with $n = 1{,}63$ into a medium of $n = 1{,}42$.

   a) What happens to the speed of the light? Does it increase, decrease, or remain the same?

   b) What happens to the wavelength of the light? Does it increase, decrease, or remain the same?

   c) Does the light bend towards the normal, away from the normal, or not at all?

6. Light is incident on a rectangular glass prism. The prism is surrounded by air. The angle of incidence is 23°. Calculate the angle of reflection and the angle of refraction.

7. Light is refracted at the interface between air and an unknown medium. If the angle of incidence is 53° and the angle of refraction is 37°, calculate the refractive index of the unknown, second medium.

8. Light is refracted at the interface between a medium of refractive index 1,5 and a second medium of refractive index 2,1. If the angle of incidence is 45°, calculate the angle of refraction.

9. A ray of light strikes the interface between air and diamond. If the incident ray makes an angle of 30° with the interface, calculate the angle made by the refracted ray with the interface.

10. The angles of incidence and refraction were measured in five unknown media and recorded in the table below. Use your knowledge about Snell's Law to identify each of the unknown media A–E. Use Table 5.1 to help you.

Medium 1	n_1	θ_1	θ_2	n_2	**Unknown Medium**
Air	1,0002926	38	27	?	A
Air	1,0002926	65	38,4	?	B
Vacuum	1	44	16,7	?	C
Air	1,0002926	15	6,9	?	D
Vacuum	1	20	13,3	?	E

11. Zingi and Tumi performed an investigation to identify an unknown liquid. They shone a ray of light into the unknown liquid, varying the angle of incidence and recording the angle of refraction. Their results are recorded in the following table:

Angle of incidence	Angle of refraction
0,0°	0,00°
5,0°	3,76°
10,0°	7,50°
15,0°	11,2°
20,0°	14,9°
25,0°	18,5°
30,0°	22,1°
35,0°	25,5°
40,0°	28,9°
45,0°	32,1°
50,0°	35,2°
55,0°	38,0°
60,0°	40,6°
65,0°	43,0°
70,0°	?
75,0°	?
80,0°	?
85,0°	?

a) Write down an aim for the investigation.

b) Make a list of all the apparatus they used.

c) Identify the unknown liquid.

12. Predict what the angle of refraction will be for 70°, 75°, 80° and 85°.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23ST 2. 23SV 3. 23SW 4. 23SX 5. 23SY 6. 23SZ
7. 23T2 8. 23T3 9. 23T4 10. 23T5 11. 23T6 12. 23T7

www.everythingscience.co.za **m.everythingscience.co.za**

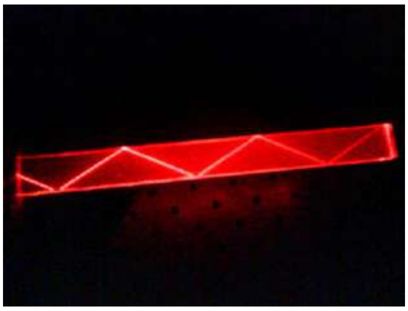
5.7 Critical angles and total internal reflection ESBN9

Total internal reflection ESBNB

See video: 23T8 at www.everythingscience.co.za

You may have noticed when experimenting with ray boxes and glass blocks in the previous section that sometimes, when you changed the angle of incidence of the light, it was not refracted out into the air, but was reflected back through the block.

When the entire incident light ray travelling through an optically denser medium is reflected back at the boundary between that medium and another of lower optical density, instead of passing through and being refracted, this is called total internal reflection.



As we increase the angle of incidence, we reach a point where the angle of refraction is 90° and the refracted ray travels along the boundary of the two media. This angle of incidence is called the critical angle.

DEFINITION: *Critical angle*

The critical angle is the angle of incidence where the angle of refraction is 90°. The light must travel from an optically more dense medium to an optically less dense medium.

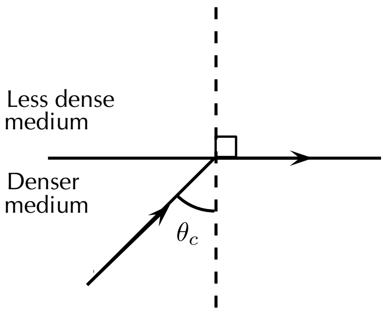


Figure 5.15: When the angle of incidence is equal to the critical angle, the angle of refraction is equal to 90°.

If the angle of incidence is bigger than this critical angle, the refracted ray will not emerge from the medium, but will be reflected back into the medium. This is called **total internal reflection**.

The conditions for total internal reflection are:

1. light is travelling from an optically denser medium (higher refractive index) to an optically less dense medium (lower refractive index).
2. the angle of incidence is greater than the critical angle.

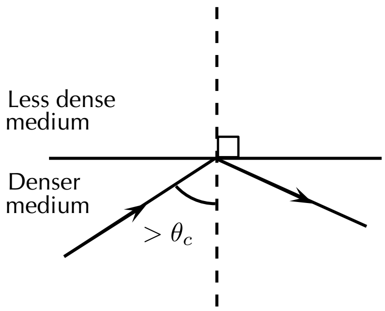


Figure 5.16: When the angle of incidence is greater than the critical angle, the light ray is reflected at the boundary of the two media and total internal reflection occurs.

Each pair of media have their own unique critical angle. For example, the critical angle for light moving from glass to air is 42°, and that of water to air is 48,8°.

Informal experiment:The critical angle of a rectangular glass block

Aim:

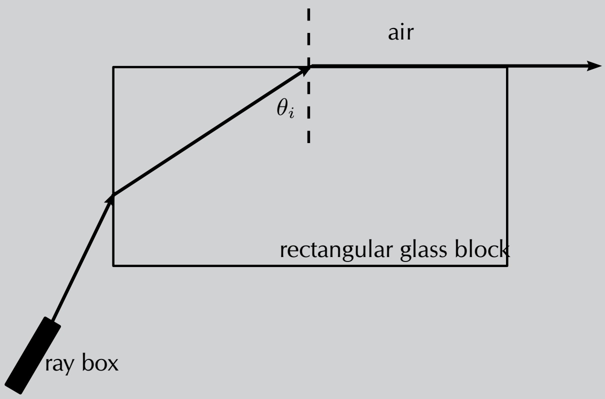
To determine the critical angle for a rectangular glass block.

Apparatus:

rectangular glass block, ray box, 360° protractor, paper, pencil, ruler

Method:

1. Place the glass block in the middle of the piece of paper and draw around the outside of the block with your pencil to make its outline.
2. Turn on the ray box and aim the light ray into the left side of the glass block. Adjust the angle at which the light strikes the glass block until you see the refracted light ray travelling along the top edge of the glass block (i.e. the angle of refraction is 90°). This situation should look something like the following diagram:



3. Draw a dot on the paper at the point where the light enters the glass block from the ray box. Then draw a dot on the paper at the point where the light is refracted (at the top of the glass block).
4. Turn off the ray box and remove the glass block from the paper.
5. Now use your ruler to draw a line between the two dots. This line represents the incident light ray.
6. When the angle of refraction is 90°, the angle of incidence is equal to the critical angle. Therefore to determine the critical angle, we need to measure this angle of incidence. Do this using your protractor.

TIP
Remember that for total internal reflection the incident ray is always in the denser medium!

7. Compare your answer with the values other members of your class obtain and discuss why they might not be identical (although they should be similar!).

Calculating the critical angle

Instead of always having to measure the critical angles of different materials, it is possible to calculate the critical angle at the surface between two media using Snell's Law. To recap, Snell's Law states:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

where n_1 is the refractive index of material 1, n_2 is the refractive index of material 2, θ_1 is the angle of incidence and θ_2 is the angle of refraction. For total internal reflection we know that the angle of incidence is the critical angle. So,

$$\theta_1 = \theta_c.$$

However, we also know that the angle of refraction at the critical angle is 90°. So we have:

$$\theta_2 = 90°.$$

We can then write Snell's Law as:

$$n_1 \sin\theta_c = n_2 \sin 90°$$

Solving for θ_c gives:

$$\begin{aligned} n_1 \sin\theta_c &= n_2 \sin 90° \\ \sin\theta_c &= \frac{n_2}{n_1}(1) \end{aligned}$$

$$\boxed{\therefore \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

Worked example 5: Critical angle

QUESTION

Given that the refractive indices of air and water are 1,00 and 1,33 respectively, find the critical angle.

SOLUTION

Step 1: Determine how to approach the problem

We can use Snell's law to determine the critical angle since we know that when the angle of incidence equals the critical angle, the angle of refraction is 90°.

Step 2: Solve the problem

$$\begin{aligned} n_1 \sin\theta_c &= n_2 \sin 90° \\ \theta_c &= \sin^{-1}\left(\frac{n_2}{n_1}\right) \\ &= \sin^{-1}\left(\frac{1}{1{,}33}\right) \\ &= 48{,}8° \end{aligned}$$

Step 3: Write the final answer

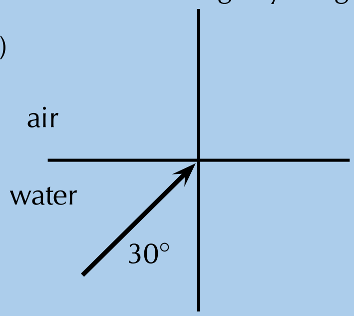
The critical angle for light travelling from water to air is 48,8°.

Worked example 6: Critical angle

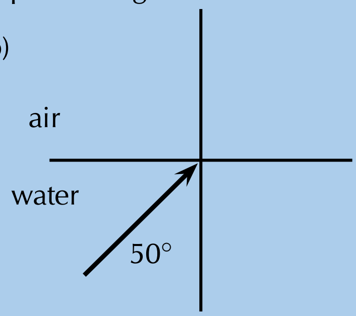
QUESTION

Complete the following ray diagrams to show the path of light in each situation.

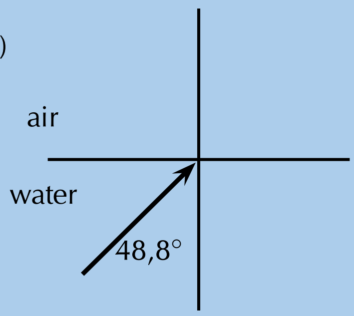
a)



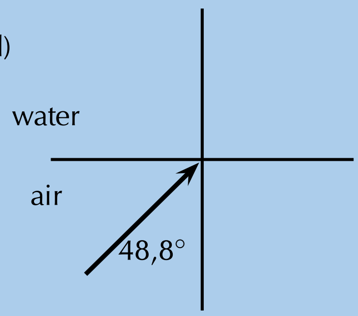
b)



c)



d)



SOLUTION

Step 1: Identify what is given and what is asked

The critical angle for water is 48,8°

We are asked to complete the diagrams.

For incident angles smaller than 48,8° refraction will occur.

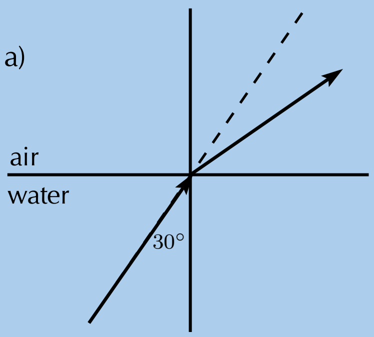
For incident angles greater than 48,8° total internal reflection will occur.

For incident angles equal to 48,8° refraction will occur at 90°.

The light must travel from a medium with a higher refractive index (higher optical density) to a medium with lower refractive index (lower optical density).

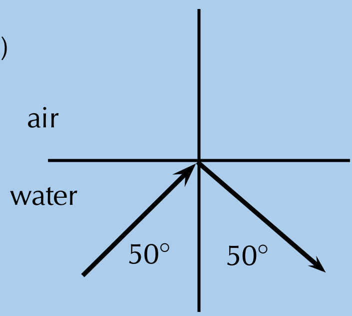
Step 2: Complete the diagrams

a)



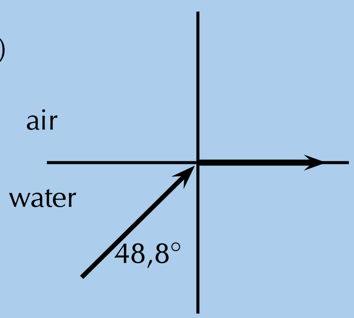
Refraction occurs (ray is bent away from the normal).

b)



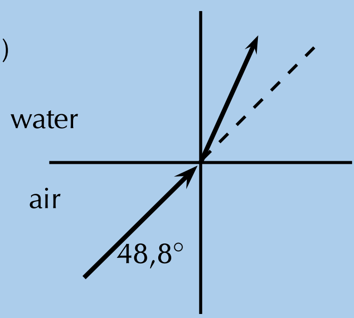
Total internal reflection occurs.

c)



$\theta_c = 48{,}8°$.

d)



Refraction towards the normal (air is less dense than water).

Fibre optics

Total internal reflection is a very useful natural phenomenon since it can be used to confine light. One of the most common applications of total internal reflection is in *fibre optics*. An optical fibre is a thin, transparent fibre, usually made of glass or plastic, for transmitting light. Optical fibres are usually thinner than a human hair! The construction of a single optical fibre is shown in Figure 5.17.

The basic functional structure of an optical fibre consists of an outer protective *cladding* and an *inner core* through which light pulses travel. The overall diameter of the fibre is about 125 μm (125×10^{-6} m) and that of the core is just about 50 μm (50×10^{-6} m). The difference in refractive index of the cladding and the core allows total internal reflection to occur in the same way as happens at an air-water surface. If light is incident on a cable end with an angle of incidence greater than the critical angle then the light will remain trapped inside the glass strand. In this way, light travels very quickly down the length of the cable.

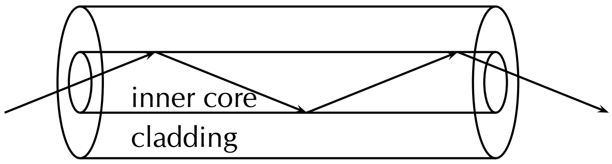


Figure 5.17: Structure of a single optical fibre.

FACT
Endoscopy means *to look inside* and refers to looking inside the human body for diagnosing medical conditions.

Fibre Optics in Telecommunications Optical fibres are most common in telecommunications, because information can be transported over long distances, with minimal loss of data. This gives optical fibres an advantage over conventional cables.

Signals are transmitted from one end of the fibre to another in the form of laser pulses. A single strand of fibre optic cable is capable of handling over 3000 transmissions at the same time which is a huge improvement over the conventional co-axial cables. Multiple signal transmission is achieved by sending individual light pulses at slightly different angles. For example if one of the pulses makes a 72,23° angle of incidence then a separate pulse can be sent at an angle of 72,26°! The transmitted signal is received almost instantaneously at the other end of the cable since the information coded onto the laser travels at the speed of light! During transmission over long distances *repeater stations* are used to amplify the signal which has weakened by the time it reaches the station. The amplified signals are then relayed towards their destination and may encounter several other repeater stations on the way.

Fibre optics in medicine Optic fibres are used in medicine in *endoscopes*.

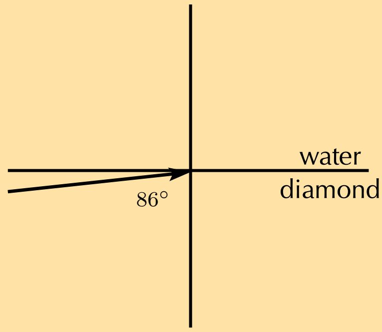
The main part of an endoscope is the optical fibre. Light is shone down the optical fibre and a medical doctor can use the endoscope to look inside the body of a patient. Endoscopes can be used to examine the inside of a patient's stomach, by inserting the endoscope down the patient's throat.

Endoscopes also allow minimally invasive surgery. This means that a person can be diagnosed and treated through a small incision (cut). This has advantages over open surgery because endoscopy is quicker and cheaper and the patient recovers more quickly. The alternative is open surgery which is expensive, requires more time and is more traumatic for the patient.

Exercise 5 – 5: Total internal reflection and fibre optics

1. Describe total internal reflection by using a diagram and referring to the conditions that must be satisfied for total internal reflection to occur.
2. Define what is meant by the *critical angle* when referring to total internal reflection. Include a ray diagram to explain the concept.
3. Will light travelling from diamond to silicon ever undergo total internal reflection?
4. Will light travelling from sapphire to diamond undergo total internal reflection?
5. What is the critical angle for light travelling from air to acetone?

6. Light travelling from diamond to water strikes the interface with an angle of incidence of 86° as shown in the picture. Calculate the critical angle to determine whether the light be totally internally reflected and so be trapped within the diamond.



7. Which of the following interfaces will have the largest critical angle?

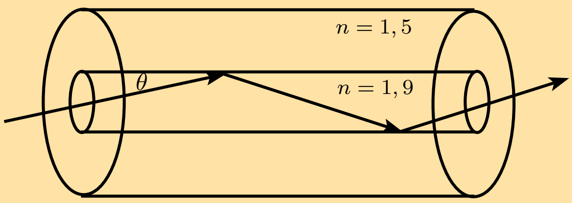
   a) a glass to water interface
   b) a diamond to water interface
   c) a diamond to glass interface

8. If a fibre optic strand is made from glass, determine the critical angle of the light ray so that the ray stays within the fibre optic strand.

9. A glass slab is inserted in a tank of water. If the refractive index of water is 1,33 and that of glass is 1,5, find the critical angle.

10. A diamond ring is placed in a container full of glycerin. If the critical angle is found to be 37,4° and the refractive index of glycerin is given to be 1,47, find the refractive index of diamond.

11. An optical fibre is made up of a core of refractive index 1,9, while the refractive index of the cladding is 1,5. Calculate the maximum angle which a light pulse can make with the wall of the core. NOTE: The question does not ask for the angle of incidence but for the angle made by the ray with the wall of the core, which will be equal to 90° − angle of incidence.



Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23T9 2. 23TB 3. 23TC 4. 23TD 5. 23TF 6. 23TG
7. 23TH 8. 23TJ 9. 23TK 10. 23TM 11. 23TN

www.everythingscience.co.za **m.everythingscience.co.za**

5.8 Summary

ESBNC

See presentation: 23TP at www.everythingscience.co.za

- Light rays are lines which are perpendicular to the light's wavefronts. In geometrical optics we represent light rays with arrows with straight lines.
- Light rays reflect off surfaces. The incident ray shines in on the surface and the reflected ray is the one that bounces off the surface. The normal is the line perpendicular to the surface where the light strikes the surface.
- The angle of incidence is the angle between the incident ray and the surface, and the incident ray, reflected ray, and the normal, all lie in the same plane.
- The Law of Reflection states the angle of incidence (θ_i) is equal to the angle of reflection (θ_r) and that the reflected ray lies in the plane of incidence.
- Light can be absorbed and transmitted.
- The speed of light, c, is constant in a given medium and has a maximum speed in vacuum of 3×10^8 m·s^{-1}
- Refraction occurs at the boundary of two media when light travels from one medium into the other and its speed changes but its frequency remains the same. If the light ray hits the boundary at an angle which is not perpendicular to or parallel to the surface, then it will change direction and appear to 'bend'.
- The refractive index (symbol n) of a material is the ratio of the speed of light in a vacuum to its speed in the material and gives an indication of how difficult it is for light to get through the material.

  $n = \frac{c}{v}$
- Optical density is a measure of the refracting power of a medium.
- The normal to a surface is the line which is perpendicular to the plane of the surface.
- The angle of incidence is the angle defined between the normal to a surface and the incoming (incident) light ray.
- The angle of refraction is the angle defined between the normal to a surface and the refracted light ray.
- Snell's Law gives the relationship between the refractive indices, angles of incidence and reflection of two media.

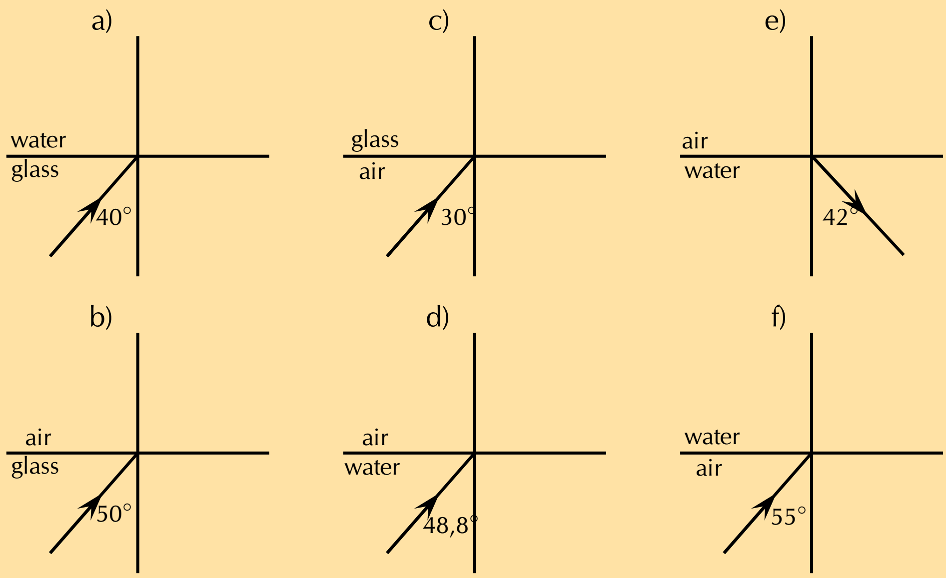
  $n_1 \sin\theta_1 = n_2 \sin\theta_2$
- Light travelling from one medium to another of lighter optical density will be refracted towards the normal.

  Light travelling from one medium to another of lower optical density will be refracted away from the normal.
- The critical angle of a medium is the angle of incidence when the angle of refraction is 90° and the refracted ray runs along the interface between the two media.

- Total internal reflection takes place when light travels from one medium to another of lower optical density. If the angle of incidence is greater than the critical angle for the medium, the light will be reflected back into the medium. No refraction takes place.
- Total internal reflection is used in optical fibres in telecommunication and in medicine in endoscopes. Optical fibres transmit information much more quickly and accurately than traditional methods.

Exercise 5 – 6: End of chapter exercises

1. Give one word for each of the following descriptions:
   a) The perpendicular line that is drawn at right angles to a reflecting surface at the point of incidence.
   b) The bending of light as it travels from one medium to another.
   c) The bouncing of light off a surface.
2. State whether the following statements are **true** or **false**. If they are false, rewrite the statement correcting it.
   a) The refractive index of a medium is an indication of how fast light will travel through the medium.
   b) Total internal refraction takes place when the incident angle is larger than the critical angle.
   c) The speed of light in a vacuum is about 3×10^{8} m·s^{-1}.
3. Complete the following ray diagrams to show the path of light.



4. A ray of light strikes a surface at 35° to the normal to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.
5. Light travels from glass ($n = 1{,}5$) to acetone ($n = 1{,}36$). The angle of incidence is 25°.

a) Describe the path of light as it moves into the acetone.

b) Calculate the angle of refraction.

c) What happens to the speed of the light as it moves from the glass to the acetone?

d) What happens to the wavelength of the light as it moves into the acetone?

6. Light strikes the interface between diamond and an unknown medium with an incident angle of 32°. The angle of refraction is measured to be 46°. Calculate the refractive index of the medium and identify the medium.

7. Explain what total internal reflection is and how it is used in medicine and telecommunications. Why is this technology much better to use?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 23TQ	1b. 23TR	1c. 23TS	2a. 23TT	2b. 23TV	2c. 23TW
3a. 23TX	3b. 23TY	3c. 23TZ	3d. 23V2	3e. 23V3	3f. 23V4
4. 23V5	5. 23V6	6. 23V7	7. 23V8		

www.everythingscience.co.za **m.everythingscience.co.za**

CHAPTER 6

2D and 3D wavefronts

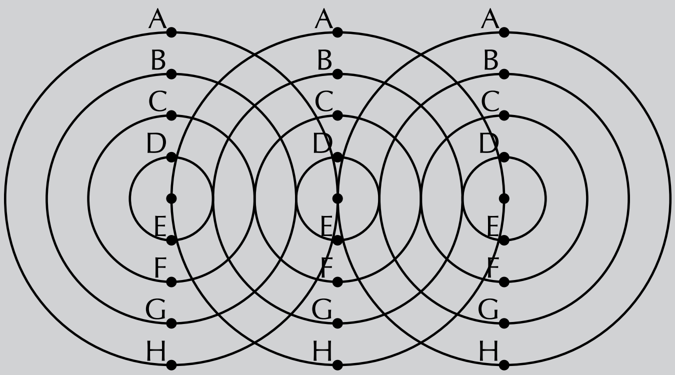
6.1 Introduction ESBND

You have learnt about the basic properties of waves before, specifically about reflection and refraction. In this chapter, you will learn about phenomena that arise with waves in two and three dimensions: diffraction. We will also build on interference which you have learnt about previously but now in more than one dimension.

6.2 Wavefronts ESBNF

Investigation: Wavefronts

The diagram below shows three identical waves being emitted by three point sources. A point source is something that generates waves and is so small that we consider it to be a point. It is not large enough to affect the waves. All points marked with the same letter are in phase. Join all points with the same letter.



What type of lines (straight, curved, etc.) do you get? How does this compare to the line that joins the sources?

Consider three point sources of waves. If each source emits waves isotropically (i.e. the same in all directions) we will get the situation shown in as shown in Figure 6.1 below.

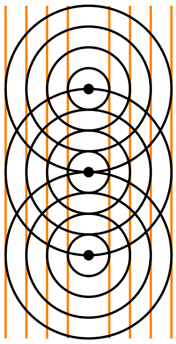


Figure 6.1:

Wavefronts are imaginary lines joining waves that are in phase. In the example, the wavefronts (shown by the orange, vertical lines) join all waves at the crest of their cycle.

We define a **wavefront** as the imaginary line that joins waves that are in phase. These are indicated by the orange, vertical lines in Figure 6.1. The points that are in phase can be peaks, troughs or anything in between, it doesn't matter which points you choose as long as they are in phase.

6.3 Huygens principle ESBNG

Christiaan Huygens described how to determine the path of waves through a medium.

DEFINITION: *The Huygens Principle*

Every point of a wave front serves as a point source of spherical, secondary waves. After a time t, the new position of the wave front will be that of a surface tangent to the secondary waves.

Huygens principle applies to any wavefront, even those that are curved as you would get from a single point source. A simple example of the Huygens Principle is to consider the single wavefront in Figure 6.2.

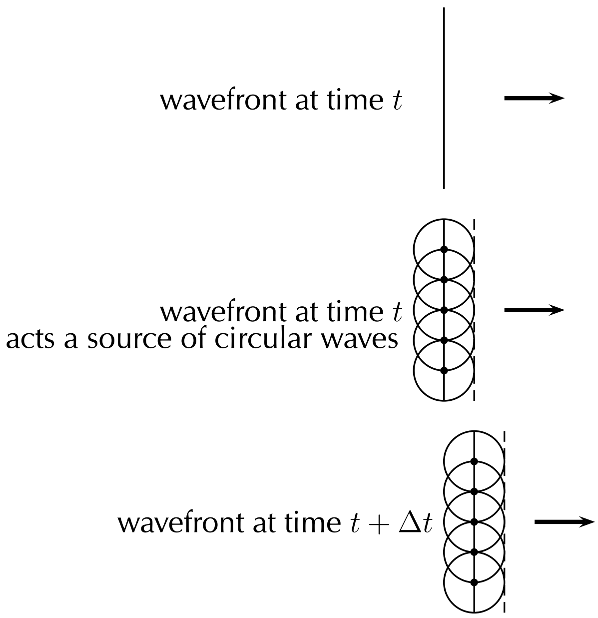
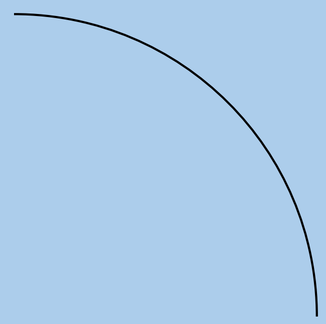


Figure 6.2: 'n Enkele golffront by tydstip t tree op soos 'n versameling puntbronne van sirkelvormige golwe wat mekaar steur om 'n nuwe golffront by 'n tydstip $t + \Delta t$. te gee. Die proses hou aan en is van toepassing op enige tipe golfvorm

Worked example 1: Application of the Huygens principle

QUESTION

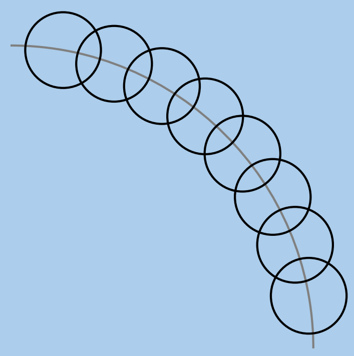
Given the wavefront,



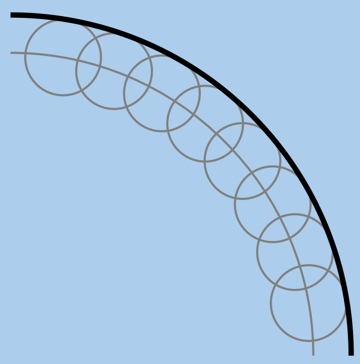
use the Huygens Principle to determine the wavefront at a later time.

SOLUTION

Step 1: Draw circles at various points along the given wavefront



Step 2: Join the circle crests to get the wavefront at a later time

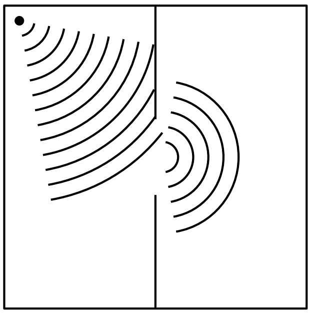


One of the most interesting, and also very useful, properties of waves is **diffraction**.

DEFINITION: *Diffraction*

Diffraction is the ability of a wave to spread out in wavefronts as the wave passes through a small aperture or around a sharp edge.

For example, if two rooms are connected by an open doorway and a sound is produced in a remote corner of one of them, a person in the other room will hear the sound as if it originated at the doorway.



As far as the second room is concerned, the vibrating air in the doorway is the source of the sound.

This means that when waves move through small holes they appear to bend around the sides because there are not enough points on the wavefront to form another straight wavefront. This is bending round the sides we call *diffraction*.

Extension

Diffraction

Diffraction effects are more clear for water waves with longer wavelengths. Diffraction can be demonstrated by placing small barriers and obstacles in a ripple tank and observing the path of the water waves as they encounter the obstacles. The waves are seen to pass around the barrier into the regions behind it; subsequently the water behind the barrier is disturbed. The amount of diffraction (the sharpness of the bending) increases with increasing wavelength and decreases with decreasing wavelength. In fact, when the wavelength of the waves are smaller than the obstacle, no noticeable diffraction occurs.

This experiment demonstrates diffraction using water waves in a ripple tank. You can also demonstrate diffraction using a single slit and a light source with coloured filters.

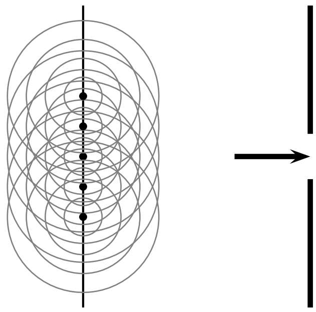
General experiment: Diffraction

Water waves in a ripple tank can be used to demonstrate diffraction and interference.

- Turn on the wave generator so that it produces waves with a high frequency (short wavelength).
  - Place a few obstacles, one at a time, (e.g. a brick or a ruler) in the ripple tank. What happens to the wavefronts as they propagate near/past the obstacles? Draw your observations.
  - How does the diffraction change when you change the size of the object?
- Now turn down the frequency of the wave generator so that it produces waves with longer wavelengths.
  - Place the same obstacles in the ripple tank (one at a time). What happens to the wavefronts as they propagate near/past the obstacles? Draw your observations.
  - How does the diffraction change from the higher frequency case?
- Remove all obstacles from the ripple tank and insert a second wave generator. Turn on both generators so that they start at the same time and have the same frequency.
  - What do you notice when the two sets of wavefronts meet each other?
  - Can you identify regions of constructive and destructive interference?
- Now turn on the generators so that they are out of phase (i.e. start them so that they do not make waves at exactly the same time).
  - What do you notice when the two sets of wavefronts meet each other?
  - Can you identify regions of constructive and destructive interference?

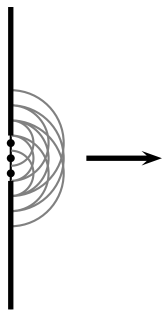
6.5 Diffraction through a single slit ESBNJ

Waves diffract when they encounter obstacles. Why does this happen? If we apply Huygens principle it becomes clear. Think about a wavefront impinging on a barrier with a slit in it, only the points on the wavefront that move into the slit can continue emitting forward moving waves - but because a lot of the wavefront has been blocked by the barrier, the points on the edges of the hole emit waves that bend round the edges. How to use this approach to understand what happens is sketched below:



Before the the wavefront strikes the barrier the wavefront generates another forward moving wavefront (applying Huygens' principle). Once the barrier blocks most of the wavefront you can see that the forward moving wavefront bends around the slit because the secondary waves they would need to interfere with to create a straight wavefront have been blocked by the barrier.

If you employ Huygens' principle you can see the effect is that the wavefronts are no longer straight lines.

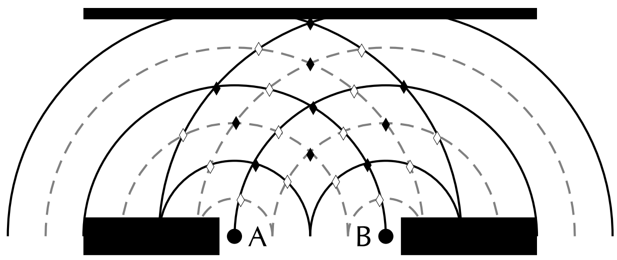


Diffraction patterns ESBNK

We can learn even more about what happens after the wavefront strikes the barrier by applying Huygens' principle further.

Each point on the wavefront moving through the slit acts like a point source. We can think about some of the effects of this if we analyse what happens when two point sources are close together and emit wavefronts with the same wavelength and frequency. These two point sources represent the point sources on the two edges of the slit and we can call the source A and source B.

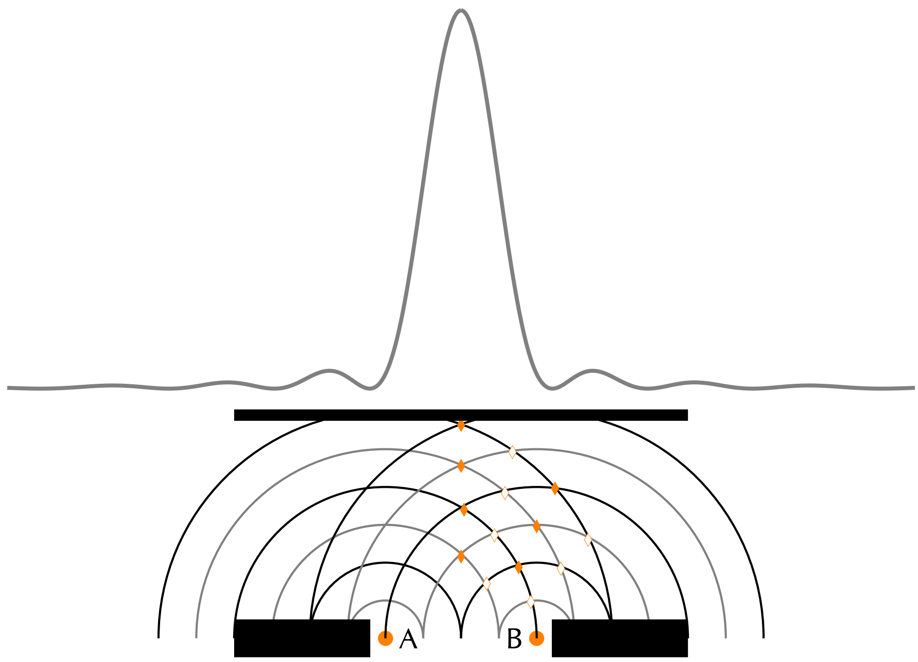
Each point source emits wavefronts from the edge of the slit. In the diagram we show a series of wavefronts emitted from each point source. The black lines show peaks in the waves emitted by the point sources and the gray lines represent troughs. We label the places where constructive interference (peak meets a peak or trough meets a trough) takes place with a solid diamond and places where destructive interference (trough meets a peak) takes place with a hollow diamond. When the wavefronts hit a barrier there will be places on the barrier where constructive interference takes place and places where destructive interference happens.



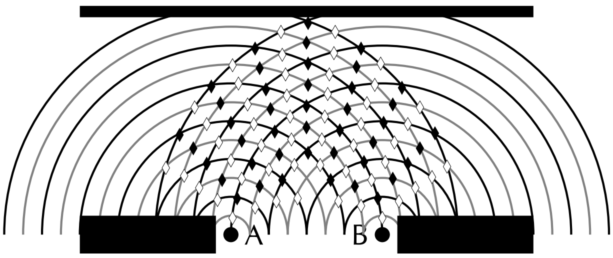
The measurable effect of the constructive or destructive interference at a barrier depends on what type of waves we are dealing with. If we were dealing with sound waves, then it would be very noisy at points along the barrier where the constructive interference is taking place and quiet where the destructive interference is taking place.

NOTE:
The pattern of constructive then destructive interference measured some distance away from a single slit is caused because of two properties of waves, diffraction **and** interference. Sometimes this pattern is called an interference pattern and sometimes it is called a diffraction pattern. Both names are correct and both properties are required for the pattern to be observed. For consistency we will call it a *diffraction pattern* in for the rest of this book.

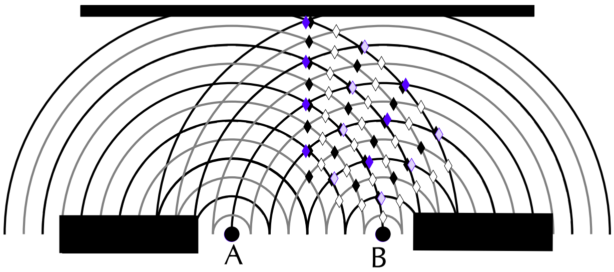
The intensity of the diffraction pattern for a single narrow slit looks like this:



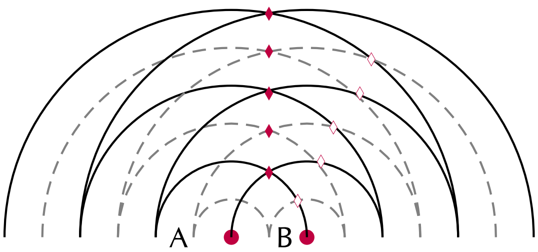
The picture above sketches how the wavefronts interfere to form the diffraction pattern. The peaks correspond to places where the waves are adding constructively and the minima are places where destructive interference is taking place. If you look at the picture you can see that if the wavelength (the distance between two consecutive peaks/troughs) of the waves were different the pattern would be different. For example, if the wavelength were halved the sketch would be:



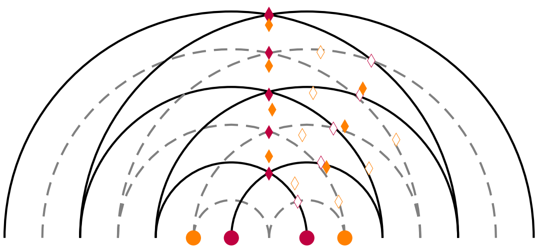
The amount that the waves diffract depends on the wavelength. We can compare the spread in the points of constructive and destructive interference by plotting the highlighted points together for the two cases. We have to line up the central maximum from the two cases to see the difference. The case where the wavelength is smaller results in smaller angles between the lines of constructive and destructive interference.



It also depends on the width of the slit, changing the width of the slit would change the distance between the points labelled A and B in the sketch. For example, if we repeat the sketch halving the distance between the points A and B we would get:



We can compare the spread in the points of constructive and destructive interference by plotting the highlighted points together for the two cases. We have to line up the central maximum from the two cases to see the difference. The case where the two points are closer together, in purple, results in bigger angles between the lines of constructive and destructive interference.



Effect of slit width and wavelength on diffraction patterns ESBNM

Using our sketches we see that the extent to which the diffracted wave passing through the slit spreads out depends on the width of the slit and the wavelength of the waves. The narrower the slit, the more diffraction there is and the shorter the wavelength the less diffraction there is. The degree to which diffraction occurs is:

$$\text{diffraction} \propto \frac{\lambda}{w}$$

where λ is the wavelength of the wave and w is the width of the slit.

We can do a sanity check on the relationship by considering some special cases, very big and very small values for each of the numerator and denominator to see what sort of behaviour we expect (this is not a calculation, just a check to see what sort outcomes we expect when we change wavelength or slit width):

- Set $\lambda = 1$ and w very large, the result will be $\frac{1}{\text{very big number}}$ which is a very small number. So for a very big slit there is very little diffraction.
- Set $\lambda = 1$ and w very small, the result will be $\frac{1}{\text{very small number}}$ which is a very big number. So for a very small slit there is large diffraction (this makes sense because eventually you are dealing with a point source which emits circular wavefronts).
- Set λ very large and $w = 1$, the result will be $\frac{\text{very big number}}{1}$ which is a very big number. So for a very big wavelength there is large diffraction.
- Set λ very small and $w = 1$, the result will be $\frac{\text{very small number}}{1}$ which is a very small number. So for a very small wavelength there is little diffraction.

Wave nature of light ESBNN

In Grade 10 we learnt about electromagnetic radiation and that visible light is a small part of the EM spectrum. EM radiation is a wave so we should see diffraction for visible light when it strikes a barrier or passes through a slit. In everyday life you don't notice diffraction of light around objects or when light passes through an open door or window. This is because the wavelength of light is very small and the ″slits″ like doors and windows are quite large.

We can put some everyday numbers into

$$\text{diffraction} \propto \frac{\lambda}{w}$$

to see how much diffraction we expect. White light is combination of light of many different colours and each colour has a different frequency or wavelength. To make things simpler lets just think about one colour, green light has a wavelength of 532×10^{-9} m. If a wavefront of green light struck the wall of a house with an open door that is 1 m wide what would we expect to see?

$$\begin{aligned}
\text{diffraction} &\propto \frac{\lambda}{w} \\
&\propto \frac{532 \times 10^{-9}\ \text{m}}{1\ \text{m}} \\
&\propto 532 \times 10^{-9}
\end{aligned}$$

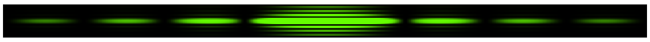
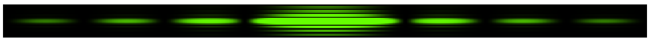


Figure 6.3: A diffraction grating reflecting green light.

The result is a very small number so we expect to see very little diffraction. In fact, the effect is so small that we cannot see it with the human eye. We can observe diffraction of green light but for us to get diffraction $\propto 1$ we need the wavelength and slit width to be the same number. So we know the effects of diffraction should become noticeable when the wavelength and slit width are similar. We can't change the wavelength of green light but there are objects called diffraction gratings that have very narrow slits that we can use to study the diffraction of light. We let wavefronts of green light strike a diffraction grating and then put a screen on the other side. We can see where the intensity of the the light on the screen is large and where it is small. For green light on a particular diffraction grating the pattern of green light on the screen looks like:



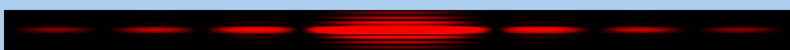
Blue light with a wavelength of 450×10^{-9} m and the same diffraction grating will produce:

Worked example 2: Diffraction

QUESTION

Two diffraction patterns are presented, determine which one has the longer wavelength based on the features of the diffraction pattern. The first pattern is for green light:

The second pattern is for red light:



The same diffraction grating is used in to generate both diffraction patterns.

SOLUTION

Step 1: Determine what is required

We need to compare the diffraction patterns to extract information about the relative wavelengths so we can decide which one is longer. We know that the diffraction pattern depends on wavelength and slit width through:

$$\text{diffraction} \propto \frac{\lambda}{w}$$

The diffraction grating is the same in both cases so we know that the slit width is fixed.

Step 2: Analyse patterns

By eye we can see that the red pattern is wider than the green pattern. There is more diffraction for the red light, this means that:

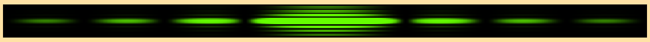
$$\text{diffraction}_{red} > \text{diffraction}_{green}$$
$$\frac{\lambda_{red}}{w} > \frac{\lambda_{green}}{w}$$
$$\lambda_{red} > \lambda_{green}$$

Step 3: Final answer

The wavelength of the red light is longer than that of the green light.

Exercise 6 – 1:

1. As the width of the slit producing a single-slit diffraction pattern is reduced, how will the diffraction pattern produced change?
2. A water break at the entrance to a harbour consists of a rock barrier with a 50 m wide opening. Ocean waves of 20 m wavelength approach the opening straight on. Light with a wavelength of 500×10^{-9} m strikes a single slit of width 30×10^{-9} m. Which waves are diffracted to a greater extent?
3. For the diffraction pattern below, sketch what you expect to change if:



a) the wavelength gets larger
b) the wavelength gets smaller
c) the slit width gets larger
d) the slit width gets smaller
e) the frequency of the wave gets smaller
f) the frequency of the wave gets larger

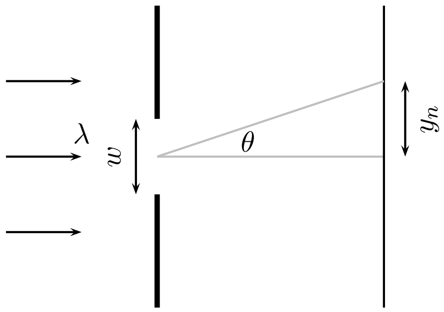
Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23V9 2. 23VB 3a. 23VC 3b. 23VD 3c. 23VF 3d. 23VG 3e. 23VH 3f. 23VJ

www.everythingscience.co.za m.everythingscience.co.za

This section is for interest and extension, **skip to end of chapter summary if you are following CAPS**.

There is a formula we can use to determine where the peaks and minima are in the interference spectrum. There will be more than one minimum. There are the same number of minima on either side of the central peak and the distances from the first one on each side are the same to the peak. The distances to the peak from the second minimum on each side is also the same, in fact the two sides are mirror images of each other. We label the first minimum that corresponds to a positive angle from the centre as $m = 1$ and the first on the other side (a negative angle from the centre) as $m = -1$, the second set of minima are labelled $m = 2$ and $m = -2$ etc.



The equation for the angle at which the minima occur is given in the definition below:

DEFINITION: *Interference Minima*

The angle at which the minima in the interference spectrum occur is:
$\sin\theta = \frac{m\lambda}{w}$
where
θ is the angle to the minimum
w is the width of the slit
λ is the wavelength of the impinging wavefronts
m is the order of the minimum, $m = \pm1, \pm2, \pm3, ...$

Worked example 3: Diffraction minimum

QUESTION

A slit with a width of 2511 nm has red light of wavelength 650 nm impinge on it. The diffracted light interferes on a surface. At which angle will the first minimum be?

SOLUTION

Step 1: Check what you are given

We know that we are dealing with diffraction patterns from the diffraction of light passing through a slit. The slit has a width of 2511 nm which is 2511×10^{-9} m and

we know that the wavelength of the light is 650 nm which is 650×10^{-9} m. We are looking to determine the angle to first minimum so we know that $m = 1$.

Step 2: Applicable principles

We know that there is a relationship between the slit width, wavelength and interference minimum angles: $\sin\theta = \frac{m\lambda}{w}$

We can use this relationship to find the angle to the minimum by substituting what we know and solving for the angle.

Step 3: Substitution

$$\begin{aligned}
\sin\theta &= \frac{650 \times 10^{-9}\text{ m}}{2511 \times 10^{-9}\text{ m}} \\
\sin\theta &= \frac{650}{2511} \\
\sin\theta &= 0{,}258861012 \\
\theta &= \sin^{-1} 0{,}258861012 \\
\theta &= 15^{\circ}
\end{aligned}$$

The first minimum is at 15° from the centre maximum.

Worked example 4: Diffraction minimum

QUESTION

A slit with a width of 2511 nm has green light of wavelength 532 nm impinge on it. The diffracted light interferes on a surface, at what angle will the first minimum be?

SOLUTION

Step 1: Check what you are given

We know that we are dealing with diffraction patterns from the diffraction of light passing through a slit. The slit has a width of 2511 nm which is 2511×10^{-9} m and we know that the wavelength of the light is 532 nm which is 532×10^{-9} m. We are looking to determine the angle to first minimum so we know that $m = 1$.

Step 2: Applicable principles

We know that there is a relationship between the slit width, wavelength and interference minimum angles: $\sin\theta = \frac{m\lambda}{w}$

We can use this relationship to find the angle to the minimum by substituting what we know and solving for the angle.

Step 3: Substitution

$$
\begin{aligned}
\sin\theta &= \frac{532 \times 10^{-9}\text{ m}}{2511 \times 10^{-9}\text{ m}} \\
\sin\theta &= \frac{532}{2511} \\
\sin\theta &= 0{,}211867782 \\
\theta &= \sin^{-1} 0{,}211867782 \\
\theta &= 12.2^{\circ}
\end{aligned}
$$

The first minimum is at 12,2° from the centre peak.

From the formula $\sin\theta = \frac{m\lambda}{w}$ you can see that a smaller wavelength for the same slit results in a smaller angle to the interference minimum. This is something you just saw in the two worked examples. Do a sanity check, go back and see if the answer makes sense. Ask yourself which light had the longer wavelength, which light had the larger angle and what do you expect for longer wavelengths from the formula.

Worked example 5: Diffraction minimum

QUESTION

A slit has a width which is unknown and has green light of wavelength 532 nm impinge on it. The diffracted light interferes on a surface, and the first minimum is measure at an angle of 20,77°?

SOLUTION

Step 1: Check what you are given

We know that we are dealing with diffraction patterns from the diffraction of light passing through a slit. We know that the wavelength of the light is 532 nm which is 532×10^{-9} m. We know the angle to first minimum so we know that $m = 1$ and $\theta = 20.77^{\circ}$.

Step 2: Applicable principles

We know that there is a relationship between the slit width, wavelength and interference minimum angles: $\sin\theta = \frac{m\lambda}{w}$

We can use this relationship to find the width by substituting what we know and solving for the width.

Step 3: Substitution

$$\sin\theta = \frac{532 \times 10^{-9} \text{ m}}{w}$$
$$\sin 20{,}77° = \frac{532 \times 10^{-9}}{w}$$
$$w = \frac{532 \times 10^{-9}}{0{,}3546666667}$$
$$w = 1500 \times 10^{-9}$$
$$w = 1500 \text{ nm}$$

The slit width is 1500 nm.

See simulation: 23VK at www.everythingscience.co.za

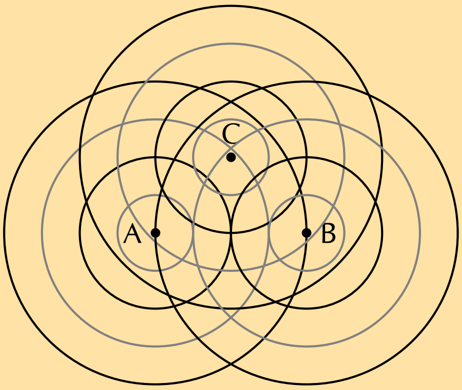
6.6 Chapter summary ESBNQ

See presentation: 23VM at www.everythingscience.co.za

- A wavefront is an imaginary line that connects waves that are in phase.
- Huygen's Principle states that every point of a wave front serves as a point source of spherical, secondary waves. After a time t, the new position of the wave front will be that of a surface tangent to the secondary waves.
- Diffraction is the ability of a wave to spread out in wavefronts as the wave passes through a small aperture or around a sharp edge.
- When a wave passes through a slit, diffraction of the wave occurs. Diffraction of the wave is when the wavefront spreads out or "bends" around corners.
- The degree of diffraction depends on the width of the slit and the wavelength of the wave with: diffraction $\propto \frac{\lambda}{w}$ where λ is the wavelength of the wave and w is the width of the slit.

Exercise 6 – 2:

1. In the diagram below the peaks of wavefronts are shown by black lines and the troughs by grey lines. Mark all the points where constructive interference between two waves is taking place and where destructive interference is taking place. Also note whether the interference results in a peak or a trough.



2. For a slit of width 1300 nm, order the following EM waves from least to most diffracted:
   a) green at 510 nm
   b) blue at 475 nm
   c) red at 650 nm
   d) yellow at 570 nm
3. For light of wavelength 540 nm, determine which of the following slits widths results in the maximum and which results in the minimum amount of diffraction
   a) 323×10^{-9} m
   b) 12,47 nm
   c) 21,1 pm
4. For light of wavelength 635 nm, determine what the width of the slit needs to be to have the diffraction be less than the angle of diffraction in each of these cases:
   a) Water waves at the entrance to a harbour which has a rock barrier with a 3 m wide opening. The waves have a wavelength of 16 m wavelength approach the opening straight on.
   b) Light with a wavelength of 786×10^{-9} m strikes a single slit of width 30×10^{-7} m.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23VN 2. 23VP 3. 23VQ 4. 23VR

www.everythingscience.co.za m.everythingscience.co.za

CHAPTER 7

Ideal gases

We are surrounded by gases in our atmosphere which support and protect life on this planet. Everyday we breathe in oxygen and release carbon dioxide. Green plants take in the carbon dioxide and release oxygen. One way or another we are surrounded by a mix of many different gases, some that we need and some that are harmful to us.
In this chapter, we are going to learn more about gases and about the different gas laws.

Key Mathematics Concepts

- Ratio and proportion — Physical Sciences, Grade 10, Science skills
- Equations — Mathematics, Grade 10, Equations and inequalities
- Graphs — Mathematics, Grade 10, Functions and graphs
- Units and unit conversions — Physical Sciences, Grade 10, Science skills

7.1 Motion of particles ESBNR

The kinetic theory of gases ESBNS

In grade 10 you learnt about the kinetic theory of matter. The kinetic theory of matter says that all matter is composed of particles which have a certain amount of energy which allows them to move at different speeds depending on the temperature (energy). There are spaces between the particles and also attractive forces between particles when they come close together.

Now we will look at applying the same ideas to gases.

The main assumptions of the kinetic theory of gases are as follows:

- Gases are made up of **particles** (e.g. atoms or molecules). The size of these particles is very small compared to the distance between the particles.
- These particles are constantly moving because they have kinetic energy. The particles move in straight lines at different speeds.
- There are **attractive forces** between particles. These forces are very weak for gases.
- The collisions between particles and the walls of the container do not change the kinetic energy of the system.
- The **temperature** of a gas is a measure of the **average kinetic energy** of the particles.

From these assumptions we can define the pressure and temperature of any gas.

DEFINITION: *Pressure*

The **pressure** of a gas is a measure of the number of collisions of the gas particles with each other and with the sides of the container that they are in.

See video: 23VS at www.everythingscience.co.za

DEFINITION: *Temperature*

The **temperature** of a substance is a measure of the **average kinetic energy** of the particles.

If the gas is heated (i.e. the temperature increases), the average kinetic energy of the gas particles will increase and if the temperature is decreased, the average kinetic energy of the particles decreases. If the energy of the particles decreases significantly, the gas liquefies (becomes a liquid).

One of the assumptions of the kinetic theory of gases is that all particles have a different speed. However, this is only the case for a **real** gas. For an ideal gas we assume that all particles in the gas have the **same** speed.

So for an ideal gas we can simply talk about the speed of particles. But for a real gas we must use the average speed of all the particles.

See video: 23VT at www.everythingscience.co.za

Ideal gases and non-ideal gas behaviour ESBNT

When we look at the gas laws in the next section we will only deal with ideal gases.

DEFINITION: *Ideal gas*

An ideal gas has identical particles of zero volume, with no intermolecular forces between them. The atoms or molecules in an ideal gas move at the same speed.

Almost all gases obey the gas laws within a limited range of pressures and temperatures. So we can use the gas laws to predict how real gases will behave.

DEFINITION: *Real gas*

Real gases behave more or less like ideal gases except at high pressures and low temperatures.

Before we go on to look at the gas laws we will first see what happens to gases at high pressures and low temperatures.

When we defined an ideal gas, we said that an ideal gas has identical particles of

zero volume and that there are no intermolecular forces between the particles in the gas. We need to look more closely at these statements because they affect how gases behave at high pressures or at low temperatures.

1. *Molecules do occupy volume*

   When pressures are very high and the molecules are compressed, the volume of the molecules becomes significant. This means that the total volume available for the gas molecules to move is reduced and collisions become more frequent. This causes the pressure of the gas to be *higher* than what would be expected for an ideal gas (Figure 7.1).

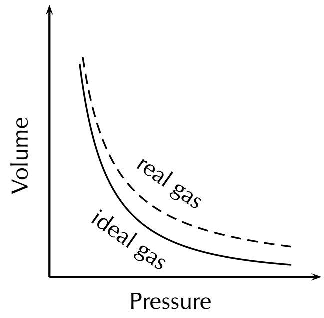


Figure 7.1:
Gases deviate from ideal gas behaviour at high pressure.

2. *Forces of attraction do exist between molecules*

   At low temperatures, when the speed of the molecules decreases and they move closer together, the intermolecular forces become more apparent. As the attraction between molecules increases, their movement decreases and there are fewer collisions between them. The pressure of the gas at low temperatures is therefore lower than what would have been expected for an ideal gas (Figure 7.2). If the temperature is low enough or the pressure high enough, a real gas will **liquefy**.

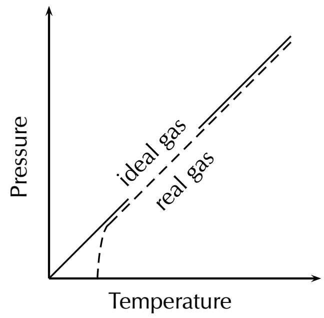


Figure 7.2:
Gases deviate from ideal gas behaviour at low temperatures.

Exercise 7 – 1:

1. Summarise the difference between a real gas and an ideal gas in the following table:

Property	Ideal gas	Real gas
Size of particles		
Attractive forces		
Speed of molecules		

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23VV

www.everythingscience.co.za **m.everythingscience.co.za**

7.2 Ideal gas laws ESBNV

There are several laws to explain the behaviour of ideal gases. The first three that we will look at apply under very strict conditions. These laws are then combined to form the general gas equation and the ideal gas equation.

Before we start looking at these laws we need to look at some common conversions for units.

The following table gives the SI units. This table also shows how to convert between common units. Do not worry if some of the units are strange to you. By the end of this chapter you will have had a chance to see all these units in action.

Variable	SI Units	Other units
Pressure (p)	Pascals (Pa)	760 mm Hg = 1 atm = 101 325 Pa = 101,325 kPa
Volume (V)	m^3	$1\ m^3 = 1\ 000\ 000\ cm^3$ $= 1000\ dm^3$ = 1000 L
Moles (n)	mol	
Universal gas constant (R)	$J{\cdot}K^{-1}{\cdot}mol^{-1}$	
Temperature (K)	Kelvin (K)	K = °C + 273

Table 7.1: Conversion table showing SI units of measurement and common conversions.

Two very useful volume relations to remember are: 1 mL = 1 cm^3 and 1 L = 1 dm^3.

Boyle's law: Pressure and volume of an enclosed gas ESBNW

If you have ever tried to force in the plunger of a syringe or a bicycle pump while sealing the opening with your finger, you will have seen Boyle's Law in action! The following experiment will allow you to see this law in action.

DEFINITION: *Boyle's Law*

The pressure of a fixed quantity of gas is inversely proportional to the volume it occupies so long as the temperature remains constant.

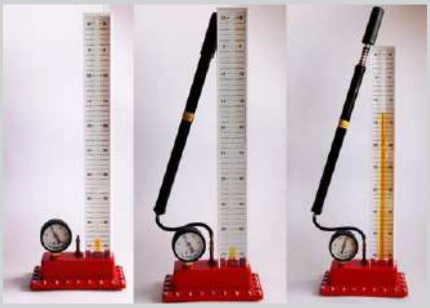
Informal experiment: Boyle's law

Aim:

To verify Boyle's law.

Apparatus:

Boyle's law apparatus (a syringe or bicycle pump attached to a pressure gauge.)



Method:

1. Use the pump to fill the syringe (or glass tube) until the pressure gauge reads the maximum value. Note the volume and the pressure reading.
2. Slowly release some of the air until the pressure has dropped by about 20 units (the units will depend on what your pressure gauge measures, e.g. kPa).
3. Let the system stabilise for about 2 min and then read the volume.
4. Repeat the above two steps until you have six pairs of pressure-volume readings.

Results:

Record your results in the following table. Remember that your pressure and volume units will be determined by the apparatus you are using.

Pressure	Volume

What happens to the volume as the pressure decreases?

Plot your results as a graph of pressure versus volume (in other words plot pressure on the x-axis and volume on the y-axis). Pressure is the independent variable, which we are changing to see what happens to volume.

Plot your results as a graph of pressure versus the inverse of volume (in other words $\frac{1}{V}$, so for each volume reading you will work out the value of 1 divided by the volume reading).

What do you notice about each graph?

Conclusion:

If the volume of the gas decreases, the pressure of the gas increases. If the volume of the gas increases, the pressure decreases. These results support Boyle's law.

In the above experiment, the volume of the gas decreased when the pressure increased, and the volume increased when the pressure decreased. This is called an **inverse relationship** (or more-less relationship). The inverse relationship between pressure and volume is shown in Figure 7.3.

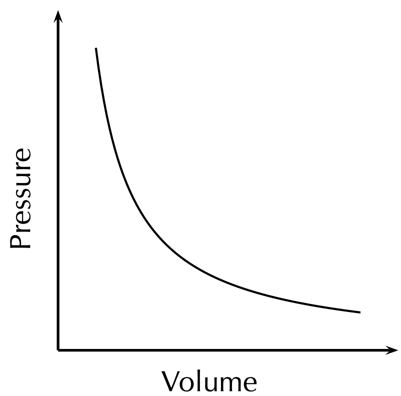
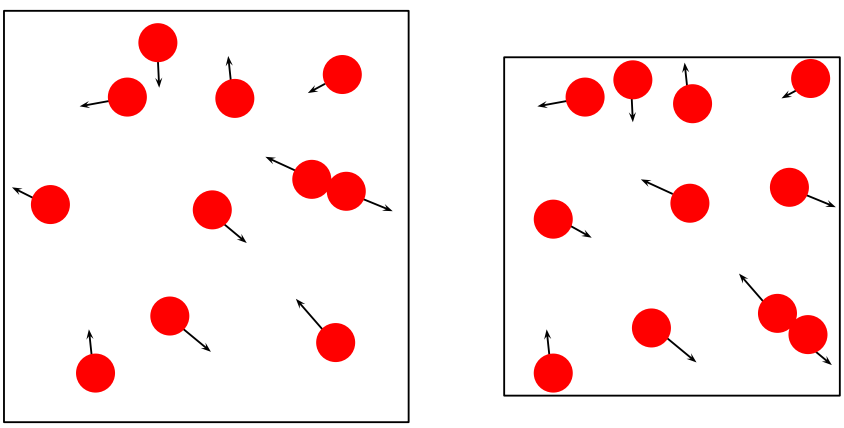


Figure 7.3: Graph showing the inverse relationship between pressure and volume.

Can you use the kinetic theory of gases to explain this inverse relationship between the pressure and volume of a gas? Let's think about it. If you decrease the volume of a gas, this means that the same number of gas particles are now going to come into contact with each other and with the sides of the container much more often. But we said that **pressure** is a measure of the *number of collisions* of gas particles with each other and with the sides of the container they are in. So, if the volume decreases, the number of collisions increases and so the pressure will naturally increase. The opposite is true if the volume of the gas is increased. Now, the gas particles collide less frequently and the pressure will decrease.



Robert Boyle is the scientist who is credited with discovering that the pressure and volume of a sample of gas are **inversely proportional**. This can be seen when a graph of pressure against the inverse of volume is plotted. When the values are plotted, the graph is a straight line. This relationship is shown in Figure 7.4.

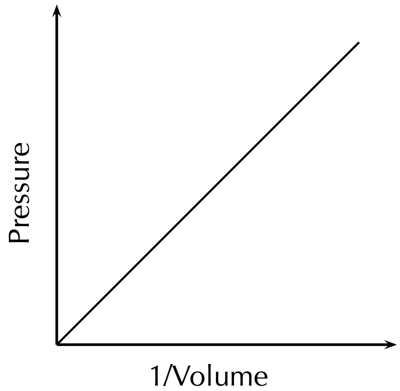


Figure 7.4: The graph of pressure plotted against the inverse of volume, produces a straight line. This shows that pressure and volume are inversely proportional.

We have just seen that the pressure of a gas is *inversely proportional* to the volume of the gas, provided the temperature stays the same. We can write this relationship symbolically as:

$$p \propto \frac{1}{V}$$

where $\propto$ means proportional and we write $\frac{1}{V}$ to show that the proportionality is inverse.

This equation can also be written as follows:

$$p = \frac{k}{V}$$

where k is a proportionality constant. If we rearrange this equation, we can say that:

$$pV = k$$

This equation means that, assuming the temperature and amount of gas is constant, multiplying any pressure and volume values for a fixed amount of gas will always give the same value (k). For example:

$$\begin{aligned} p_1V_1 &= k \\ p_2V_2 &= k \end{aligned}$$

where the subscripts 1 and 2 refer to two pairs of pressure and volume readings for the same mass of any gas at the same temperature.

From this, we can then say that:

$$\boxed{p_1V_1 = p_2V_2}$$

And we can also generalise this to say that:

$$\begin{aligned} p_1V_1 &= p_2V_2 \\ &= p_3V_3 \\ &= p_nV_n \end{aligned}$$

In other words we can use any two pairs of readings, it does not have to be the first and second readings, it can be the first and third readings or the second and fifth readings.

If you work out the value of k for any pair of pressure readings from the experiment above and then work out k for any other pair of pressure readings you should find they are the same.

For example if you have:

$$\begin{aligned}p_1 &= 1 \text{ atm}\\ p_2 &= 2 \text{ atm}\\ V_1 &= 4 \text{ cm}^3\\ V_2 &= 2 \text{ cm}^3\end{aligned}$$

then using the first pressure and volume gives:

$$\begin{aligned}k &= p_1V_1\\ k &= (1 \text{ atm})(4 \text{ cm}^3)\\ &= 4 \text{ atm}\cdot\text{cm}^3\end{aligned}$$

and using the second pressure and volume gives:

$$\begin{aligned}k &= p_2V_2\\ &= (2 \text{ atm})(2 \text{ cm}^3)\\ &= 4 \text{ atm}\cdot\text{cm}^3\end{aligned}$$

Remember that Boyle's Law requires two conditions. First, the **amount** of gas must stay constant. If you let a little of the air escape from the container in which it is enclosed, the pressure of the gas will decrease along with the volume, and the inverse proportion relationship is broken. Second, the **temperature** must stay constant. Cooling or heating matter generally causes it to contract or expand, or the pressure to decrease or increase. In the experiment, if we were to heat up the gas, it would expand and require you to apply a greater force to keep the plunger at a given position. Again, the proportionality would be broken.

See simulation: 23VW at www.everythingscience.co.za

Before we look at some calculations using Boyle's law we first need to know some different units for volume and pressure. Volume units should be familiar to you from earlier grades and will usually be cm^3 or dm^3 or m^3 or L. The SI unit for volume is m^3.

Pressure is measured in several different units. We can measure pressure in millimetres of mercury (mm Hg) or pascals (Pa) or atmospheres (atm). The SI unit for pressure is Pa. See Table 7.1 for converting between units.

FACT

Did you know that the mechanisms involved in *breathing* also relate to Boyle's Law? Just below the lungs is a muscle called the **diaphragm**. When a person breathes in, the diaphragm moves down and becomes more "flattened" so that the volume of the lungs can increase. When the lung volume *increases*, the pressure in the lungs *decreases* (Boyle's law). Since air always moves from areas of high pressure to areas of lower pressure, air will now be drawn into the lungs because the air pressure *outside* the body is higher than the pressure *in* the lungs. The opposite process happens when a person breathes out. Now, the diaphragm moves upwards and causes the volume of the lungs to *decrease*. The pressure in the lungs will *increase*, and the air that was in the lungs will be forced out towards the lower air pressure outside the body.

Worked example 1: Boyle's law

QUESTION

A sample of helium occupies a volume of 160 cm^3 at 100 kPa and 25 °C. What volume will it occupy if the pressure is adjusted to 80 kPa and the temperature remains unchanged?

TIP

It is not necessary to convert to SI units for Boyle's law. Changing pressure and volume into different units involves *multiplication*. If you were to change the units in the above equation, this would involve multiplication on both sides of the equation, and so the conversions cancel each other out. However, although *SI* units don't have to be used, you must make sure that for each variable you use the *same* units throughout the equation. This is not true for some of the calculations we will do at a later stage, where SI units *must* be used.

SOLUTION

Step 1: Write down all the information that you know about the gas.

$$\begin{aligned}p_1 &= 100 \text{ kPa}\\ p_2 &= 80 \text{ kPa}\\ V_1 &= 160 \text{ cm}^3\\ V_2 &= ?\end{aligned}$$

Step 2: Use an appropriate gas law equation to calculate the unknown variable.

Because the temperature of the gas stays the same, the following equation can be used:

$$p_2V_2 = p_1V_1$$

Step 3: Substitute the known values into the equation, making sure that the units for each variable are the *same*. Calculate the unknown variable.

$$\begin{aligned}(80)V_2 &= (100)(160)\\ (80)V_2 &= 16\ 000\\ V_2 &= 200 \text{ cm}^3\end{aligned}$$

The volume occupied by the gas at a pressure of 80 kPa is 200 cm^3

Step 4: Check your answer

Boyle's law states that the pressure is inversely proportional to the volume. Since the pressure decreased the volume must increase. Our answer for the volume is greater than the initial volume and so our answer is reasonable.

Worked example 2: Boyle's law

QUESTION

The volume of a sample of gas is increased from 2,5 L to 2,8 L while a constant temperature is maintained. What is the final pressure of the gas under these volume conditions, if the initial pressure is 695 Pa?

SOLUTION

Step 1: Write down all the information that you know about the gas.

$$\begin{aligned} V_1 &= 2{,}5 \text{ L} \\ V_2 &= 2{,}8 \text{ L} \\ p_1 &= 695 \text{ Pa} \\ p_2 &= ? \end{aligned}$$

Step 2: Choose a relevant gas law equation to calculate the unknown variable.

The sample of gas is at constant temperature and so we can use Boyle's law:

$$p_2V_2 = p_1V_1$$

Step 3: Substitute the known values into the equation, making sure that the units for each variable are the *same*. Calculate the unknown variable.

$$\begin{aligned} (2{,}8)p_2 &= (695)(2{,}5) \\ (2{,}8)p_2 &= 1737{,}5 \\ p_2 &= 620{,}5 \text{ kPa} \end{aligned}$$

The pressure of the gas at a volume of 2,8 L is 620,5 kPa

Step 4: Check your answer

Boyle's law states that the pressure is inversely proportional to the volume. Since the volume increased the pressure must decrease. Our answer for the pressure is less than the initial pressure and so our answer is reasonable.

Exercise 7 – 2: Boyle's law

1. An unknown gas has an initial pressure of 150 kPa and a volume of 1 L. If the volume is increased to 1,5 L, what will the pressure be now?
2. A bicycle pump contains 250 cm^3 of air at a pressure of 90 kPa. If the air is compressed, the volume is reduced to 200 cm^3. What is the pressure of the air inside the pump?

3. The air inside a syringe occupies a volume of 10 cm^3 and exerts a pressure of 100 kPa. If the end of the syringe is sealed and the plunger is pushed down, the pressure increases to 120 kPa. What is the volume of the air in the syringe?

4. During an investigation to find the relationship between the pressure and volume of an enclosed gas at constant temperature, the following results were obtained.

Volume (dm^3)	12	16	20	24	28	32	36	40
Pressure (kPa)	400	300	240	200	171	150	133	120

   a) Plot a graph of pressure (p) against volume (V). Volume will be on the x-axis and pressure on the y-axis. Describe the relationship that you see.

   b) Plot a graph of p against $\frac{1}{V}$. Describe the relationship that you see.

   c) Do your results support Boyle's Law? Explain your answer.

5. Masoabi and Justine are experimenting with Boyle's law. They both used the same amount of gas. Their data is given in the table below:

	Masoabi		**Justine**	
	Initial	**Final**	**Initial**	**Final**
Temperature (K)	325	350	325	325
Volume (dm^3)	1	3	1	3
Pressure (Pa)	650	233	650	217

   Masoabi and Justine argue about who is correct.

   a) Calculate the final pressure that would be expected using the initial pressure and volume and the final volume.

   b) Who correctly followed Boyle's law and why?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23VX 2. 23VY 3. 23VZ 4. 23W2 5. 23W3

www.everythingscience.co.za m.everythingscience.co.za

Charles' law: Volume and temperature of an enclosed gas ESBNX

Charles' law describes the relationship between the **volume** and **temperature** of a gas. The law was first published by Joseph Louis Gay-Lussac in 1802, but he referenced unpublished work by Jacques Charles from around 1787. This law states that at constant pressure, the volume of a given mass of an ideal gas increases or decreases by the same factor as its temperature (in Kelvin) increases or decreases. Another way of

saying this is that temperature and volume are **directly proportional**.

DEFINITION: *Charles' Law*

The volume of an enclosed sample of gas is directly proportional to its Kelvin temperature provided the pressure and amount of gas is kept constant.

Informal experiment: Charles's law

Aim:

To demonstrate Charles's Law.

Apparatus:

glass bottle (e.g. empty glass coke bottle), balloon, beaker or pot, water, hot plate

Method:

1. Place the balloon over the opening of the empty bottle.
2. Fill the beaker or pot with water and place it on the hot plate.
3. Stand the bottle in the beaker or pot and turn the hot plate on.
4. Observe what happens to the balloon.

Results:

You should see that the balloon starts to expand. As the air inside the bottle is heated, the pressure also increases, causing the volume to increase. Since the volume of the glass bottle can't increase, the air moves into the balloon, causing it to expand.

Conclusion:

The temperature and volume of the gas are directly related to each other. As one increases, so does the other.

You can also see this if you place the balloon and bottle into a freezer. The balloon will shrink after being in the freezer for a short while.

Mathematically, the relationship between temperature and pressure can be represented as follows:

$$V \propto T$$

In other words, the volume is directly proportional to the temperature.

Or, replacing the proportionality symbol with the constant of proportionality (k):

$$V = kT$$

If the equation is rearranged, then:

$$\frac{V}{T} = k$$

or:

$$\frac{V_1}{T_1} = k$$
$$\frac{V_2}{T_2} = k$$
$$\frac{V_n}{T_n} = k$$

So we can say that:

$$\boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}}$$

The equation relating volume and temperature produces a straight line graph.

However, if we plot this graph using the **Celsius** temperature scale (i.e. using °C), the zero point of temperature doesn't correspond to the zero point of volume. When the volume is zero, the temperature is actually -273 °C (Figure 7.5).

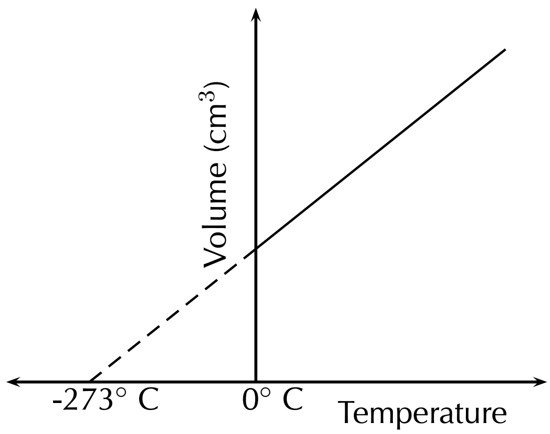


Figure 7.5: The relationship between volume and temperature, shown on the Celsius temperature scale.

A new temperature scale, the **Kelvin scale** must be used instead. Since zero on the Celsius scale corresponds with a Kelvin temperature of -273 °C, it can be said that:

$\text{Kelvin temperature } (T_K) = \text{Celsius temperature } (T_C) + 273$

We can write:

$T_K = T_C + 273$ or $T_C = T_K - 273$

We can now plot the graph of temperature versus volume on the Kelvin scale. This is shown in Figure 7.6.

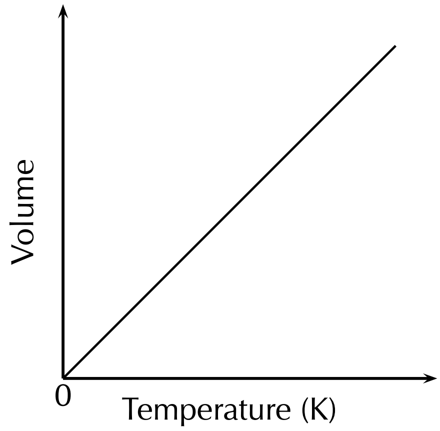


Figure 7.6: The volume of a gas is directly proportional to its temperature, provided the pressure of the gas is constant.

See video: 23W4 at www.everythingscience.co.za

Can you explain Charles' law in terms of the kinetic theory of gases? When the temperature of a gas increases, so does the average speed of its molecules. The molecules collide with the walls of the container more often and with greater impact. These collisions will push back the walls, so that the gas occupies a greater volume than it did at the start. We saw this in the first demonstration. Because the glass bottle couldn't expand, the gas pushed out the balloon instead.

Worked example 3: Charles' law

QUESTION

At a temperature of 298 K , a certain amount of CO_2 gas occupies a volume of 6 L. What temperature will the gas be at if its volume is reduced to 5,5 L? The pressure remains constant.

SOLUTION

Step 1: Write down all the information that you know about the gas.

$$V_1 = 6 \text{ L}$$
$$V_2 = 5{,}5 \text{ L}$$
$$T_1 = 298 \text{ K}$$
$$T_2 = ?$$

Step 2: Convert the known values to SI units if necessary.

Temperature data is already in Kelvin, and so no conversions are necessary.

Step 3: Choose a relevant gas law equation that will allow you to calculate the unknown variable.

The pressure is kept constant while the volume and temperature are being varied. The amount of gas is also kept constant and so we can use Charles' law:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Step 4: Substitute the known values into the equation. Calculate the unknown variable.

$$\begin{aligned}\frac{6}{298} &= \frac{5,5}{T_2}\\ 0,0201\ldots &= \frac{5,5}{T_2}\\ (0,0201\ldots)T_2 &= 5,5\\ T_2 &= 273,2 \text{ K}\end{aligned}$$

The gas will be at a temperature of 273,2 K.

Step 5: Check your answer

Charles' law states that the temperature is directly proportional to the volume. In this example the volume decreases and so the temperature must decrease. Our answer gives a lower final temperature than the initial temperature and so is correct.

Worked example 4: Charles' law

QUESTION

Ammonium chloride and calcium hydroxide are allowed to react. The ammonia that is released in the reaction is collected in a gas syringe (a syringe that has very little friction so that the plunger can move freely) and sealed in. This gas is allowed to come to room temperature which is 20 °C. The volume of the ammonia is found to be 122 mL. It is now placed in a water bath set at 32 °C. What will be the volume reading after the syringe has been left in the bath for 1 hour (assume the plunger moves completely freely)? (By leaving the syringe for this length of time, we can be certain that the sample of gas is at the higher temperature.)

SOLUTION

Step 1: Write down all the information that you know about the gas.

$$V_1 = 122 \text{ mL}$$
$$V_2 = ?$$
$$T_1 = 20\ °\text{C}$$
$$T_2 = 7\ °\text{C}$$

Step 2: Convert the known values to SI units if necessary.

Here, temperature must be converted into Kelvin, therefore:

$$T_1 = 20 + 273 = 293 \text{ K}$$
$$T_2 = 32 + 273 = 305 \text{ K}$$

Step 3: Choose a relevant gas law equation that will allow you to calculate the unknown variable.

The pressure is kept constant while the volume and temperature are being varied. The amount of gas is also kept constant and so we can use Charles' law:

$$\frac{V_2}{T_2} = \frac{V_1}{T_1}$$

Step 4: Substitute the known values into the equation. Calculate the unknown variable.

$$\frac{V_2}{305} = \frac{122}{293}$$
$$\frac{V_2}{305} = 0{,}416\ldots$$
$$V_2 = 127 \text{ mL}$$

The volume reading on the syringe will be 127 mL after the syringe has been left in the water bath for one hour.

Step 5: Check your answer

Charles' law states that the temperature is directly proportional to the volume. In this example the temperature increases and so the volume must increase. Our answer gives a higher final volume than the initial volume and so is correct.

TIP
Note that here the temperature must be converted to Kelvin since the change from degrees Celsius involves addition, not multiplication by a fixed conversion ratio (as is the case with pressure and volume.)

TIP
You may see this law referred to as Gay-Lussac's law or as Amontons' law. Many scientists were working on the same problems at the same time and it is often difficult to know who actually discovered a particular law.

Exercise 7 – 3: Charles' law

1. The table below gives the temperature (in °C) of helium gas under different volumes at a constant pressure.

Volume (L)	Temperature (°C)
1,0	−161,9
1,5	−106,7
2	−50,8
2,5	4,8
3,0	60,3
3,5	115,9

   a) Draw a graph to show the relationship between temperature and volume.
   b) Describe the relationship you observe.
   c) If you extrapolate the graph (in other words, extend the graph line even though you may not have the exact data points), at what temperature does it intersect the x-axis?
   d) What is significant about this temperature?
   e) What conclusions can you draw? Use Charles' law to help you.

2. A sample of nitrogen monoxide (NO) gas is at a temperature of 8 °C and occupies a volume of 4,4 dm^3. What volume will the sample of gas have if the temperature is increased to 25 °C?

3. A sample of oxygen (O_2) gas is at a temperature of 340 K and occupies a volume of 1,2 dm^3. What temperature will the sample of gas be at if the volume is decreased to 200 cm^3?

4. Explain what would happen if you were verifying Charles' law and you let some of the gas escape.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23W5 2. 23W6 3. 23W7 4. 23W8

www.everythingscience.co.za m.everythingscience.co.za

Pressure-temperature relation ESBNY

The pressure of a gas is directly proportional to its temperature, if the volume is kept constant (Figure 7.7). Recall that as the temperature of a gas increases, so does the kinetic energy of the particles in the gas. This causes the particles in the gas to move more rapidly and to collide with each other and with the side of the container more often. Since pressure is a measure of these collisions, the pressure of the gas increases with an increase in temperature. The pressure of the gas will decrease if its temperature decreases.

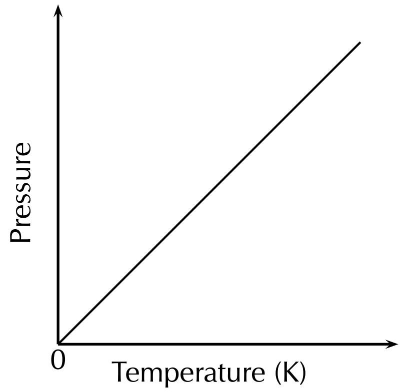


Figure 7.7: The relationship between the temperature and pressure of a gas.

In the same way that we have done for the other gas laws, we can describe the relationship between temperature and pressure using symbols, as follows:

$T \propto p$,

therefore:

$p = kT$

Rearranging this we get:

$$\frac{p}{T} = k$$

and that, provided the **amount** of gas stays the same (and the volume also stays the same):

$$\boxed{\frac{p_1}{T_1} = \frac{p_2}{T_2}}$$

Worked example 5: Pressure-temperature relation

QUESTION

At a temperature of 298 K , a certain amount of oxygen (O_2) gas has a pressure of 0,4 atm. What temperature will the gas be at if its pressure is increased to 0,7 atm?

SOLUTION

Step 1: Write down all the information that you know about the gas.

$$T_1 = 298 \text{ K}$$
$$T_2 = ?$$
$$p_1 = 0{,}4 \text{ atm}$$
$$p_2 = 0{,}7 \text{ atm}$$

Step 2: Convert the known values to SI units if necessary.

The temperature is already in Kelvin. We do not need to convert the pressure to pascals.

Step 3: Choose a relevant gas law equation that will allow you to calculate the unknown variable.

The volume is kept constant while the pressure and temperature are being varied. The amount of gas is also kept constant and so we can use the pressure-temperature relation:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

Step 4: Substitute the known values into the equation. Calculate the unknown variable.

$$\begin{aligned} \frac{0{,}4}{298} &= \frac{0{,}7}{T_2} \\ 0{,}0013\ldots &= \frac{0{,}7}{T_2} \\ (0{,}0013\ldots)T_2 &= 0{,}7 \\ T_2 &= 521{,}5 \text{ K} \end{aligned}$$

The temperature will be 521,5 K.

Step 5: Check your answer

The pressure-temperature relation states that the pressure is directly proportional to the temperature. In this example the pressure increases and so the temperature must increase. Our answer gives a higher final temperature than the initial temperature and so is correct.

Worked example 6: Pressure-temperature relation

QUESTION

A fixed volume of carbon monoxide (CO) gas has a temperature of 32 °C and a pressure of 680 Pa. If the temperature is decreased to 15 °C what will the pressure be?

SOLUTION

Step 1: Write down all the information that you know about the gas.

$$T_1 = 32\ °C$$
$$T_2 = 15\ °C$$
$$p_1 = 680\ Pa$$
$$p_2 = ?$$

Step 2: Convert the known values to SI units if necessary.

We need to convert the given temperatures to Kelvin temperature.

$$\begin{aligned} T_1 &= 32 + 273 \\ &= 305\ K \\ T_2 &= 15 + 273 \\ &= 288\ K \end{aligned}$$

Step 3: Choose a relevant gas law equation that will allow you to calculate the unknown variable.

$$\frac{p_2}{T_2} = \frac{p_1}{T_1}$$

Step 4: Substitute the known values into the equation. Calculate the unknown variable.

$$\begin{aligned} \frac{p_2}{288} &= \frac{680}{305} \\ \frac{p_2}{288} &= 2{,}2295\ldots \\ p_2 &= 642{,}1\ Pa \end{aligned}$$

The pressure will be 642,1 Pa.

Step 5: Check your answer

The pressure-temperature relation states that the pressure is directly proportional to the temperature. In this example the temperature decreases and so the pressure must decrease. Our answer gives a lower final pressure than the initial pressure and so is correct.

Exercise 7 – 4: Pressure-temperature relation

1. The table below gives the temperature (in °C) of helium under different pressures

at a constant volume.

Pressure (atm)	Temperature (°C)	Temperature (K)
1,0	20	
1,2	78,6	
1,4	137,2	
1,6	195,8	
1,8	254,4	
2,0	313	

a) Convert all the temperature data to Kelvin.

b) Draw a graph to show the relationship between temperature and pressure.

c) Describe the relationship you observe.

2. A cylinder that contains methane gas is kept at a temperature of 15 °C and exerts a pressure of 7 atm. If the temperature of the cylinder increases to 25 °C, what pressure does the gas now exert?

3. A cylinder of propane gas at a temperature of 20 °C exerts a pressure of 8 atm. When a cylinder has been placed in sunlight, its temperature increases to 25 °C. What is the pressure of the gas inside the cylinder at this temperature?

4. A hairspray can is a can that contains a gas under high pressures. The can has the following warning written on it: "Do not place near open flame. Do not dispose of in a fire. Keep away from heat." Use what you know about the pressure and temperature of gases to explain why it is dangerous to not follow this warning.

5. A cylinder of acetylene gas is kept at a temperature of 291 K. The pressure in the cylinder is 5 atm. This cylinder can withstand a pressure of 8 atm before it explodes. What is the maximum temperature that the cylinder can safely be stored at?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23W9 2. 23WB 3. 23WC 4. 23WD 5. 23WF

www.everythingscience.co.za **m.everythingscience.co.za**

The general gas equation ESBNZ

All the gas laws we have described so far rely on the fact that the amount of gas and one other variable (temperature, pressure or volume) remains constant. Since this is unlikely to be the case most times, it is useful to combine the relationships into one equation. We will use Boyle's law and the pressure-temperature relation to work out the general gas equation.

Boyle's law: $p \propto \frac{1}{V}$ (constant T)

In other words pressure is inversely proportional to the volume at constant temperature.

Pressure-temperature relation: $p \propto T$ (constant V)

In other words pressure is also directly proportional to the temperature at constant volume.

If we now vary both the volume and the temperature, the two proportionalities will still hold, but will be equal to a different proportionality constant.

So we can combine these relationships, to get:

$$p \propto \frac{T}{V}$$

Note that this says that the pressure is still directly proportional to the temperature and inversely proportional to the volume.

If we introduce the proportionality constant, k, we get:

$$p = k\frac{T}{V}$$

or, rearranging the equation:

$$pV = kT$$

We can also rewrite this relationship as follows:

$$\frac{pV}{T} = k$$

Provided the mass of the gas stays the same, we can also say that:

$$\boxed{\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}}$$

In the above equation, the subscripts 1 and 2 refer to two pressure and volume readings for the same mass of gas under different conditions. This is known as the **general gas equation**. Temperature is always in Kelvin and the units used for pressure and volume must be the same on both sides of the equation.

TIP

Remember that the general gas equation only applies if the mass (or number of moles) of the gas is fixed.

Worked example 7: General gas equation

QUESTION

At the beginning of a journey, a truck tyre has a volume of 30 dm^3 and an internal pressure of 170 kPa. The temperature of the tyre is 16 °C. By the end of the trip, the volume of the tyre has increased to 32 dm^3 and the temperature of the air inside the tyre is 40 °C. What is the tyre pressure at the end of the journey?

SOLUTION

Step 1: Write down all the information that you know about the gas.

$$T_1 = 16\ ^\circ\text{C}$$
$$T_2 = 40\ ^\circ\text{C}$$
$$V_1 = 30\ \text{dm}^3$$
$$V_2 = 32\ \text{dm}^3$$
$$p_1 = 170\ \text{kPa}$$
$$p_2 = ?$$

Step 2: Convert the known values to SI units if necessary.

We need to convert the given temperatures to Kelvin temperature.

$$\begin{aligned} T_1 &= 16 + 273 \\ &= 289\ \text{K} \\ T_2 &= 40 + 273 \\ &= 313\ \text{K} \end{aligned}$$

Step 3: Choose a relevant gas law equation that will allow you to calculate the unknown variable.

Temperature, pressure and volume are all varying so we must use the general gas equation:

$$\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1}$$

Step 4: Substitute the known values into the equation. Calculate the unknown variable.

$$\begin{aligned} \frac{(32)p_2}{313} &= \frac{(170)(30)}{289} \\ \frac{(32)p_2}{313} &= 17{,}647\ldots \\ (32)p_2 &= 5523{,}529\ldots \\ p_2 &= 172{,}6\ \text{kPa} \end{aligned}$$

The pressure will be 172,6 kPa.

Worked example 8: General gas equation

QUESTION

A sample of a gas exerts a pressure of 100 kPa at 15 °C. The volume under these conditions is 10 dm^3. The pressure increases to 130 kPa and the temperature increases to 32 °C. What is the new volume of the gas?

SOLUTION

Step 1: Write down all the information that you know about the gas.

$$\begin{aligned} T_1 &= 15\ °\text{C} \\ T_2 &= 32\ °\text{C} \\ p_1 &= 100\ \text{kPa} \\ p_2 &= 130\ \text{kPa} \\ V_1 &= 10\ \text{dm}^3 \\ V_2 &= ? \end{aligned}$$

Step 2: Convert the known values to SI units if necessary.

Here, temperature must be converted into Kelvin, therefore:

$$\begin{aligned} T_1 &= 15 + 273 = 288\ \text{K} \\ T_2 &= 32 + 273 = 305\ \text{K} \end{aligned}$$

Step 3: Choose a relevant gas law equation that will allow you to calculate the unknown variable.

We use the general gas equation:

$$\frac{p_2V_2}{T_2} = \frac{p_1V_1}{T_1}$$

Step 4: Substitute the known values into the equation. Calculate the unknown variable.

$$\begin{aligned} \frac{(130)V_2}{305} &= \frac{(100)(10)}{288} \\ \frac{(130)V_2}{305} &= 3{,}47\ldots \\ (130)V_2 &= 1059{,}027\ldots \\ V_2 &= 8{,}15\ \text{dm}^3 \end{aligned}$$

The volume will be 8,15 dm^3.

Worked example 9: General gas equation

QUESTION

A cylinder of propane gas is kept at a temperature of 298 K. The gas exerts a pressure of 5 atm and the cylinder holds 4 dm^3 of gas. If the pressure of the cylinder increases to 5,2 atm and 0,3 dm^3 of gas leaks out, what temperature is the gas now at?

SOLUTION

Step 1: Write down all the information that you know about the gas.

$$
\begin{aligned}
T_1 &= 298 \text{ K} \\
T_2 &= ? \\
V_1 &= 4 \text{ dm}^3 \\
V_2 &= 4 - 0{,}3 = 3{,}7 \text{ dm}^3 \\
p_1 &= 5 \text{ atm} \\
p_2 &= 5{,}2 \text{ atm}
\end{aligned}
$$

Step 2: Convert the known values to SI units if necessary.

Temperature data is already in Kelvin. All other values are in the same units.

Step 3: Choose a relevant gas law equation that will allow you to calculate the unknown variable.

Since volume, pressure and temperature are varying, we must use the general gas equation:

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

Step 4: Substitute the known values into the equation. Calculate the unknown variable.

$$
\begin{aligned}
\frac{(5)(4)}{298} &= \frac{(5{,}2)(3{,}7)}{T_2} \\
0{,}067\ldots &= \frac{19{,}24}{T_2} \\
(0{,}067\ldots)T_2 &= 19{,}24 \\
T_2 &= 286{,}7 \text{ K}
\end{aligned}
$$

The temperature will be 286,7 K.

Exercise 7 – 5: The general gas equation

1. A closed gas system initially has a volume of 8 L and a temperature of 100 °C. The pressure of the gas is unknown. If the temperature of the gas decreases to 50 °C, the gas occupies a volume of 5 L and the pressure of the gas is 1,2 atm. What was the initial pressure of the gas?
2. A balloon is filled with helium gas at 27 °C and a pressure of 1,0 atm. As the balloon rises, the volume of the balloon increases by a factor of 1,6 and the temperature decreases to 15 °C. What is the final pressure of the gas (assuming none has escaped)?
3. 25 cm^3 of gas at 1 atm has a temperature of 25 °C. When the gas is compressed to 20 cm^3, the temperature of the gas increases to 28 °C. Calculate the final pressure of the gas.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23WG 2. 23WH 3. 23WJ

www.everythingscience.co.za m.everythingscience.co.za

TIP

A joule can be defined as Pa·m^3. So when you are using the ideal gas equation you must use the SI units to ensure that you get the correct answer.

The ideal gas equation ESBP2

In the early 1800's, Amedeo Avogadro noted that if you have samples of different gases, of the same volume, at a fixed temperature and pressure, then the samples must contain the same number of freely moving particles (i.e. atoms or molecules).

DEFINITION: *Avogadro's Law*

Equal volumes of gases, at the same temperature and pressure, contain the same number of molecules.

You will remember from the previous section, that we combined different gas law equations to get *one* that included temperature, volume and pressure. In this equation

$$\frac{pV}{T} = k$$

the value of k is different for different masses of gas.

We find that when we calculate k for 1 mol of gas that we get:

$$\frac{pV}{T} = 8{,}314$$

This result is given a special name. It is the universal gas constant, R. R is measured in units of J·K^{-1}·mol^{-1}. No matter which gas we use, 1 mol of that gas will have the same constant.

TIP
All quantities in the equation $pV = nRT$ must be in the same units as the value of R. In other words, SI units must be used throughout the equation.

If we now extend this result to any number of moles of a gas we get the following:

$$\frac{pV}{T} = nR$$

where n is the number of moles of gas.

Rearranging this equation gives:

$$\boxed{pV = nRT}$$

This is the ideal gas equation. When you work with this equation you must have all units in SI units.

See video: 23WK at www.everythingscience.co.za

Worked example 10: Ideal gas equation

QUESTION

Two moles of oxygen (O_2) gas occupy a volume of 25 dm^3 at a temperature of 40 °C. Calculate the pressure of the gas under these conditions.

SOLUTION

Step 1: Write down all the information that you know about the gas.

$$\begin{aligned} p &= ? \\ V &= 25 \text{ dm}^3 \\ n &= 2 \text{ mol} \\ T &= 40 \text{ °C} \\ R &= 8{,}314 \text{ J·K·mol}^{-1} \end{aligned}$$

Step 2: Convert the known values to SI units if necessary.

We need to convert the temperature to Kelvin and the volume to m^3:

$$\begin{aligned} V &= \frac{25}{1000} = 0{,}025 \text{ dm}^3 \\ T &= 40 + 273 = 313 \text{ K} \end{aligned}$$

Step 3: Choose a relevant gas law equation that will allow you to calculate the unknown variable.

We are varying everything (temperature, pressure, volume and amount of gas) and so we must use the ideal gas equation.

$$pV = nRT$$

Step 4: Substitute the known values into the equation. Calculate the unknown variable.

$$
\begin{aligned}
(0{,}025\ \text{m}^3)(p) &= (2\ \text{mol})(8{,}314\ \text{J·K}^{-1}\text{·mol}^{-1})(313\ \text{K})\\
(0{,}025\ \text{m}^3)(p) &= 5204{,}564\ \text{Pa·m}^3\\
p &= 208\ 182{,}56\ \text{Pa}
\end{aligned}
$$

The pressure will be 208 182,56 Pa or 208,2 kPa.

Worked example 11: Ideal gas equation

QUESTION

Carbon dioxide (CO_2) gas is produced as a result of the reaction between calcium carbonate and hydrochloric acid. The gas that is produced is collected in a container of unknown volume. The pressure of the gas is 105 kPa at a temperature of 20 °C. If the number of moles of gas collected is 0,86 mol, what is the volume?

SOLUTION

Step 1: Write down all the information that you know about the gas.

$$
\begin{aligned}
p &= 105\ \text{kPa}\\
V &= ?\\
n &= 0{,}86\ \text{mol}\\
T &= 20\ ^\circ\text{C}\\
R &= 8{,}314\ \text{J·K·mol}^{-1}
\end{aligned}
$$

Step 2: Convert the known values to SI units if necessary.

We need to convert the temperature to Kelvin and the pressure to Pa:

$$
\begin{aligned}
p &= 105 \times 1000 = 105\ 000\ \text{Pa}\\
T &= 20 + 273 = 293\ \text{K}
\end{aligned}
$$

Step 3: Choose a relevant gas law equation that will allow you to calculate the unknown variable.

We are varying everything (temperature, pressure, volume and amount of gas) and so we must use the ideal gas equation.

$$pV = nRT$$

Step 4: Substitute the known values into the equation. Calculate the unknown variable.

$$\begin{aligned}(105\ 000\ \text{Pa})V &= (8{,}314\ \text{J}\cdot\text{K}^{-1}\cdot\text{mol}^{-1})(293\ \text{K})(0{,}86\ \text{mol})\\(105\ 000\ \text{Pa})V &= 2094{,}96\ \text{Pa}\cdot\text{m}^3\\V &= 0{,}020\ \text{m}^3\\&= 20\ \text{dm}^3\end{aligned}$$

The volume is 20 dm^3.

Worked example 12: Ideal gas equation

QUESTION

Nitrogen (N_2) reacts with hydrogen (H_2) according to the following equation:

$$N_2 + 3H_2 \rightarrow 2NH_3$$

2 mol ammonia (NH_3) gas is collected in a separate gas cylinder which has a volume of 25 dm^3. The pressure of the gas is 195,89 kPa. Calculate the temperature of the gas inside the cylinder.

SOLUTION

Step 1: Write down all the information that you know about the gas.

$$\begin{aligned}p &= 195{,}98\ \text{Pa}\\V &= 25\ \text{dm}^3\\n &= 2\ \text{mol}\\R &= 8{,}3\ \text{J}\cdot\text{K}^{-1}\text{mol}^{-1}\\T &= ?\end{aligned}$$

Step 2: Convert the known values to SI units if necessary.

We must convert the volume to m^3 and the pressure to Pa:

$$\begin{aligned} V &= \frac{25}{1000} \\ &= 0{,}025 \text{ m}^3 \\ p &= 195{,}89 \times 1000 \\ &= 195\ 890 \text{ Pa} \end{aligned}$$

Step 3: Choose a relevant gas law equation that will allow you to calculate the unknown variable.

$$pV = nRT$$

Step 4: Substitute the known values into the equation. Calculate the unknown variable.

$$\begin{aligned} (195\ 890)(0{,}025) &= (2)(8{,}314)T \\ 4897{,}25 &= 16{,}628(T) \\ T &= 294{,}52 \text{ K} \end{aligned}$$

The temperature is 294,52 K.

Worked example 13: Ideal gas equation

QUESTION

Calculate the number of moles of air particles in a classroom of length 10 m, a width of 7 m and a height of 2 m on a day when the temperature is 23 °C and the air pressure is 98 kPa .

SOLUTION

Step 1: Calculate the volume of air in the classroom

The classroom is a rectangular prism (recall grade 10 maths on measurement). We can calculate the volume using:

$$\begin{aligned} V &= \text{length} \times \text{width} \times \text{height} \\ &= (10)(7)(2) \\ &= 140 \text{ m}^3 \end{aligned}$$

Step 2: Write down all the information that you know about the gas.

$$\begin{aligned} p &= 98 \text{ kPa} \\ V &= 140 \text{ m}^3 \\ n &= ? \\ R &= 8{,}314 \text{ J·K}^{-1}\text{mol}^{-1} \\ T &= 23\ ^\circ\text{C} \end{aligned}$$

Step 3: Convert the known values to SI units if necessary.

We must convert the temperature to K and the pressure to Pa:

$$\begin{aligned} T &= 25 + 273 = 298 \text{ K} \\ p &= 98 \times 1000 = 98\,000 \text{ Pa} \end{aligned}$$

Step 4: Choose a relevant gas law equation that will allow you to calculate the unknown variable.

$$pV = nRT$$

Step 5: Substitute the known values into the equation. Calculate the unknown variable.

$$\begin{aligned} (98\,000)(140) &= n(8{,}314)(298) \\ 13\,720\,000 &= 2477{,}572(n) \\ n &= 5537{,}7 \text{ mol} \end{aligned}$$

The number of moles in the classroom is 5537,7 mol.

Exercise 7 – 6: The ideal gas equation

1. An unknown gas has a pressure of 0,9 atm, a temperature of 120 °C and the number of moles is 0,28 mol. What is the volume of the sample?
2. 6 g of chlorine (Cl_2) occupies a volume of 0,002 m^3 at a temperature of 26 °C. What is the pressure of the gas under these conditions?

3. An average pair of human lungs contains about 3,5 L of air after inhalation and about 3,0 L after exhalation. Assuming that air in your lungs is at 37 °C and 1,0 atm, determine the number of moles of air in a typical breath.

4. A learner is asked to calculate the answer to the problem below:

   Calculate the pressure exerted by 1,5 moles of nitrogen gas in a container with a volume of 20 dm³ at a temperature of 37 °C.

   The learner writes the solution as follows:

$$\begin{aligned} V &= 20 \text{ dm}^3 \\ n &= 1{,}5 \text{ mol} \\ R &= 8{,}314 \text{ J}\cdot\text{K}^{-1}\cdot\text{mol}^{-1} \\ T &= 37 + 273 = 310 \text{ K} \end{aligned}$$

$$\begin{aligned} pT &= nRV \\ p(310) &= (1{,}5)(8{,}314)(20) \\ p(310) &= (249{,}42) \\ &= 0{,}8 \text{ kPa} \end{aligned}$$

   a) Identify 2 mistakes the learner has made in the calculation.
   b) Are the units of the final answer correct?
   c) Rewrite the solution, correcting the mistakes to arrive at the right answer.

5. Most modern cars are equipped with airbags for both the driver and the passenger. An airbag will completely inflate in 0,05 s. This is important because a typical car collision lasts about 0,125 s. The following reaction of sodium azide (a compound found in airbags) is activated by an electrical signal:

   $2NaN_3(s) \rightarrow 2Na\ (s) + 3N_2(g)$

   a) Calculate the mass of $N_2(g)$ needed to inflate a sample airbag to a volume of 65 dm³ at 25 °C and 99,3 kPa. Assume the gas temperature remains constant during the reaction.
   b) The above reaction produces heat, which raises the temperature in the airbag. Describe, in terms of the kinetic theory of gases, how the pressure in the sample airbag will change, if at all, as the gas temperature returns to 25 °C.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23WM 2. 23WN 3. 23WP 4. 23WQ 5. 23WR

www.everythingscience.co.za m.everythingscience.co.za

7.3 Chapter summary ESBP3

See presentation: 23WS at www.everythingscience.co.za

- The **kinetic theory of gases** helps to explain the behaviour of gases under different conditions.
- The **kinetic theory of gases** states that gases are made up of constantly moving particles that have attractive forces between them.
- The **pressure** of a gas is a measure of the number of collisions of the gas particles with each other and with the sides of the container that they are in.
- The **temperature** of a substance is a measure of the average kinetic energy of the particles.
- An **ideal gas** has identical particles of zero volume, with no intermolecular forces between the particles. The atoms or molecules in an ideal gas move at the same speed.
- A **real gas** behaves like an ideal gas, except at high pressures and low temperatures. At low temperatures, the forces between molecules become significant and the gas will liquefy. At high pressures, the volume of the particles becomes significant.
- **Boyle's law** states that the pressure of a fixed quantity of gas is inversely proportional to the volume it occupies so long as the temperature remains constant. In other words, $pV = k$ or:

$$p_1V_1 = p_2V_2$$

- **Charles' law** states that the volume of an enclosed sample of gas is directly proportional to its Kelvin temperature provided the pressure and amount of gas remains constant. In other words, $\frac{V}{T} = k$ or:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

- The **pressure** of a fixed mass of gas is directly proportional to its temperature, if the volume is constant. In other words, $\frac{p}{T} = k$ or:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

- For Charles' law and for the pressure-temperature relation the temperature must be written in **Kelvin**. Temperature in degrees Celsius (°C) can be converted to temperature in Kelvin (K) using the following equation:

$$T_K = T_C + 273$$

- Combining Boyle's law and the relationship between the temperature and pressure of a gas, gives the **general gas equation**, which applies as long as the amount of gas remains constant. The general gas equation is $\frac{pV}{T} = k$, or:

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

- **Avogadro's** law states that equal volumes of gases, at the same temperature and pressure, contain the same number of molecules.
- The **universal gas constant** (R) is 8,314 J·K^{-1}·mol^{-1}. This constant is found by calculating $\frac{pV}{T}$ for 1 mol of any gas.
- Extending the above calculation to apply to any number of moles of gas gives the **ideal gas equation**:

$$pV = nRT$$

In this equation, **SI units** must be used. The SI unit for volume is m^3, for pressure it is Pa and for temperature it is K.

Physical Quantities		
Quantity	**Unit name**	**Unit symbol**
Moles (n)	moles	mol
Pressure (p)	pascals	Pa
Temperature (T)	kelvin	K
Volume (V)	meters cubed	m^3

Exercise 7 – 7:

1. Give one word or term for each of the following definitions.
   a) A gas is that has identical particles of zero volume, with no intermolecular forces between the particles.
   b) The law that states that the volume of a gas is directly proportional to the temperature of the gas, provided that the pressure and the amount of the gas remain constant.
   c) A measure of the average kinetic energy of gas particles.
2. Which one of the following properties of a fixed quantity of a gas must be kept constant during an investigation of Boyle's law?
   a) density
   b) pressure
   c) temperature
   d) volume

   (*IEB 2003 Paper 2*)
3. Three containers of *equal volume* are filled with *equal masses* of helium, nitrogen and carbon dioxide gas respectively. The gases in the three containers are all at the same *temperature*. Which one of the following statements is correct regarding the pressure of the gases?
   a) All three gases will be at the same pressure
   b) The helium will be at the greatest pressure
   c) The nitrogen will be at the greatest pressure

d) The carbon dioxide will be at the greatest pressure

(*IEB 2004 Paper 2*)

4. The ideal gas equation is given by **pV = nRT**. Which one of the following conditions is true according to Avogadro's hypothesis?

a	$p \propto \frac{1}{V}$	(T = constant)
b	$V \propto T$	(p = constant)
c	$V \propto n$	(p, T = constant)
d	$p \propto T$	(n = constant)

(*DoE Exemplar paper 2, 2007*)

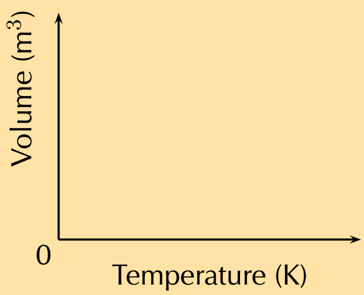
5. Complete the following table by stating whether or not the property is constant or variable for the given gas law.

Law	Pressure (p)	Volume (V)	Temperature (T)	Moles (n)
Boyle's law				
Charles' law				
Gay-Lussac's law				
General gas equation				
Ideal gas equation				

6. Use your knowledge of the gas laws to explain the following statements.

   a) It is dangerous to put an aerosol can near heat.
   b) A pressure vessel that is poorly designed and made can be a serious safety hazard (a pressure vessel is a closed, rigid container that is used to hold gases at a pressure that is higher than the normal air pressure).
   c) The volume of a car tyre increases after a trip on a hot road.

7. Copy the following set of labelled axes and answer the questions that follow:



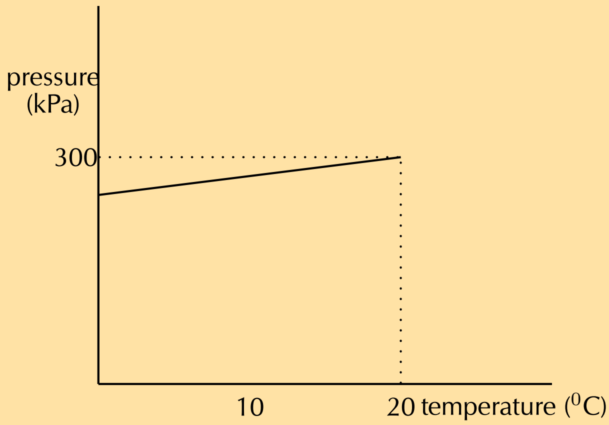
   a) On the axes, **using a solid line**, draw the graph that would be obtained for a fixed mass of an ideal gas if the pressure is kept constant.
   b) If the gradient of the above graph is measured to be 0,008 $m^3 \cdot K^{-1}$, calculate the pressure that 0,3 mol of this gas would exert.

(*IEB 2002 Paper 2*)

8. Two gas cylinders, A and B, have a volume of 0,15 m^3 and 0,20 m^3 respectively. Cylinder A contains 35 mol He gas at pressure p and cylinder B contains 40 mol He gas at 5 atm. The ratio of the Kelvin temperatures A:B is 1,80:1,00. Calculate the pressure of the gas (in kPa) in cylinder A.

   (*IEB 2002 Paper 2*)

9. A learner investigates the relationship between the Celsius temperature and the pressure of a fixed amount of helium gas in a 500 cm^3 closed container. From the results of the investigation, she draws the graph below:



   a) Under the conditions of this investigation, helium gas behaves like an ideal gas. Explain briefly why this is so.
   b) From the shape of the graph, the learner concludes that the pressure of the helium gas is directly proportional to the Celsius temperature. Is her conclusion correct? Briefly explain your answer.
   c) Calculate the pressure of the helium gas at 0 °C.
   d) Calculate the mass of helium gas in the container.

   (*IEB 2003 Paper 2*)

10. One of the cylinders of a motor car engine, before compression contains 450 cm^3 of a mixture of air and petrol in the gaseous phase, at a temperature of 30 °C and a pressure of 100 kPa. If the volume of the cylinder after compression decreases to one tenth of the original volume, and the temperature of the gas mixture rises to 140 °C, calculate the pressure now exerted by the gas mixture.

11. A gas of unknown volume has a temperature of 14 °C. When the temperature of the gas is increased to 100 °C, the volume is found to be 5,5 L. What was the initial volume of the gas?

12. A gas has an initial volume of 2600 mL and a temperature of 350 K.

   a) If the volume is reduced to 1500 mL, what will the temperature of the gas be in Kelvin?
   b) Has the temperature *increased* or *decreased*?
   c) Explain this change, using the kinetic theory of gases.

13. In an experiment to determine the relationship between pressure and temperature of a fixed mass of gas, a group of learners obtained the following results:

Pressure (kPa)	101	120	130,5	138
Temperature (°C)	0	50	80	100
Total gas volume (cm^3)	250	250	250	250

a) Draw a straight-line graph of pressure (on the dependent, y-axis) versus temperature (on the independent, x-axis) on a piece of graph paper. Plot the points. Give your graph a suitable heading.

b) *A straight-line graph passing through the origin is essential to obtain a mathematical relationship between pressure and temperature..*

   Extrapolate (extend) your graph and determine the temperature (in °C) at which the graph will pass through the temperature axis.

c) Write down, in words, the relationship between pressure and Kelvin temperature.

d) From your graph, determine the pressure (in kPa) at 173 K. Indicate on your graph how you obtained this value.

e) How would the gradient of the graph be affected (if at all) if a larger mass of the gas is used? Write down ONLY **increases**, **decreases** or **stays the same**.

(*DoE Exemplar Paper 2, 2007*)

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 23WT	1b. 23WV	1c. 23WW	2. 23WX	3. 23WY	4. 23WZ
5. 23X2	6a. 23X3	6b. 23X4	6c. 23X5	7. 23X6	8. 23X7
9. 23X8	10. 23X9	11. 23XB	12. 23XC	13. 23XD	

www.everythingscience.co.za **m.everythingscience.co.za**

CHAPTER 8

Quantitative aspects of chemical change

Wherever we look in real life we see the importance of mixing things in precise quantities. Cooking and baking, the medicines you take when sick, the products that you buy, all of these rely on the ingredients being mixed in specific amounts. And even the amount of product formed relies on how much of each ingredient is used. In this chapter we will look at some of these quantities and how they can be calculated.

In grade 10 we learnt about writing chemical equations and about the information that can be obtained from a balanced chemical equation. In this chapter we are going to explore these concepts further and learn more about gases, solutions and reactions. We will explore the concept of theoretical yield in greater detail and learn about limiting reagents.

Key Mathematics Concepts

- Ratio and proportion — Physical Sciences, Grade 10, Science skills
- Equations — Mathematics, Grade 10, Equations and inequalities
- Units and unit conversions — Physical Sciences, Grade 10, Science skills

8.1 Gases and solutions ESBP4

We will begin by taking a closer look at gases and solutions and work out how to solve problems relating to them.

Molar volumes of gases ESBP5

It is possible to calculate the volume of one mole of gas at standard temperature and pressure (STP) using what we now know about gases.

NOTE:
STP is a temperature of 273 K and a pressure of 101,3 kPa. The amount of gas is usually 1 mol.

We write down all the values that we know about one mole of gas at STP:

$$\begin{aligned} p &= 101{,}3 \text{ kPa} = 101\ 300 \text{ Pa} \\ n &= 1 \text{ mol} \\ R &= 8{,}31 \text{ J·K}^{-1}\text{·mol}^{-1} \\ T &= 273 \text{ K} \end{aligned}$$

Now we can substitute these values into the ideal gas equation:

$$\begin{aligned} pV &= nRT \\ (101\ 300)V &= (1)(8{,}31)(273) \\ (101\ 300)V &= 2265{,}9 \\ V &= 0{,}0224\ \text{m}^3 \\ V &= 22{,}4\ \text{dm}^3 \end{aligned}$$

TIP

The standard units used for this equation are p in Pa, V in m^3 and T in K. Remember also that 1000 cm^3 = 1 dm^3 and 1000 dm^3 = 1 m^3.

The volume of 1 mole of gas at STP is 22,4 dm^3.

And if we had any number of moles of gas, not just one mole then we would get:

$$\boxed{V_g = 22{,}4n_g}$$

Worked example 1: Molar gas volume

QUESTION

What is the volume of 2,3 mol of hydrogen gas at STP?

SOLUTION

Step 1: Find the volume

$$\begin{aligned} V_g &= (22{,}4)n_g \\ &= (22{,}4)(2{,}3) \\ &= 51{,}52\ \text{dm}^3 \end{aligned}$$

Reactions and gases ESBP6

Some reactions take place between gases. For these reactions we can work out the volumes of the gases using the fact that volume is proportional to the number of moles.

We can use the following formula:

$$V_\text{A} = \frac{a}{b}V_\text{B}$$

where:

$$\begin{aligned} V_A &= \text{volume of A} \\ V_B &= \text{volume of B} \\ a &= \text{stoichiometric coefficient of A} \\ b &= \text{stoichiometric coefficient of B} \end{aligned}$$

TIP
The number in front of a reactant or a product in a balanced chemical equation is called the stoichiometric coefficient or stoichiometric ratio.

Worked example 2: Volume and gases

QUESTION

Hydrogen and oxygen react to form water according to the following equation:

$$2H_2(g) + O_2(g) \rightarrow 2H_2O(g)$$

If 3 dm^3 of oxygen is used, what volume of water is produced?

SOLUTION

Step 1: Determine the volume of water produced in the reaction.

We use the equation given above to work out the volume of water needed:

$$\begin{aligned} V_A &= \frac{a}{b}V_B \\ V_{H_2O} &= \frac{2}{1}V_{O_2} \\ &= 2(3) \\ &= 6 \text{ dm}^3 \end{aligned}$$

We can interpret the chemical equation in the worked example above ($2H_2(g) + O_2(g) \rightarrow 2H_2O(g)$) as:

2 moles of hydrogen react with 1 mole of oxygen to produce 2 moles of water. We can also say that 2 volumes of hydrogen react with 1 volume of oxygen to produce 2 volumes of water.

Worked example 3: Gas phase calculations

QUESTION

What volume of oxygen at STP is needed for the complete combustion of 3,3 dm^3 of propane (C_3H_8)? (Hint: CO_2 and H_2O are the products as in all combustion reactions)

SOLUTION

Step 1: Write a balanced equation for the reaction.

$C_3H_8(g) + 5O_2(g) \rightarrow 3CO_2(g) + 4H_2O\ (g)$

Step 2: Determine the volume of oxygen needed for the reaction.

We use the equation given above to work out the volume of oxygen needed:

$$\begin{aligned} V_A &= \frac{a}{b} V_B \\ V_{O_2} &= \frac{5}{1} V_{C_3H_8} \\ &= 5(3{,}3) \\ &= 16{,}5 \text{ dm}^3 \end{aligned}$$

TIP

When you are busy with these calculations, you will need to remember the following: 1 dm^3 = 1 L = 1000 mL = 1000 cm^3, therefore dividing a volume in cm^3 by 1000 will give you the equivalent volume in dm^3.

Exercise 8 – 1:

1. Methane burns in oxygen, forming water and carbon dioxide according to the following equation:

$$CH_4(g) + 2O_2(g) \rightarrow 2H_2O(g) + CO_2(g)$$

If 4 dm^3 of methane is used, what volume of water is produced?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23XF

www.everythingscience.co.za m.everythingscience.co.za

Solutions ESBP7

In grade 10 you learnt how to calculate the molar concentration of a solution. The molar concentration of a solution is the number of moles of solute per litre of solvent (mol·L^{-1}). This is more commonly given as moles of solute per cubic decimetre of solution (mol·dm^{-3}).

To calculate concentration we use $C = \frac{n}{V}$, where C is the molar concentration, n is the number of moles and V is the volume of the solution.

Calculating molar concentrations is useful to determine how much solute we need to add to a given volume of solvent in order to make a standard solution.

A standard solution is a solution in which the concentration is known to a high degree of precision. When we work with standard solutions we can take the concentration to be constant.

TIP
When performing a titration we say that the substance of unknown concentration is **titrated** with the standard solution. A pipette is a measuring device that is used to measure an exact amount of a liquid. If you use a pipette to add liquid to a flask then you would say that the liquid was **pipetted** into a flask.

Worked example 4: Concentration calculations

QUESTION

How much sodium chloride (in g) will one need to prepare 500 cm^3 of a standard solution with a concentration of 0,01 $mol{\cdot}dm^{-3}$?

SOLUTION

Step 1: Convert all quantities into the correct units for this equation.

The volume must be converted to dm^3:

$$\begin{aligned} V &= \frac{500}{1000} \\ &= 0{,}5\ \text{dm}^3 \end{aligned}$$

Step 2: Calculate the number of moles of sodium chloride needed.

$$\begin{aligned} C &= \frac{n}{V} \\ 0{,}01 &= \frac{n}{0{,}5} \\ n &= 0{,}005\ \text{mol} \end{aligned}$$

Step 3: Convert moles of NaCl to mass.

To find the mass of NaCl we need the molar mass of NaCl. We can get this from the periodic table (recall from grade 10 how to calculate the molar mass of a compound).

$$\begin{aligned} m &= nM \\ &= (0{,}005)(58) \\ &= 0{,}29\ \text{g} \end{aligned}$$

The mass of sodium chloride needed is 0,29 g

We will now look at another use of concentration which is for titration calculations.

Titrations

A titration is a technique for determining the concentration of an unknown solution. Titrations can be done using many different types of reactions. Acid-base reactions and redox reactions are both commonly used for titrations.

In grade 10 you did a simple acid-base titration. Now we will look at how to calculate the concentration of an unknown solution using an acid-base titration.

We can reduce the number of calculations that we have to do in titration calculations

by using the following:

$$\frac{C_A V_A}{a} = \frac{C_B V_B}{b}$$

The a and b are the stoichiometric coefficients of compounds A and B respectively.

Worked example 5: Titration calculation

QUESTION

Given the equation:

$\mathrm{NaOH\ (aq) + HCl\ (aq) \rightarrow NaCl\ (aq) + H_2O\ (l)}$

25 cm^3 of a 0,2 $mol{\cdot}dm^{-3}$ hydrochloric acid solution was pipetted into a conical flask and titrated with sodium hydroxide. It was found that 15 cm^3 of the sodium hydroxide was needed to neutralise the acid. Calculate the concentration of the sodium hydroxide.

SOLUTION

Step 1: Write down all the information you know about the reaction, and make sure that the equation is balanced.

NaOH: $V = 15\ cm^3$

HCl: $V = 25\ cm^3$; $C = 0{,}2\ mol{\cdot}dm^{-3}$

The equation is already balanced.

Step 2: Convert the volume to dm^3

$$\begin{aligned} V_{\mathrm{NaOH}} &= \frac{15}{1000} \\ &= 0{,}015\ dm^3 \end{aligned}$$

$$\begin{aligned} V_{\mathrm{HCl}} &= \frac{25}{1000} \\ &= 0{,}025\ dm^3 \end{aligned}$$

Step 3: Calculate the concentration of the sodium hydroxide

$$\begin{aligned} \frac{C_A V_A}{a} &= \frac{C_B V_B}{b} \\ \frac{(0{,}2)(0{,}025)}{1} &= \frac{(C_{\mathrm{NaOH}})(0{,}015)}{1} \\ 0{,}005 &= (0{,}015)C_{\mathrm{NaOH}} \\ C_{\mathrm{NaOH}} &= 0{,}33\ mol{\cdot}dm^{-3} \end{aligned}$$

The concentration of the NaOH solution is 0,33 $mol{\cdot}dm^{-3}$

Worked example 6: Titration calculation

QUESTION

4,9 g of sulfuric acid is dissolved in water and the final solution has a volume of 220 cm^3. Using an acid-base titration, it was found that 20 cm^3 of this solution was able to completely neutralise 10 cm^3 of a sodium hydroxide solution. Calculate the concentration of the sodium hydroxide in mol·dm^{-3}.

SOLUTION

Step 1: Write a balanced equation for the titration reaction.

$H_2SO_4(aq) + 2NaOH\ (aq) \rightarrow Na_2SO_4(aq) + 2H_2O\ (l)$

Step 2: Calculate the concentration of the sulfuric acid solution.

First convert the volume into dm^3:

$$V = \frac{220}{1000} = 0{,}22 \text{ dm}^3$$

Then calculate the number of moles of sulfuric acid:

$$\begin{aligned} n &= \frac{m}{M} \\ &= \frac{4{,}9}{98} \\ &= 0{,}05 \text{ mol} \end{aligned}$$

Now we can calculate the concentration of the sulfuric acid:

$$\begin{aligned} C &= \frac{n}{V} \\ &= \frac{0{,}05}{0{,}22} \\ &= 0{,}227 \text{ mol·dm}^{-3} \end{aligned}$$

Step 3: Calculate the concentration of the sodium hydroxide solution.

Remember that only 20 cm^3 or 0,02 dm^3 of the sulfuric acid solution is used.

$$\begin{aligned} \frac{C_1V_1}{n_1} &= \frac{C_2V_2}{n_2} \\ \frac{(0{,}227)(0{,}02)}{1} &= \frac{(C_{\text{NaOH}})(0{,}01)}{2} \\ 0{,}00454 &= (0{,}005)C_{\text{NaOH}} \\ C_{\text{NaOH}} &= 0{,}909 \text{ mol·dm}^{-3} \end{aligned}$$

Exercise 8 – 2: Gases and solutions

1. Acetylene (C_2H_2) burns in oxygen according to the following reaction:

$$2C_2H_2(g) + 5O_2(g) \rightarrow 4CO_2(g) + 2H_2O\ (g)$$

If 3,5 dm^3 of acetylene gas is burnt, what volume of carbon dioxide will be produced?

2. 130 g of magnesium chloride ($MgCl_2$) is dissolved in 300 mL of water.

   a) Calculate the concentration of the solution.

   b) What mass of magnesium chloride would need to be added for the concentration to become 6,7 $mol{\cdot}dm^{-3}$?

3. Given the equation:

   $KOH\ (aq) + HNO_3(aq) \rightarrow KNO_3(aq) + H_2O\ (l)$

   20 cm^3 of a 1,3 $mol{\cdot}dm^{-3}$ potassium hydroxide (KOH) solution was pipetted into a conical flask and titrated with nitric acid (HNO_3). It was found that 17 cm^3 of the nitric acid was needed to neutralise the base. Calculate the concentration of the nitric acid.

4. Given the equation:

   $3Ca(OH)_2(aq) + 2H_3PO_4(aq) \rightarrow Ca_3(PO_4)_2(aq) + 6H_2O\ (l)$

   10 cm^3 of a 0,4 $mol{\cdot}dm^{-3}$ calcium hydroxide ($Ca(OH)_2$) solution was pipetted into a conical flask and titrated with phosphoric acid (H_3PO_4). It was found that 11 cm^3 of the phosphoric acid was needed to neutralise the base. Calculate the concentration of the phosphoric acid.

5. A 3,7 g sample of an antacid (which contains mostly calcium carbonate) is dissolved in water. The final solution has a volume of 500 mL. 25 mL of this solution is then pipetted into a conical flask and titrated with hydrochloric acid. It is found that 20 mL of the hydrochloric acid completely neutralises the antacid solution. What is the concentration of the hydrochloric acid?

   The equation for this reaction is:

$$CaCO_3(aq) + 2HCl\ (aq) \rightarrow CaCl_2(aq) + H_2O\ (l) + CO_2(g)$$

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1\. 23XG 2. 23XH 3. 23XJ 4. 23XK 5. 23XM

www.everythingscience.co.za m.everythingscience.co.za

8.2 Stoichiometric calculations ESBP8

In grade 10 you learnt how to write balanced chemical equations and started looking at stoichiometric calculations. By knowing the ratios of substances in a reaction, it is possible to use stoichiometry to calculate the amount of either reactants or products that are involved in the reaction.

The following figure highlights the relation between the balanced chemical equation and the number of moles:

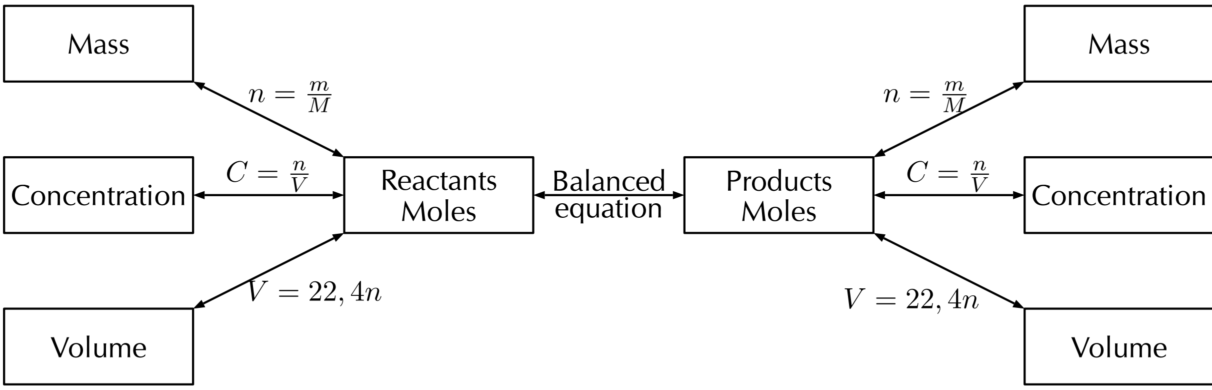


Figure 8.1: Stoichiometric flow diagram

In grade 10 we explored some of the concepts of stoichiometry. We looked at how to calculate the number of moles of a substance and how to find the molar mass. We also looked at how to find the molecular and empirical formulae of substances. Now we will explore more of these concepts such as limiting reagents, percent purity and percent yield.

Limiting reagents ESBP9

Activity: What is a limiting reagent?

For this activity you will need A4 sheets of paper in white, red, blue, yellow, green and pink. (or you can use several sheets of white paper and colour them using kokis or crayons).

Tear the white sheet into five pieces, the red sheet into ten pieces, the blue sheet into eight pieces, the yellow sheet into seven pieces, the green sheet into nine pieces and the pink piece into four pieces.

1. Stick two red pieces to each white piece. Do you have any red or white pieces left?
2. Stick one yellow piece to each blue piece. Do you have any yellow or blue pieces left?

3. Stick three green pieces to each pink piece. Do you have any green or pink pieces left?

You should find that you had no red or white pieces left. For the blue and yellow pieces you should have one blue piece left. And for the green and pink pieces you should have had one pink piece left.

We say that the pink and blue pieces were in excess while the green and yellow sheets were limiting. In other words you would have had to tear the green and yellow sheets into **more** pieces or you would have had to tear the blue and pink pieces into **less** pieces.

In the above activity we could solve the problem of having too many or too few pieces of paper by simply tearing the pieces of paper into more pieces. In chemistry we also encounter this problem when mixing different substances. Often we will find that we added too much or too little of a particular substance. It is important to know that this happens and to know how much (i.e. the quantities) of different reactants are used in the reaction. This knowledge is used in industrial reactions.

DEFINITION: *Limiting reagent*

A limiting reagent (or reactant) is a reagent that is completely used up in a chemical reaction.

DEFINITION: *Excess reagent*

An excess reagent (or reactant) is a reagent that is not completely used up in a chemical reaction.

See simulation: 23XN at www.everythingscience.co.za

Worked example 7: Limiting reagents

QUESTION

Sulfuric acid (H_2SO_4) reacts with ammonia (NH_3) to produce the fertiliser ammonium sulfate ($(NH_4)_2SO_4$) according to the following equation:

$$H_2SO_4(aq) + 2NH_3(g) \rightarrow (NH_4)_2SO_4(aq)$$

What is the maximum mass of ammonium sulfate that can be obtained from 2,0 kg of sulfuric acid and 1,0 kg of ammonia?

SOLUTION

Step 1: Convert the mass of sulfuric acid and ammonia into moles

Moles of sulfuric acid:

$$
\begin{aligned}
n &= \frac{m}{M} \\
&= \frac{2000}{98} \\
&= 20{,}4 \text{ mol}
\end{aligned}
$$

Moles of ammonia:

$$
\begin{aligned}
n &= \frac{m}{M} \\
&= \frac{1000}{17} \\
&= 58{,}8 \text{ mol}
\end{aligned}
$$

Step 2: Use the balanced equation to determine which of the reactants is limiting.

We need to look at how many moles of product we can get from each reactant. Then we compare these two results. The smaller number is the amount of product that we can produce and the reactant that gives the smaller number, is the limiting reagent.

The mole ratio of H_2SO_4 to $(NH_4)_2SO_4$ is $1 : 1$. So the number of moles of $(NH_4)_2SO_4$ that can be produced from the sulfuric acid is:

$$
\begin{aligned}
n_{(NH_4)_2SO_4} &= n_{H_2SO_4} \times \frac{\text{stoichiometric coefficient } (NH_4)_2SO_4}{\text{stoichiometric coefficient } H_2SO_4} \\
&= 20{,}4 \text{ mol } H_2SO_4 \times \frac{1 \text{ mol } (NH_4)_2SO_4}{1 \text{ mol } H_2SO_4} \\
&= 20{,}4 \text{ mol } (NH_4)_2SO_4
\end{aligned}
$$

The mole ratio of NH_3 to $(NH_4)_2SO_4$ is $2 : 1$. So the number of moles of $(NH_4)_2SO_4$ that can be produced from the ammonia is:

$$
\begin{aligned}
n_{(NH_4)_2SO_4} &= n_{NH_3} \times \frac{\text{stoichiometric coefficient } (NH_4)_2SO_4}{\text{stoichiometric coefficient } NH_3} \\
&= 58{,}8 \text{ mol } NH_3 \times \frac{1 \text{ mol } (NH_4)_2SO_4}{2 \text{ mol } NH_3} \\
&= 29{,}4 \text{ mol } (NH_4)_2SO_4
\end{aligned}
$$

Since we get less $(NH_4)_2SO_4$ from H_2SO_4 than is produced from NH_3, the sulfuric acid is the limiting reactant.

Step 3: Calculate the maximum mass of ammonium sulfate that can be produced

From the step above we saw that we have 20,4 mol of $(NH_4)_2SO_4$.

The maximum mass of ammonium sulfate that can be produced is calculated as follows:

$$
\begin{aligned}
m &= nM \\
&= (20{,}4)(132) \\
&= 2692{,}8 \text{ g} \\
&= 2{,}6928 \text{ kg}
\end{aligned}
$$

The maximum mass of ammonium sulfate that can be produced is 2,69 kg.

Exercise 8 – 3:

1. When an electrical current is passed through a sodium chloride solution, sodium hydroxide can be produced according to the following equation:

   $2NaCl\ (aq) + H_2O\ (l) \rightarrow Cl_2(g) + H_2(g) + 2NaOH\ (aq)$

   What is the maximum mass of sodium hydroxide that can be obtained from 4,0 kg of sodium chloride and 3,0 kg of water?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23XP

www.everythingscience.co.za m.everythingscience.co.za

Percent yield ESBPB

The percent yield of a reaction is very important as it tells us how efficient a reaction is. A reaction that has a low percent yield is not very useful in industry. If you are making a new medicine or pesticide and your reaction has a low percent yield then you would search for a different way of doing the reaction. This reduces the amount of (often very expensive) chemicals that you use and reduces waste.

The percent yield can be calculated using:

$$\%\text{yield} = \frac{\text{actual yield}}{\text{theoretical yield}} \times 100$$

where the actual yield is the amount of product that is produced when you carry out the reaction and the theoretical yield is the amount of product that you calculate for the reaction using stoichiometric methods.

Worked example 8: Percent yield

QUESTION

Sulfuric acid (H_2SO_4) reacts with ammonia (NH_3) to produce the fertiliser ammonium sulfate ($(NH_4)_2SO_4$) according to the following equation:

$H_2SO_4(aq) + 2NH_3(g) \rightarrow (NH_4)_2SO_4(aq)$

A factory worker carries out the above reaction (using 2,0 kg of sulfuric acid and 1,0 kg of ammonia) and gets 2,5 kg of ammonium sulfate. What is the percentage yield of the reaction?

SOLUTION

Step 1: Determine which is the limiting reagent

We determined the limiting reagent for this reaction with the same amounts of reactants in the previous worked example, so we will just use the result from there.

Sulfuric acid is the limiting reagent. The number of moles of ammonium sulfate that can be produced is 20,4 mol.

Step 2: Calculate the theoretical yield of ammonium sulphate

From the previous worked example we found the maximum mass of ammonium sulfate that could be produced.

The theoretical yield (or maximum mass) of ammonium sulfate that can be produced is 2,69 kg.

Step 3: Calculate the percentage yield

$$\begin{aligned}\%\text{yield} &= \frac{\text{actual yield}}{\text{theoretical yield}} \times 100 \\ &= \frac{2,5}{2,694}(100) \\ &= 92,8\%\end{aligned}$$

This reaction has a high percent yield and so would therefore be a useful reaction to use in industry.

Exercise 8 – 4:

1. When an electrical current is passed through a sodium chloride solution, sodium hydroxide can be produced according to the following equation:

   $2NaCl + H_2O \rightarrow Cl_2 + H_2 + 2NaOH$

   A chemist carries out the above reaction using 4,0 kg of sodium chloride and 3,0 kg of water. The chemist finds that they get 1,8 kg of sodium hydroxide. What is the percentage yield?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23XQ

www.everythingscience.co.za **m.everythingscience.co.za**

Molecular and empirical formulae were introduced in grade 10. The empirical formula is the simplest formula of a compound (and represents the ratio of atoms of each element in a compound). The molecular formula is the full formula of the compound (and represents the total number of atoms of each element in a compound). You should also recall from grade 10 the percent composition of a substance. This is the percentage by molecular mass that each element contributes to the overall formula. For example water (H_2O) has the following percentage composition: 89% oxygen and 11% hydrogen.

Worked example 9: Empirical and molecular formula

QUESTION

Vinegar, which is used in our homes, is a dilute form of acetic acid. A sample of acetic acid has the following percentage composition: 39,9% carbon, 6,7% hydrogen and 53,4% oxygen.

1. Determine the empirical formula of acetic acid.
2. Determine the molecular formula of acetic acid if the molar mass of acetic acid is 60,06 $g{\cdot}mol^{-1}$.

SOLUTION

Step 1: Find the mass

In 100 g of acetic acid, there is: 39,9 g C, 6,7 g H and 53,4 g O.

Step 2: Find the moles

$n = \frac{m}{M}$

$$n_{\text{C}} = \frac{39{,}9}{12} = 3{,}325 \text{ mol}$$
$$n_{\text{H}} = \frac{6{,}7}{1{,}01} = 6{,}6337 \text{ mol}$$
$$n_{\text{O}} = \frac{53{,}4}{16} = 3{,}3375 \text{ mol}$$

Step 3: Find the empirical formula

To find the empirical formula we first note how many moles of each element we have. Then we divide the moles of each element by the smallest of these numbers, to get the ratios of the elements. This ratio is rounded off to the nearest whole number.

C	H	O
3,325	6,6337	3,3375
$\frac{3,325}{3,325}$	$\frac{6,6337}{3,325}$	$\frac{3,3375}{3,325}$
1	2	1

The empirical formula is CH_2O.

Step 4: Find the molecular formula

The molar mass of acetic acid using the empirical formula (CH_2O) is 30,02 $g{\cdot}mol^{-1}$. However the question gives the molar mass as 60,06 $g{\cdot}mol^{-1}$. Therefore the actual number of moles of each element must be double what it is in the empirical formula $\left(\frac{60,06}{30,02} = 2\right)$. The molecular formula is therefore $C_2H_4O_2$ or CH_3COOH

Exercise 8 – 5:

1. A sample of oxalic acid has the following percentage composition: 26,7% carbon, 2,2% hydrogen and 71,1% oxygen.

   Determine the molecular formula of oxalic acid if the molar mass of oxalic acid is 90 $g{\cdot}mol^{-1}$.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23XR

www.everythingscience.co.za m.everythingscience.co.za

Percent purity ESBPD

The final use of stoichiometric calculations that we will look at is to determine the percent purity of a sample. Percent purity is important since when you make a compound you may have a small amount of impurity in the sample and you would need to keep this below a certain level. Or you may need to know how much of a particular ion is dissolved in water to determine if it is below the legally allowed level.

Percent purity can be calculated using:

$$\%\text{purity} = \frac{\text{mass of compound}}{\text{mass of sample}} \times 100$$

Worked example 10: Percent purity

QUESTION

Shells contain calcium carbonate ($CaCO_3$) as well as other minerals. Faarah wants to know how much calcium carbonate is in a shell. She finds that the shell weighs 5 g. After performing some more experiments she finds that the mass of calcium carbonate and the crucible (a container that is used to heat compounds in) is 3,2 g. The mass of the crucible is 0,5 g. How much calcium carbonate is in the shell?

SOLUTION

Step 1: Write down an equation for percent purity

Percent purity is given by:

$$\%\text{purity} = \frac{\text{mass of compound}}{\text{mass of sample}} \times 100$$

Step 2: Find the mass of the product

We are given the mass of the crucible and the mass of the crucible with the product. We need to subtract the mass of the crucible from the mass of the crucible with the product to obtain only the mass of the product.

$$\begin{aligned}\text{Mass product} &= 3{,}2 \text{ g} - 0{,}5 \text{ g}\\ &= 2{,}7 \text{ g}\end{aligned}$$

Step 3: Calculate the answer.

Substituting the calculated mass into the equation for percent purity gives:

$$\begin{aligned}\%\text{purity} &= \frac{\text{mass of compound}}{\text{mass of sample}} \times 100\\ &= \frac{2{,}7}{5}(100)\\ &= 54\%\end{aligned}$$

Worked example 11: Percent purity

QUESTION

Limestone is mostly calcium carbonate ($CaCO_3$). Jake wants to know how much calcium carbonate is in a sample of limestone. He finds that the sample weighs 3,5 g. He then adds concentrated hydrochloric acid (HCl) to the sample. The equation for this

reaction is:

$$CaCO_3(s) + 2HCl\ (aq) \rightarrow CO_2(g) + CaCl_2(aq) + H_2O\ (l)$$

If the mass of calcium chloride produced is 3,6 g, what is the percent purity of the limestone sample?

SOLUTION

Step 1: Calculate the number of moles of calcium chloride

The number of moles of calcium chloride is:

$$\begin{aligned} n &= \frac{m}{M} \\ &= \frac{3{,}6}{111} \\ &= 0{,}032 \text{ mol} \end{aligned}$$

Step 2: Calculate the number of moles of calcium carbonate

The molar ratio of calcium chloride to calcium carbonate is 1:1. Therefore the number of moles of calcium carbonate is 0,032 mol.

Step 3: Calculate the mass of calcium carbonate

The mass of calcium carbonate is:

$$\begin{aligned} m &= nM \\ &= (0{,}032)(100) \\ m &= 3{,}24 \text{ g} \end{aligned}$$

Step 4: Calculate the percent purity

Substituting the calculated mass into the equation for percent purity gives:

$$\begin{aligned} \%\text{purity} &= \frac{\text{mass of compound}}{\text{mass of sample}} \times 100 \\ &= \frac{3{,}3}{3{,}5} \times (100) \\ &= 94{,}3\% \end{aligned}$$

Exercise 8 – 6:

1. Hematite contains iron oxide (Fe_2O_3) as well as other compounds. Thembile

wants to know how much iron oxide is in a sample of hematite. He finds that the sample of hematite weighs 6,2 g. After performing some experiments he finds that the mass of iron oxide and the crucible (a container that is used to heat compounds in) is 4,8 g. The mass of the crucible is 0,5 g. How much iron oxide is in the sample of hematite?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23XS

www.everythingscience.co.za **m.everythingscience.co.za**

See video: 23XT at www.everythingscience.co.za

General experiment: The thermal decomposition of lead(II) nitrate

Aim:

To observe what happens when lead(II) nitrate is heated.

Apparatus:

- Bunsen burner
- Test tube (glass)
- Test tube holder
- 0,5 g lead(II) nitrate

Method:

WARNING!

Lead(II) nitrate produces toxic nitrogen dioxide on heating.

---

1. Place the lead(II) nitrate sample in the test tube.
2. Light the Bunsen burner and carefully hold the test tube in the flame. Remember to point the mouth of the test tube away from you.
3. Observe what happens.

Results:

A crackling noise is heard on heating the sample and a small amount of a brownish coloured gas is noted. The white powder became yellow.

If a glowing splint is held at the mouth of the test tube the splint reignites.

The balanced chemical for this reaction is:

$$2Pb(NO_3)_2(s) + \text{ heat } \rightarrow 2PbO\ (s) + 4NO_2(g) + O_2(g)$$

Informal experiment:Stoichiometry

Aim:

To determine the percentage yield of magnesium carbonate from magnesium sulfate and sodium carbonate.

Apparatus:

- magnesium sulfate (Epsom salts)
- sodium carbonate
- mass meter
- hot plate
- small glass beakers (heat resistant)
- funnel
- filter paper

Method:

1. Weigh out 5 g of magnesium sulfate.
2. Dissolve the magnesium sulfate in 20 ml of water. Heat the solution until all the solid dissolves.
3. Dissolve 5 g of sodium carbonate in 20 ml of warm water. If necessary heat the solution to dissolve all the sodium carbonate.
4. Carefully pour the sodium carbonate solution into the hot magnesium sulfate solution. You should have a milky looking solution.
5. Weigh the filter paper (if your mass meter is not very accurate then assume the mass of the filter paper is 0). Carefully filter the final solution.
6. Leave the filter paper to dry slightly (or overnight) and then weigh it. The solid that stays on the filter paper is magnesium carbonate.

Results:

The equation for this reaction is: $MgSO_4(aq) + Na_2CO_3(aq) \rightarrow Na_2SO_4(aq) + MgCO_3(s) + H_2O\ (l)$.

Use the above equation to work out the percentage yield of the magnesium sulfate. Remember that you need to determine which of the reactants is limiting. (You may get a percentage yield of greater than 100% if your sample is not completely dry.)

Conclusion:

By performing an experiment we were able to calculate the percentage yield of a reaction.

Exercise 8 – 7: Stoichiometry

1. Given the following reaction:

$$3Fe_2O_3(s) + CO\ (g) \rightarrow 2Fe_2O_4(s) + CO_2(g)$$

   If 2,3 kg of Fe_2O_3 and 1,7 kg of CO is used, what is the maximum mass of Fe_2O_4 that can be produced?

2. Sodium nitrate decomposes on heating to produce sodium nitrite and oxygen according to the following equation:

$$2NaNO_3(s) \rightarrow 2NaNO_2(s) + O_2(g)$$

   Nombusa carries out the above reaction using 50 g of sodium nitrate. Nombusa finds that they get 36 g of sodium nitrite. What is the percentage yield?

3. Benzene has the following percentage composition: 92,31% carbon and 7,69% hydrogen

   Determine the molecular formula of benzene if the molar mass of benzene is 78 $g{\cdot}mol^{-1}$.

4. Cuprite is a minor ore of copper. Cuprite is mainly composed of copper(I) oxide (Cu_2O). Jennifer wants to know how much copper oxide is in a sample of cuprite. She has a sample of cuprite that weighs 7,7 g. She performs some experiments and finds that the mass of iron oxide and crucible (a container that is used to heat compounds in) is 7,4 g. The mass of the crucible is 0,2 g. What is the percent purity of the sample of cuprite?

5. A sample containing tin dioxide (SnO_2) is to be tested to see how much tin dioxide it contains. The sample weighs 6,2 g. Sulfuric acid (H_2SO_4) is added to the sample and tin sulfate ($Sn(SO_4)_2$) forms. The equation for this reaction is:

$$SnO_2(s) + 2H_2SO_4(aq) \rightarrow Sn(SO_4)_2(s) + 2H_2O(l)$$

   If the mass of tin sulfate produced is 4,7 g, what is the percent purity of the sample?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23XV 2. 23XW 3. 23XX 4. 23XY 5. 23XZ

www.everythingscience.co.za m.everythingscience.co.za

8.3 Volume relationships in gaseous reactions ESBPF

Using what we have learnt about stoichiometry and about gases we can now apply these principles to reactions involving gases.

We will use explosions as an example.

Worked example 12: Explosions

QUESTION

Ammonium nitrate is used as an explosive in mining. The following reaction occurs when ammonium nitrate is heated:

$$2NH_4NO_3(s) \rightarrow 2N_2(g) + 4H_2O\ (g) + O_2(g)$$

If 750 g of ammonium nitrate is used, what volume of oxygen gas would we expect to produce (at STP)?

SOLUTION

Step 1: Work out the number of moles of ammonium nitrate

The number of moles of ammonium nitrate used is:

$$\begin{aligned} n &= \frac{m}{M} \\ &= \frac{750}{80} \\ &= 9{,}375 \text{ mol} \end{aligned}$$

Step 2: Work out the amount of oxygen

The mole ratio of NH_4NO_3 to O_2 is $2:1$. So the number of moles of O_2 is:

$$\begin{aligned} n_{O_2} &= n_{NH_4NO_3} \times \frac{\text{stoichiometric coefficient } O_2}{\text{stoichiometric coefficient } NH_4NO_3} \\ &= 9{,}375 \text{ mol} NH_4NO_3 \times \frac{1 \text{ mol} O_2}{2 \text{ mol } NH_4NO_3} \\ &= 4{,}6875 \text{ mol} \end{aligned}$$

Step 3: Work out the volume of oxygen

Recall from earlier in the chapter that we said that one mole of any gas occupies 22,4 dm^3 at STP.

$$\begin{aligned} V &= (22{,}4)n \\ &= (22{,}4)(4{,}6875) \\ &= 105 \text{ dm}^3 \end{aligned}$$

Airbags in cars use a controlled explosion to inflate the bag. When a car hits another car or an object, various sensors trigger the airbag. A chemical reaction then produces a large volume of gas which inflates the airbag.

Worked example 13: Controlled explosion

QUESTION

Sodium azide is sometimes used in airbags. When triggered, it has the following reaction:

$$2NaN_3(s) \rightarrow 2Na(s) + 3N_2(g)$$

If 55 grams of sodium azide is used, what volume of nitrogen gas would we expect to produce?

SOLUTION

Step 1: Work out the number of moles of sodium azide

The number of moles of sodium azide used is:

$$\begin{aligned} n &= \frac{m}{M} \\ &= \frac{55}{65} \\ &= 0{,}85 \text{ mol} \end{aligned}$$

Step 2: Work out the amount of nitrogen

The mole ratio of NaN_3 to N_2 is 2 : 3. So the number of moles of N_2 is:

$$\begin{aligned} n_{N_2} &= n_{NaN_3} \times \frac{\text{stoichiometric coefficient } N_2}{\text{stoichiometric coefficient } NaN_3} \\ &= 0{,}85 \text{ mol} NaN_3 \times \frac{3 \text{ mol} N_2}{2 \text{ mol } NaN_3} \\ &= 1{,}27 \text{ mol } N_2 \end{aligned}$$

Step 3: Work out the volume of nitrogen

$$\begin{aligned} V &= (22{,}4)n \\ &= (22{,}4)(1{,}27) \\ &= 28{,}4 \text{ dm}^3 \end{aligned}$$

Exercise 8 – 8: Gases 2

1. What volume of oxygen is needed for the complete combustion of 5 g of magnesium to form magnesium oxide?
2. Annalize is making a mini volcano for her science project. She mixes baking soda (mostly $NaHCO_3$) and vinegar (mostly CH_3COOH) together to make her volcano erupt. The reaction for this equation is:

   $$NaHCO_3(s) + CH_3COOH\ (aq) \rightarrow CH_3COONa\ (aq) + H_2O\ (l) + CO_2(g)$$

   What volume of carbon dioxide is produced if Annalize uses 50 ml of 0,2 mol·dm^3 acetic acid?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23Y2   2. 23Y3

www.everythingscience.co.za   m.everythingscience.co.za

8.4 Chapter summary   ESBPG

See presentation: 23Y4 at www.everythingscience.co.za

- The volume of **one mole** of gas at STP is 22,4 dm^3

- For any number of moles of gas at STP we can use $V_g = 22{,}4n_g$ to find the volume.
- The volume relationship for two gases in a reaction is given by: $V_A = \frac{a}{b}V_B$.

  where V_A is the volume of gas A, V_B is the gas B, a is the stoichiometric coefficient of gas A and b is the stoichiometric coefficient of gas B.
- The **concentration** of a solution can be calculated using: $C = \frac{n}{V}$

  where C is the concentration (in mol·dm^{3}), n is the number of moles of solute dissolved in the solution and V is the volume of the solution (in dm^{3}).
- A **standard solution** is a solution in which the concentration is known to a high degree of precision. For the purposes of calculations, a standard solution can be thought of as one in which the concentration is a set value.
- A titration is a technique for determining the concentration of an unknown solution. We can calculate the unknown concentration using:

$$\frac{C_A V_A}{a} = \frac{C_B V_B}{b}$$

- A **limiting reagent** is a reagent that is completely used up in a chemical reaction.
- An **excess reagent** is a reagent that is not completely used up in a chemical reaction.
- **Percent yield** is calculated using:

$$\%\text{yield} = \frac{\text{actual yield}}{\text{theoretical yield}} \times 100$$

  Where the actual yield is the amount of product that is produced when you carry out the reaction and the theoretical yield is the amount of product that you calculate for the reaction using stoichiometric methods.
- The **empirical formula** is the simplest formula of a compound.
- The **molecular formula** is the full formula of a compound.
- **Percent purity** is calculated using:

$$\%\text{purity} = \frac{\text{mass of compound}}{\text{mass of sample}} \times 100$$

Physical Quantities		
Quantity	**Unit name**	**Unit symbol**
Concentration (C)	moles per cubic decimetre	mol·dm^{-3}
Mass (m)	kilogram	kg
Molar mass (m)	gram per mole	g·mol^{-1}
Moles (n)	moles	mol

Exercise 8 – 9:

1. Write only the word/term for each of the following descriptions:

   a) A reagent that is completely used up in a chemical reaction.

   b) The simplest formula of a compound.

   c) The amount of product that is calculated for a reaction using stoichiometric methods.

   d) A technique for determining the concentration of an unknown solution.

2. What is the volume of 3 mol of N_2 gas at STP?

   a) 67,2 dm^3

   b) 22,4 dm^3

   c) 33,6 dm^3

   d) 63,2 dm^3

3. Given the following reaction:

$$3NO_2(g) + H_2O\ (g) \rightarrow 2HNO_3(g) + NO\ (g)$$

   If 2,7 dm^3 of NO_2 is used, what volume of HNO_3 is produced?

   a) 4,1 dm^3

   b) 2,7 dm^3

   c) 1,8 dm^3

   d) 3,4 dm^3

4. If 4,2 g of magnesium sulfate is dissolved in 350 cm^3 of water, what is the concentration of the solution?

   a) 0,1 $mol{\cdot}dm^{-3}$

   b) 0,05 $mol{\cdot}dm^{-3}$

   c) 0,003 $mol{\cdot}dm^{-3}$

   d) 0,0001 $mol{\cdot}dm^{-3}$

5. Gold is occasionally found as the mineral calaverite. Calaverite contains a telluride of gold ($AuTe_2$). Phumza wants to know how much calaverite is in a sample of rock. She finds that the rock sample weighs 3,6 g. After performing some experiments she finds that the mass of calaverite and the crucible is 2,4 g. The mass of the crucible is 0,3 g. What is the percent purity of calaverite in the sample?

   a) 63%

   b) 54%

   c) 58%

   d) 67%

6. 300 cm^3 of a 0,1 $mol{\cdot}dm^{-3}$ solution of sulfuric acid is added to 200 cm^3 of a 0,5 $mol{\cdot}dm^{-3}$ solution of sodium hydroxide.

a) Write down a balanced equation for the reaction which takes place when these two solutions are mixed.

b) Calculate the number of moles of sulfuric acid which were added to the sodium hydroxide solution.

c) Is the number of moles of sulfuric acid enough to fully neutralise the sodium hydroxide solution? Support your answer by showing all relevant calculations.

(IEB Paper 2 2004)

7. Given the equation:

$2NaOH\ (aq) + H_2SO_4(aq) \rightarrow Na_2SO_4(aq) + 2H_2O\ (l)$

25 cm^3 of a 0,7 $mol{\cdot}dm^{-3}$ sulfuric acid (H_2SO_4) solution was pipetted into a conical flask and titrated with sodium hydroxide (NaOH). It was found that 23 cm^3 of the sodium hydroxide was needed to neutralise the acid. Calculate the concentration of the sodium hydroxide.

8. Ozone (O_3) reacts with nitrogen monoxide gas (NO) to produce NO_2 gas. The NO gas forms largely as a result of emissions from the exhausts of motor vehicles and from certain jet planes. The NO_2 gas also contributes to the brown smog (smoke and fog), which is seen over most urban areas. This gas is also harmful to humans, as it causes breathing (respiratory) problems. The following equation indicates the reaction between ozone and nitrogen monoxide:

$$O_3(g) + NO\ (g) \rightarrow O_2(g) + NO_2(g)$$

In one such reaction 0,74 g of O_3 reacts with 0,67 g NO.

a) Calculate the number of moles of O_3 and of NO present at the start of the reaction.

b) Identify the limiting reagent in the reaction and justify your answer.

c) Calculate the mass of NO_2 produced from the reaction.

(DoE Exemplar Paper 2, 2007)

9. Calcium carbonate decomposes on heating to produce calcium oxide and oxygen according to the following equation:

$$CaCO_3(s) \rightarrow CaO\ (s) + O_2(g)$$

Thabang carries out the above reaction using 127 g of calcium carbonate. He finds that he gets 68,2 g of calcium oxide. What is the percentage yield?

10. Some airbags contain a mixture of sodium azide (NaN_3) and potassium nitrate (KNO_3). When a car crash is detected by the signalling system, the sodium azide is heated until it decomposes to form nitrogen gas and sodium metal:

$$2NaN_3(s) \rightarrow 2Na\ (s) + 3N_2(g)$$

The potassium nitrate then reacts with the sodium metal forming more nitrogen:

$$10Na\ (s) + 2KNO_3(s) \rightarrow K_2O\ (s) + 5Na_2O\ (s) + N_2(g)$$

A typical passenger side airbag contains 250 g of sodium azide.

a) What mass of sodium metal is formed in the first reaction?

b) What is the total volume of nitrogen gas formed from both reactions?

c) How much potassium nitrate (in g) is needed for all the sodium to be used up in the second reaction?

11. Chlorofluorocarbons (CFC's) are a class of compounds that have a long history of use in refrigerators. CFC's are slowly being phased out as they deplete the amount of ozone in the ozone layer. Jabu has a sample of a CFC that has the following percentage composition: 14,05% carbon, 41,48% chlorine and 44,46% fluorine.

    Determine the molecular formula of this CFC if the molar mass is 171 $g{\cdot}mol^{-1}$.

12. A sample containing tin dioxide (SnO_2) is to be tested to see how much tin dioxide it contains. The sample weighs 6,2 g. Sulfuric acid (H_2SO_4) is added to the sample and tin sulfate ($Sn(SO_4)_2$) forms. The equation for this reaction is:

$$SnO_2(s) + 2H_2SO_4(aq) \rightarrow Sn(SO_4)_2(s) + 2H_2O\ (l)$$

    If the mass of tin sulfate produced is 4,7 g, what is the percent purity of the sample?

13. Syngas (synthesis gas) is a mixture of carbon monoxide and hydrogen. Syngas can be produced from methane using:

$$CH_4(g) + H_2O\ (g) \rightarrow CO\ (g) + 3H_2(g)$$

    Neels wants to make a mixture of syngas that has three times the volume of hydrogen gas.

    a) If the volume of methane used is 4 dm^3, what volume of carbon monoxide and hydrogen will be produced?

    b) Will this amount of methane produce the correct mixture?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 23Y5 1b. 23Y6 1c. 23Y7 1d. 23Y8 2. 23Y9 3. 23YB
4. 23YC 5. 23YD 6. 23YF 7. 23YG 8. 23YH 9. 23YJ
10. 23YK 11. 23YM 12. 23YN 13. 23YP

www.everythingscience.co.za m.everythingscience.co.za

CHAPTER 9

Electrostatics

9.1 Introduction ESBPH

In Grade 10, you learnt about the force between charges. In this chapter you will learn exactly how to determine this force and about a basic law of electrostatics.

Key Mathematics Concepts

- Ratio and proportion — Physical Sciences, Grade 10, Science skills
- Equations — Mathematics, Grade 10, Equations and inequalities
- Units and unit conversions — Physical Sciences, Grade 10, Science skills
- Scientific notation — Physical Sciences, Grade 10, Science skills

9.2 Coulomb's law ESBPJ

Like charges repel each other while unlike charges attract each other. If the charges are at rest then the force between them is known as the **electrostatic force**. The electrostatic force between charges increases when the magnitude of the charges increases or the distance between the charges decreases.

The electrostatic force was first studied in detail by Charles-Augustin de Coulomb around 1784. Through his observations he was able to show that the **magnitude** of the electrostatic force between two point-like charges is inversely proportional to the square of the distance between the charges. He also discovered that the **magnitude** of the force is proportional to the product of the charges. That is:

$$F \propto \frac{Q_1 Q_2}{r^2},$$

where Q_1 and Q_2 are the magnitudes of the two charges respectively and r is the distance between them. The magnitude of the electrostatic force between two point-like charges is given by *Coulomb's law*.

DEFINITION: *Coulomb's law*

Coulomb's law states that the magnitude of the electrostatic force between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.

$$F = \frac{kQ_1 Q_2}{r^2},$$

The proportionality constant k is called the *electrostatic constant* and has the value: 9,0 $\times$ 10^9 N·m^2·C^{-2} in free space.

Similarity of Coulomb's law to Newton's universal law of gravitation.

Notice how similar in form Coulomb's law is to Newton's universal law of gravitation between two point-like particles:

$$F_G = \frac{Gm_1m_2}{d^2},$$

where m_1 and m_2 are the masses of the two point-like particles, d is the distance between them, and G is the gravitational constant. Both are inverse-square laws.

Both laws represent the force exerted by particles (point masses or point charges) on each other that interact by means of a field.

See video: 23YQ at www.everythingscience.co.za

Worked example 1: Coulomb's law

QUESTION

Two point-like charges carrying charges of $+3 \times 10^{-9}$ C and -5×10^{-9} C are 2 m apart. Determine the magnitude of the force between them and state whether it is attractive or repulsive.

SOLUTION

Step 1: Determine what is required

We are required to determine the force between two point charges given the charges and the distance between them.

Step 2: Determine how to approach the problem

We can use Coulomb's law to calculate the magnitude of the force. $F = k\frac{Q_1Q_2}{r^2}$

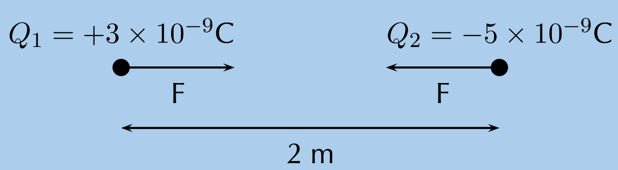
Step 3: Determine what is given

We are given:

- $Q_1 = +3 \times 10^{-9}$ C
- $Q_2 = -5 \times 10^{-9}$ C
- $r = 2$ m

We know that $k = 9{,}0 \times 10^{9}$ N·m^2·C^{-2}.

We can draw a diagram of the situation.



Step 4: Check units

All quantities are in SI units.

Step 5: Determine the magnitude of the force

Using Coulomb's law we have

$$
\begin{aligned}
F &= \frac{kQ_1Q_2}{r^2} \\
&= \frac{(9{,}0 \times 10^9)(3 \times 10^{-9})(5 \times 10^{-9})}{(2)^2} \\
&= 3{,}37 \times 10^{-8}\ \text{N}
\end{aligned}
$$

Thus the *magnitude* of the force is $3{,}37 \times 10^{-8}$ N. However since the point charges have opposite signs, the force will be attractive.

Step 6: Free body diagram

We can draw a free body diagram to show the forces. Each charge experiences a force with the same magnitude and the forces are attractive, so we have:

$Q_1 = +3 \times 10^{-9}\text{C}$ $\qquad$ $Q_2 = -5 \times 10^{-9}\text{C}$

$\bullet\!\longrightarrow$ $\qquad\qquad$ $\longleftarrow\!\bullet$

$\text{F} = 3,37 \times 10^{-8}$ N $\qquad$ $\text{F} = 3,37 \times 10^{-8}$ N

Next is another example that demonstrates the difference in magnitude between the gravitational force and the electrostatic force.

Worked example 2: Coulomb's law

QUESTION

Determine the magnitudes of the electrostatic force and gravitational force between two electrons 10^{-10} m apart (i.e. the forces felt inside an atom) and state whether the forces are attractive or repulsive.

SOLUTION

Step 1: Determine what is required

We are required to calculate the electrostatic and gravitational forces between two electrons, a given distance apart.

Step 2: Determine how to approach the problem

We can use:

$$F_e = \frac{kQ_1Q_2}{r^2}$$

to calculate the electrostatic force and

$$F_g = \frac{Gm_1m_2}{d^2}$$

to calculate the gravitational force.

Step 3: Determine what is given

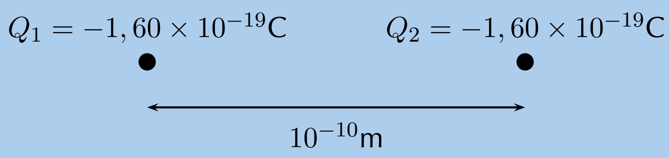
- $Q_1 = Q_2 = 1{,}6 \times 10^{-19}$ C (The charge on an electron)
- $m_1 = m_2 = 9{,}1 \times 10^{-31}$ kg (The mass of an electron)
- $r = d = 1 \times 10^{-10}$ m

We know that:

- $k = 9{,}0 \times 10^{9}$ N·m^2·C^{-2}
- $G = 6{,}67 \times 10^{-11}$ N·m^2·kg^{-2}

All quantities are in SI units.

We can draw a diagram of the situation.



Step 4: Calculate the electrostatic force

$$\begin{aligned} F_e &= \frac{kQ_1Q_2}{r^2} \\ &= \frac{(9{,}0 \times 10^{9})(1{,}60 \times 10^{-19})(1{,}60 \times 10^{-19})}{(10^{-10})^2} \\ &= 2{,}30 \times 10^{-8} \text{ N} \end{aligned}$$

Hence the *magnitude* of the electrostatic force between the electrons is $2{,}30 \times 10^{-8}$ N. Since electrons carry like charges, the force is repulsive.

Step 5: Calculate the gravitational force

$$\begin{aligned} F_g &= \frac{Gm_1m_2}{d^2} \\ &= \frac{(6{,}67 \times 10^{-11})(9{,}11 \times 10^{-31})(9{,}11 \times 10^{-31})}{(10^{-10})^2} \\ &= 5{,}54 \times 10^{-51} \text{ N} \end{aligned}$$

TIP
We can apply Newton's third law to charges because two charges exert forces of equal magnitude on one another in opposite directions.

TIP
Choosing a positive direction
When substituting into the Coulomb's law equation, one may choose a positive direction thus making it unnecessary to include the signs of the charges. Instead, select a positive direction. Those forces that tend to move the charge in this direction are added, while forces acting in the opposite direction are subtracted.

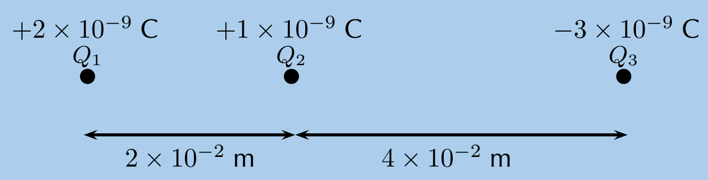
The magnitude of the gravitational force between the electrons is $5{,}54 \times 10^{-51}$ N. Remember that the gravitational force is always an attractive force.

Notice that the gravitational force between the electrons is **much smaller** than the electrostatic force.

Worked example 3: Coulomb's law

QUESTION

Three point charges are in a straight line. Their charges are $Q_1 = +2 \times 10^{-9}$ C, $Q_2 = +1 \times 10^{-9}$ C and $Q_3 = -3 \times 10^{-9}$ C. The distance between Q_1 and Q_2 is 2×10^{-2} m and the distance between Q_2 and Q_3 is 4×10^{-2} m. What is the net electrostatic force on Q_2 due to the other two charges?



SOLUTION

Step 1: Determine what is required

We need to calculate the net force on Q_2. This force is the sum of the two electrostatic forces - the forces between Q_1 on Q_2 and Q_3 on Q_2.

Step 2: Determine how to approach the problem

- We need to calculate the two electrostatic forces on Q_2, using Coulomb's law.
- We then need to add up the two forces using our rules for adding vector quantities, because force is a vector quantity.

Step 3: Determine what is given

We are given all the charges and all the distances.

Step 4: Calculate the magnitude of the forces.

Force on Q_2 due to Q_1:

$$
\begin{aligned}
F_1 &= k\frac{Q_1Q_2}{r^2} \\
&= (9{,}0 \times 10^9)\frac{(2 \times 10^{-9})(1 \times 10^{-9})}{(2 \times 10^{-2})^2} \\
&= (9{,}0 \times 10^9)\frac{(2 \times 10^{-9})(1 \times 10^{-9})}{(4 \times 10^{-4})} \\
&= 4{,}5 \times 10^{-5} \text{ N}
\end{aligned}
$$

Force on Q_2 due to Q_3:

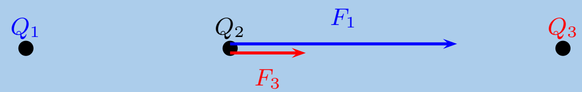
$$
\begin{aligned}
F_3 &= k\frac{Q_2Q_3}{r^2} \\
&= (9{,}0 \times 10^9)\frac{(1 \times 10^{-9})(3 \times 10^{-9})}{(4 \times 10^{-2})^2} \\
&= (9{,}0 \times 10^9)\frac{(1 \times 10^{-9})(3 \times 10^{-9})}{(16 \times 10^{-4})} \\
&= 1{,}69 \times 10^{-5} \text{ N}
\end{aligned}
$$

Step 5: Vector addition of forces

We know the force magnitudes but we need to use the charges to determine whether the forces are repulsive or attractive. It is helpful to draw the force diagram to help determine the final direction of the net force on Q_2. We choose the positive direction to be to the right (the positive x-direction).

The force between Q_1 and Q_2 is repulsive (like charges). This means that it pushes Q_2 to the right, or in the positive direction.

The force between Q_2 and Q_3 is attractive (unlike charges) and pulls Q_2 to the right.



Therefore both forces are acting in the positive direction.

Therefore,

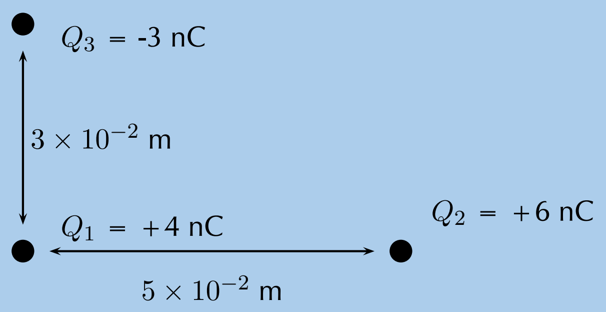
$$
\begin{aligned}
F_R &= 4{,}5 \times 10^{-5} \text{ N} + 1{,}69 \times 10^{-5} \text{ N} \\
&= 6{,}19 \times 10^{-5} \text{ N}
\end{aligned}
$$

The resultant force acting on Q_2 is $6{,}19 \times 10^{-5}$ N to the right.

Worked example 4: Coulomb's law

QUESTION

Three point charges form a right-angled triangle. Their charges are $Q_1 = 4 \times 10^{-9}$ C = 4 nC, $Q_2 = 6 \times 10^{-9}$ C = 6 nC and $Q_3 = -3 \times 10^{-9}$ C = -3 nC. The distance between Q_1 and Q_2 is 5×10^{-2} m and the distance between Q_1 and Q_3 is 3×10^{-2} m. What is the net electrostatic force on Q_1 due to the other two charges if they are arranged as shown?



SOLUTION

Step 1: Determine what is required

We need to calculate the net force on Q_1. This force is the sum of the two electrostatic forces - the forces of Q_2 on Q_1 and Q_3 on Q_1.

Step 2: Determine how to approach the problem

- We need to calculate, using Coulomb's law, the electrostatic force exerted on Q_1 by Q_2, and the electrostatic force exerted on Q_1 by Q_3.
- We then need to add up the two forces using our rules for adding vector quantities, because force is a vector quantity.

Step 3: Determine what is given

We are given all the charges and two of the distances.

Step 4: Calculate the magnitude of the forces.

The magnitude of the force exerted by Q_2 on Q_1, which we will call F_2, is:

$$\begin{aligned}
F_2 &= k\frac{Q_1Q_2}{r^2} \\
&= (9{,}0 \times 10^9)\frac{(4 \times 10^{-9})(6 \times 10^{-9})}{(5 \times 10^{-2})^2} \\
&= (9{,}0 \times 10^9)\frac{(4 \times 10^{-9})(6 \times 10^{-9})}{(25 \times 10^{-4})} \\
&= 8{,}630 \times 10^{-5}\ \text{N}
\end{aligned}$$

The magnitude of the force exerted by Q_3 on Q_1, which we will call F_3, is:

$$\begin{aligned}F_3 &= k\frac{Q_1Q_3}{r^2}\\ &= (9{,}0\times 10^{9})\frac{(4\times 10^{-9})(3\times 10^{-9})}{(3\times 10^{-2})^2}\\ &= (9{,}0\times 10^{9})\frac{(4\times 10^{-9})(3\times 10^{-9})}{(9\times 10^{-4})}\\ &= 1{,}199\times 10^{-4}\ \text{N}\end{aligned}$$

Step 5: Vector addition of forces

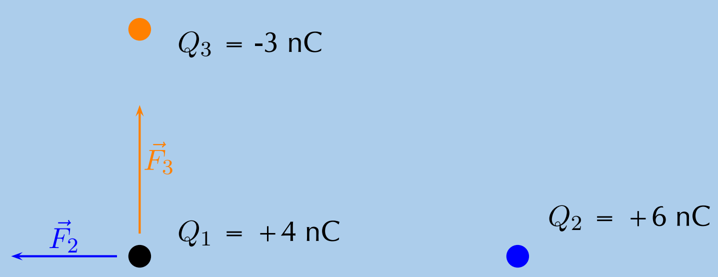
This is a two-dimensional problem involving vectors. We have already solved many two-dimensional force problems and will use precisely the **same procedure** as before. Determine the vectors on the Cartesian plane, break them into components in the x- and y-directions, and then sum components in each direction to get the components of the resultant.

We choose the positive directions to be to the right (the positive x-direction) and up (the positive y-direction). We know the magnitudes of the forces but we need to use the signs of the charges to determine whether the forces are repulsive or attractive. Then we can use a diagram to determine the directions.

The force between Q_1 and Q_2 is repulsive (like charges). This means that it pushes Q_1 to the left, or in the negative x-direction.

The force between Q_1 and Q_3 is attractive (unlike charges) and pulls Q_1 in the positive y-direction.

We can redraw the diagram as a free-body diagram illustrating the forces to make sure we can visualise the situation:

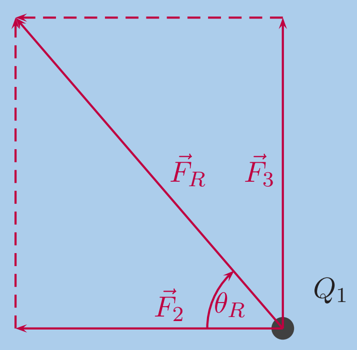


Step 6: Resultant force

The magnitude of the resultant force acting on Q_1 can be calculated from the forces using Pythagoras' theorem because there are only two forces and they act in the x- and y-directions:

$$\begin{aligned}F_R^2 &= F_2^2 + F_3^2 \text{ by Pythagoras' theorem}\\ F_R &= \sqrt{(8{,}630\times 10^{-5})^2 + (1{,}199\times 10^{-4})^2}\\ F_R &= 1{,}48\times 10^{-4}\ \text{N}\end{aligned}$$

and the angle, θ_R made with the x-axis can be found using trigonometry.



$$\tan(\theta_R) = \frac{\text{y-component}}{\text{x-component}}$$

$$\tan(\theta_R) = \frac{1{,}199 \times 10^{-4}}{8{,}630 \times 10^{-5}}$$

$$\theta_R = \tan^{-1}(\frac{1{,}199 \times 10^{-4}}{8{,}630 \times 10^{-5}})$$

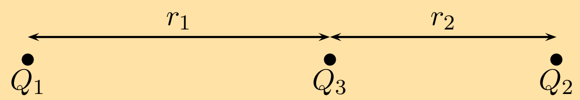
$$\theta_R = 54{,}25^\circ \text{ to 2 decimal places}$$

The final resultant force acting on Q_1 is $1{,}48 \times 10^{-4}$ N acting at an angle of $54{,}25^\circ$ to the negative x-axis or $125{,}75^\circ$to the positive x-axis.

We mentioned in grade 10 that charge placed on a spherical conductor spreads evenly along the surface. As a result, if we are far enough from the charged sphere, electrostatically, it behaves as a point-like charge. Thus we can treat spherical conductors (e.g. metallic balls) as point-like charges, with all the charge acting at the centre.

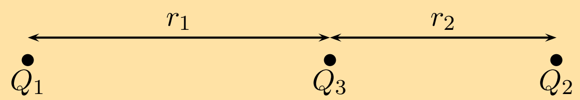
Exercise 9 – 1: Electrostatic forces

1. Calculate the electrostatic force between two charges of +6 nC and +1 nC if they are separated by a distance of 2 mm.
2. What is the magnitude of the repulsive force between two pith balls (a pith ball is a small, light ball that can easily be charged) that are 8 cm apart and have equal charges of −30 nC?
3. How strong is the attractive force between a glass rod with a 0,7 μC charge and a silk cloth with a −0,6 μC charge, which are 12 cm apart, using the approximation that they act like point charges?
4. Two point charges exert a 5 N force on each other. What will the resulting force be if the distance between them is increased by a factor of three?
5. Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?
6. If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them?
7. Calculate the distance between two charges of +4 nC and −3 nC if the electrostatic force between them is 0,005 N.
8. For the charge configuration shown, calculate the resultant force on Q_2 if:
   - $Q_1 = 2{,}3 \times 10^{-7}$ C
   - $Q_2 = 4 \times 10^{-6}$ C
   - $Q_3 = 3{,}3 \times 10^{-7}$ C
   - $r_1 = 2{,}5 \times 10^{-1}$ m
   - $r_2 = 3{,}7 \times 10^{-2}$ m

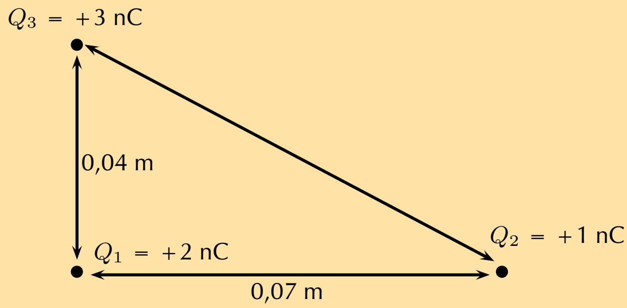


9. For the charge configuration shown, calculate the charge on Q_3 if the resultant force on Q_2 is $6{,}3 \times 10^{-1}$ N to the right and:

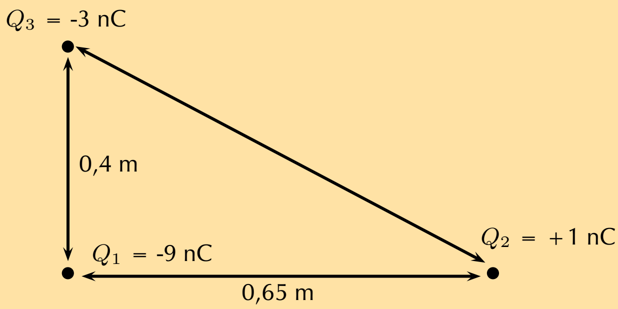
- $Q_1 = 4{,}36 \times 10^{-6}$ C
- $Q_2 = -7 \times 10^{-7}$ C
- $r_1 = 1{,}85 \times 10^{-1}$ m
- $r_2 = 4{,}7 \times 10^{-2}$ m



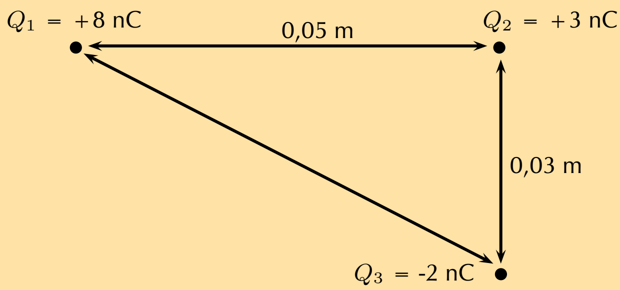
10. Calculate the resultant force on Q_1 given this charge configuration:



11. Calculate the resultant force on Q_1 given this charge configuration:



12. Calculate the resultant force on Q_2 given this charge configuration:



Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23YR 2. 23YS 3. 23YT 4. 23YV 5. 23YW 6. 23YX
7. 23YY 8. 23YZ 9. 23Z2 10. 23Z3 11. 23Z4 12. 23Z5

www.everythingscience.co.za m.everythingscience.co.za

9.3 Electric field ESBPK

We have seen in the previous section that point charges exert forces on each other even when they are far apart and not touching each other. How do the charges 'know' about the existence of other charges around them?

The answer is that you can think of every charge as being surrounded in space by an electric field. The electric field is the region of space in which an electric charge will experience a force. The direction of the electric field represents the direction of the force a positive test charge would experience if placed in the electric field. In other words, the direction of an electric field at a point in space is the same direction in which a positive test charge would move if placed at that point.

DEFINITION: *Electric field*

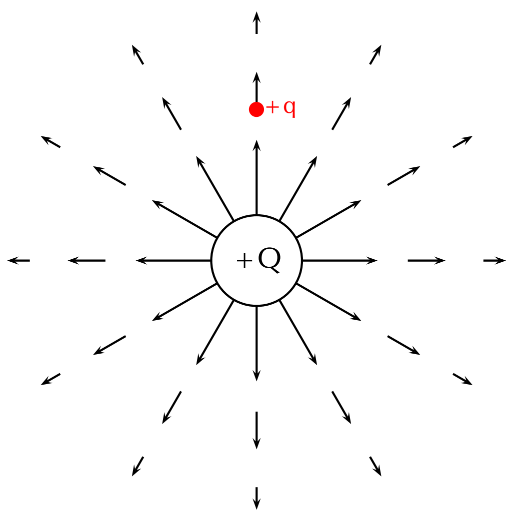
A region of space in which an electric charge will experience a force. The direction of the field at a point in space is the direction in which a positive test charge would moved if placed at that point.

Representing electric fields ESBPM

We can represent the strength and direction of an electric field at a point using **electric field lines**. This is similar to representing magnetic fields around magnets using magnetic field lines as you studied in Grade 10. In the following we will study what the electric fields look like around isolated charges.

Positive charge acting on a test charge

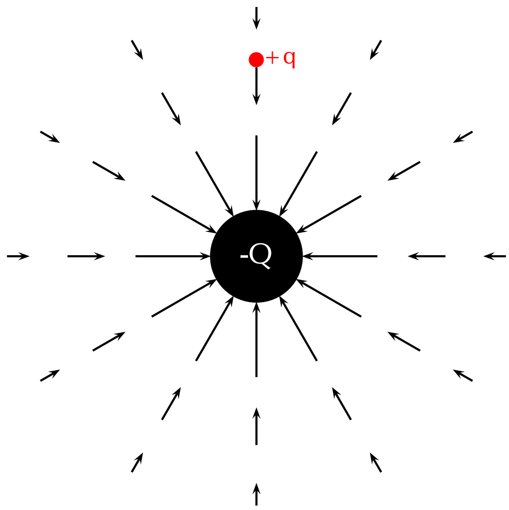
The magnitude of the force that a test charge experiences due to another charge is governed by Coulomb's law. In the diagram below, at each point around the positive charge, $+Q$, we calculate the force a positive test charge, $+q$, would experience, and represent this force (a vector) with an arrow. The force vectors for some points around $+Q$ are shown in the diagram along with the positive test charge $+q$ (in red) located at one of the points.



At every point around the charge $+Q$, the positive test charge, $+q$, will experience a force pushing it away. This is because both charges are positive and so they repel each other. We cannot draw an arrow at every point but we include enough arrows to illustrate what the field would look like. The arrows represent the force the test charge would experience at each point. Coulomb's law is an inverse-square law which means that the force gets weaker the greater the distance between the two charges. This is why the arrows get shorter further away from $+Q$.

Negative charge acting on a test charge

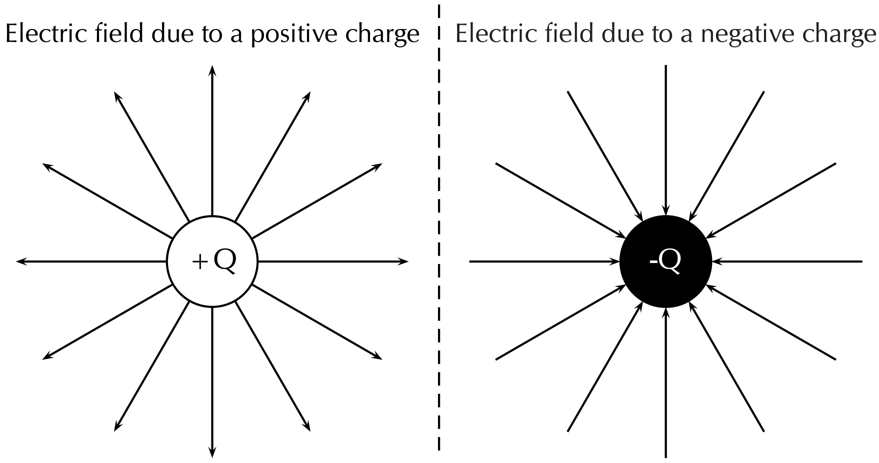
For a negative charge, $-Q$, and a positive test charge, $+q$, the force vectors would look like:



Notice that it is almost identical to the positive charge case. The arrows are the same lengths as in the previous diagram because the absolute magnitude of the charge is the same and so is the magnitude of the test charge. Thus the magnitude of the force is the same at the same points in space. However, the arrows point in the opposite direction because the charges now have opposite signs and attract each other.

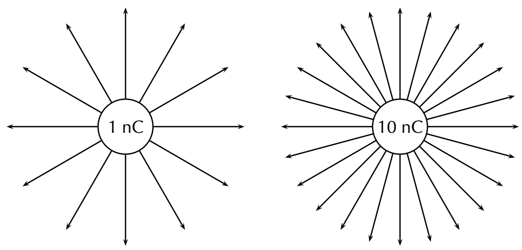
Electric fields around isolated charges - summary

Now, to make things simpler, we draw continuous lines that are tangential to the force that a test charge would experience at each point. The field lines are closer together where the field is stronger. Look at the diagram below: close to the central charges, the field lines are close together. This is where the electric field is strongest. Further away from the central charges where the electric field is weaker, the field lines are more spread out from each other.



We use the following conventions when drawing electric field lines:

- Arrows on the field lines indicate the direction of the field, i.e. the direction in which a positive test charge would move if placed in the field.
- Electric field lines point away from positive charges (like charges repel) and towards negative charges (unlike charges attract).
- Field lines are drawn closer together where the field is stronger.
- Field lines do not touch or cross each other.
- Field lines are drawn perpendicular to a charge or charged surface.
- The greater the magnitude of the charge, the stronger its electric field. We represent this by drawing more field lines around the greater charge than for charges with smaller magnitudes.



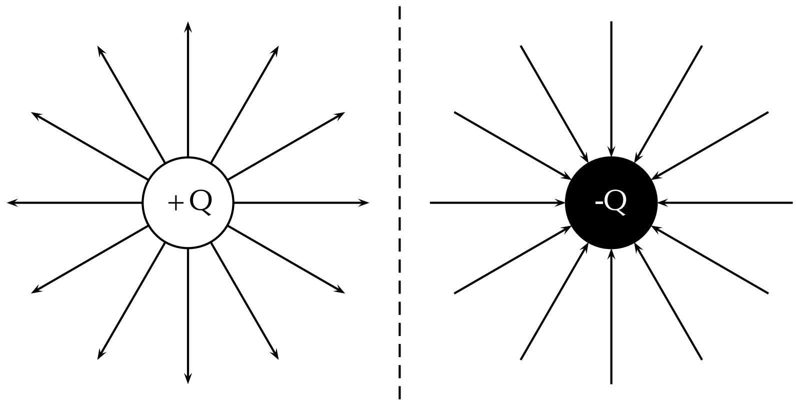
Some important points to remember about electric fields:

- There is an electric field at **every point** in space surrounding a charge.
- Field lines are merely a **representation** – they are not real. When we draw them, we just pick convenient places to indicate the field in space.
- Field lines exist in three dimensions, not only in two dimension as we've drawn them.
- The number of field lines passing through a surface is proportional to the charge contained inside the surface.

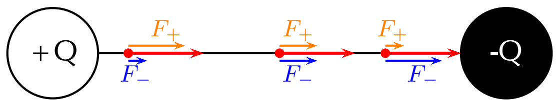
We have seen what the electric fields look like around isolated positive and negative charges. Now we will study what the electric fields look like around combinations of charges placed close together.

Electric field around two unlike charges

We will start by looking at the electric field around a positive and negative charge placed next to each other. Using the rules for drawing electric field lines, we will sketch the electric field one step at a time. The net resulting field is the sum of the fields from each of the charges. To start off let us sketch the electric fields for each of the charges separately.

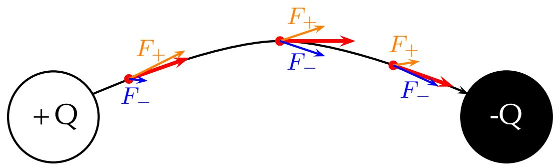


A positive test charge (red dots) placed at different positions directly between the two charges would be pushed away (orange force arrows) from the positive charge and pulled towards (blue force arrows) the negative charge in a straight line. The orange and blue force arrows have been drawn slightly offset from the dots for clarity. In reality they would lie on top of each other. Notice that the further from the positive charge, the smaller the repulsive force, F_+ (shorter orange arrows) and the closer to the negative charge the greater the attractive force, F_- (longer blue arrows). The resultant forces are shown by the red arrows. The electric field line is the black line which is tangential to the resultant forces and is a straight line between the charges pointing from the positive to the negative charge.

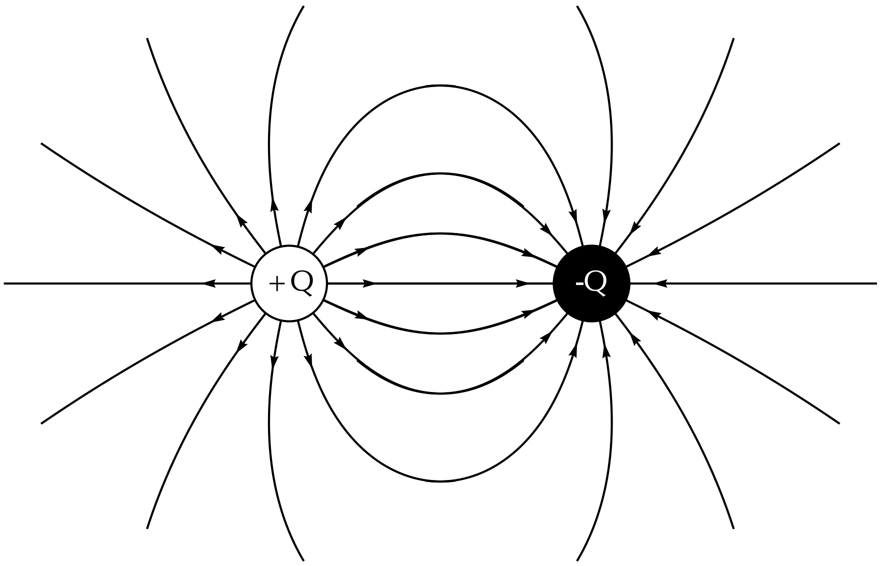


Now let's consider a positive test charge placed slightly higher than the line joining the two charges. The test charge will experience a repulsive force (F_+ in orange) from the positive charge and an attractive force (F_- in blue) due to the negative charge. As before, the magnitude of these forces will depend on the distance of the test charge from each of the charges according to Coulomb's law. Starting at a position closer to the positive charge, the test charge will experience a larger repulsive force due to the positive charge and a weaker attractive force from the negative charge. At a position half-way between the positive and negative charges, the magnitudes of the repulsive and attractive forces are the same. If the test charge is placed closer to the negative charge, then the attractive force will be greater and the repulsive force it experiences

due to the more distant positive charge will be weaker. At each point we add the forces due to the positive and negative charges to find the resultant force on the test charge (shown by the red arrows). The resulting electric field line, which is tangential to the resultant force vectors, will be a curve.

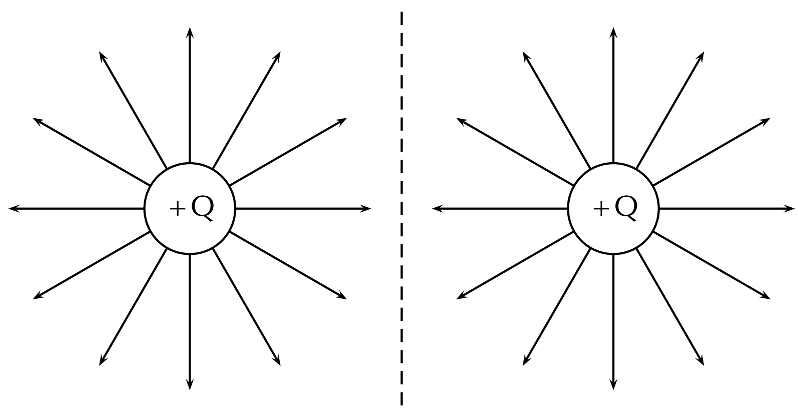


Now we can fill in the other field lines quite easily using the same ideas. The electric field lines look like:



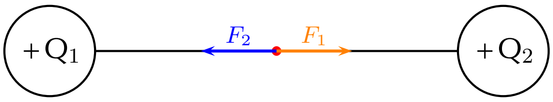
Electric field around two like charges (both positive)

For the case of two positive charges Q_1 and Q_2 of the same magnitude, things look a little different. We can't just turn the arrows around the way we did before. In this case the positive test charge is repelled by both charges. The electric fields around each of the charges in isolation looks like.



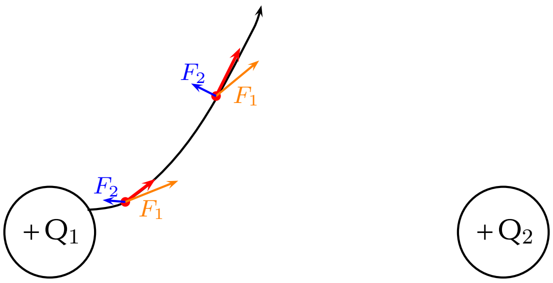
Now we can look at the resulting electric field when the charges are placed next to each other. Let us start by placing a positive test charge directly between the two

charges. We can draw the forces exerted on the test charge due to Q_1 and Q_2 and determine the resultant force.

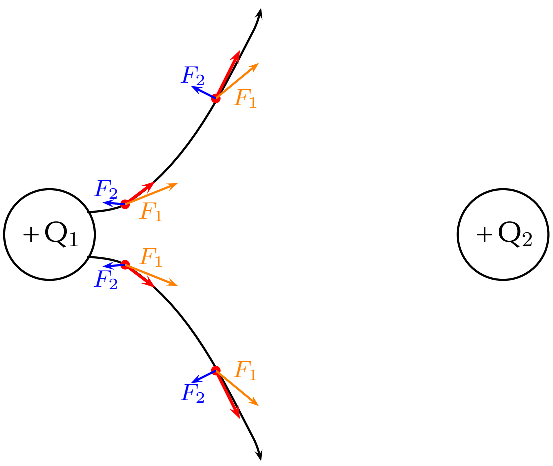


The force F_1 (in orange) on the test charge (red dot) due to the charge Q_1 is equal in magnitude but opposite in direction to F_2 (in blue) which is the force exerted on the test charge due to Q_2. Therefore they cancel each other out and there is no resultant force. This means that the electric field directly between the charges cancels out in the middle. A test charge placed at this point would not experience a force.

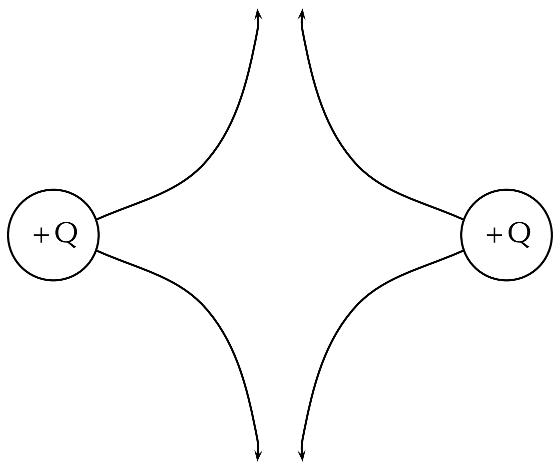
Now let's consider a positive test charge placed close to Q_1 and above the imaginary line joining the centres of the charges. Again we can draw the forces exerted on the test charge due to Q_1 and Q_2 and sum them to find the resultant force (shown in red). This tells us the direction of the electric field line at each point. The electric field line (black line) is tangential to the resultant forces.



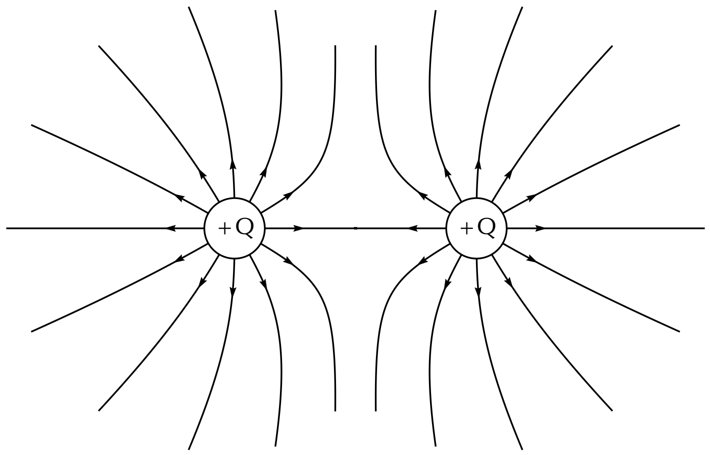
If we place a test charge in the same relative positions but *below* the imaginary line joining the centres of the charges, we can see in the diagram below that the resultant forces are reflections of the forces above. Therefore, the electric field line is just a reflection of the field line above.



Since Q_2 has the same charge as Q_1, the forces at the same relative points close to Q_2 will have the same magnitudes but opposite directions i.e. they are also reflections . We can therefore easily draw the next two field lines as follows:

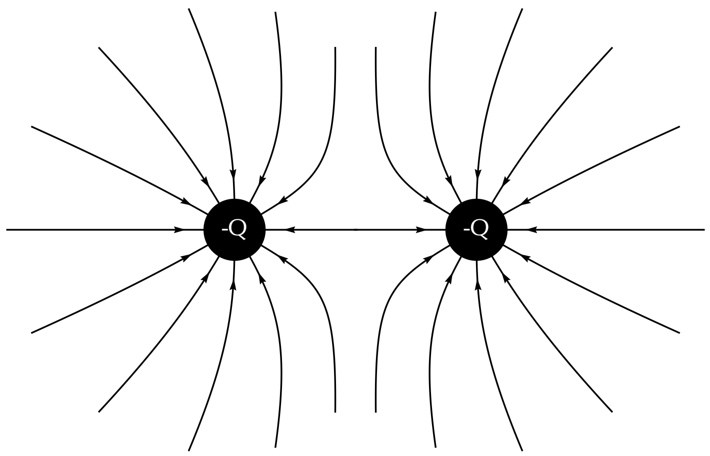


Working through a number of possible starting points for the test charge we can show the electric field can be represented by:



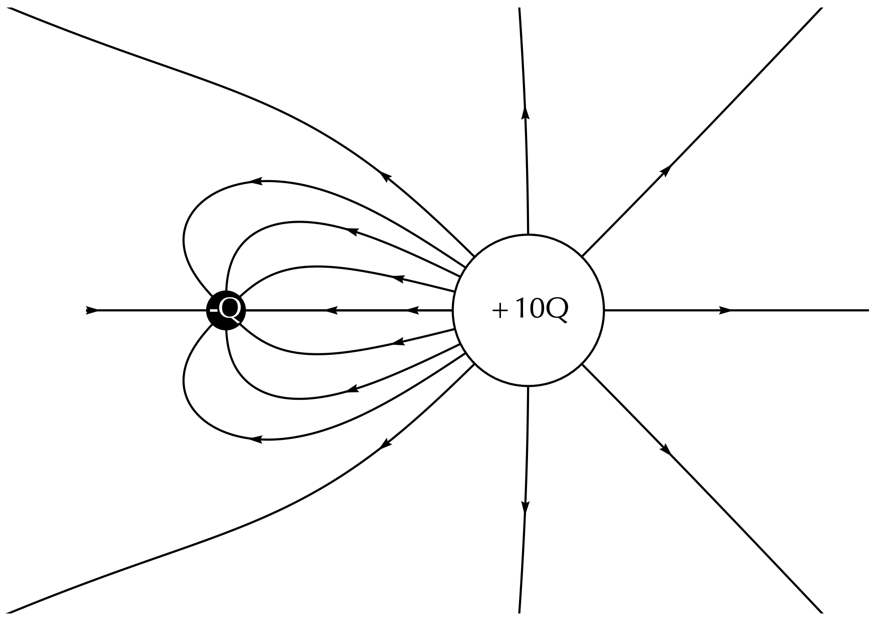
Electric field around two like charges (both negative)

We can use the fact that the direction of the force is reversed for a test charge if you change the sign of the charge that is influencing it. If we change to the case where both charges are negative we get the following result:



Charges of different magnitudes

When the magnitudes are not equal the larger charge will influence the direction of the field lines more than if they were equal. For example, here is a configuration where the positive charge is much larger than the negative charge. You can see that the field lines look more similar to that of an isolated charge at greater distances than in the earlier example. This is because the larger charge gives rise to a stronger field and therefore makes a larger relative contribution to the force on a test charge than the smaller charge.



Electric field strength ESBPP

In the previous sections we have studied how we can represent the electric fields around a charge or combination of charges by means of electric field lines. In this representation we see that the electric field strength is represented by how close together the field lines are. In addition to the drawings of the electric field, we would also like to be able to quantify (put a number to) how strong an electric field is and what its direction is at any point in space.

A small test charge q placed near a charge Q will experience a force due to the electric field surrounding Q. The magnitude of the force is described by Coulomb's law and depends on the magnitude of the charge Q and the distance of the test charge from Q. The closer the test charge q is to the charge Q, the greater the force it will experience. Also, at points closer to the charge Q, the stronger is its electric field. We define the electric field at a point as the force per unit charge.

DEFINITION: *Electric field*

The magnitude of the electric field, E, at a point can be quantified as the force per unit charge We can write this as:

$$E = \frac{F}{q}$$

where F is the Coulomb force exerted by a charge on a test charge q.
The units of the electric field are newtons per coulomb: N·C^{-1}.

Since the force F is a vector and q is a scalar, the electric field, E, is also a vector; it has a magnitude and a direction at every point.

Given the definition of electric field above and substituting the expression for Coulomb's law for F:

$$
\begin{aligned}
E &= \frac{F}{q} \\
&= \frac{kQq}{r^2 q} \\
E &= \frac{kQ}{r^2}
\end{aligned}
$$

we can see that the electric field E only depends on the charge Q and not the magnitude of the test charge.

If the electric field is known, then the electrostatic force on any charge q placed into the field is simply obtained by rearranging the definition equation:

$$F = qE.$$

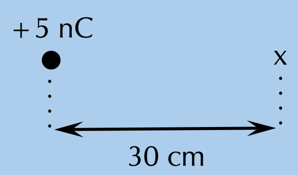
See video: 23Z6 at www.everythingscience.co.za

See simulation: 23Z7 at www.everythingscience.co.za

Worked example 5: Electric field 1

QUESTION

Calculate the electric field strength 30 cm from a 5 nC charge.



SOLUTION

Step 1: Determine what is required

We need to calculate the electric field a distance from a given charge.

Step 2: Determine what is given

We are given the magnitude of the charge and the distance from the charge.

Step 3: Determine how to approach the problem

We will use the equation: $E = k\frac{Q}{r^2}$.

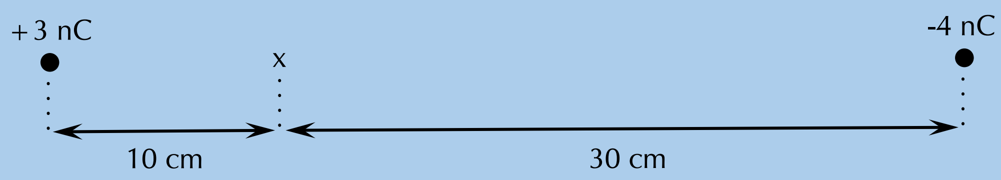
Step 4: Solve the problem

$$
\begin{aligned}
E &= \frac{kQ}{r^2} \\
&= \frac{(9{,}0 \times 10^9)(5 \times 10^{-9})}{0{,}3^2} \\
&= 4{,}99 \times 10^2 \text{ N·C}^{-1}
\end{aligned}
$$

Worked example 6: Electric field 2

QUESTION

Two charges of $Q_1 = +3\text{nC}$ and $Q_2 = -4\text{nC}$ are separated by a distance of 50 cm. What is the electric field strength at a point that is 20 cm from Q_1 and 50 cm from Q_2? The point lies between Q_1 and Q_2.



SOLUTION

Step 1: Determine what is required

We need to calculate the electric field a distance from two given charges.

Step 2: Determine what is given

We are given the magnitude of the charges and the distances from the charges.

Step 3: Determine how to approach the problem

We will use the equation: $E = k\frac{Q}{r^2}$.

We need to calculate the electric field for each charge separately and then add them to determine the resultant field.

Step 4: Solve the problem

We first solve for Q_1:

$$
\begin{aligned}
E &= \frac{kQ}{r^2} \\
&= \frac{(9{,}0 \times 10^9)(3 \times 10^{-9})}{0{,}2^2} \\
&= 6{,}74 \times 10^2 \text{ N·C}^{-1}
\end{aligned}
$$

Then for Q_2:

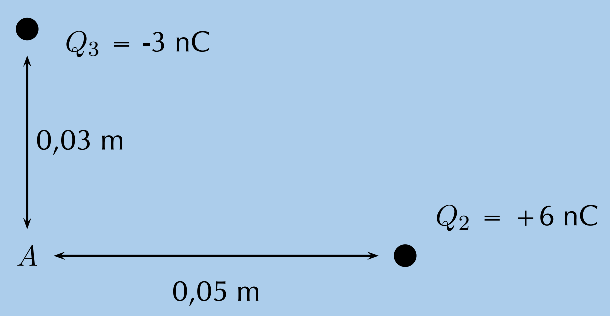
$$
\begin{aligned}
E &= \frac{kQ}{r^2} \\
&= \frac{(9{,}0 \times 10^9)(4 \times 10^{-9})}{0{,}3^2} \\
&= 3{,}99 \times 10^2 \text{ N·C}^{-1}
\end{aligned}
$$

We need to add the two electric fields because both are in the same direction. The field is away from Q_1 and towards Q_2. Therefore, $E_{\text{total}} = 6{,}74 \times 10^2 + 3{,}99 \times 10^2 = 1{,}08 \times 10^2$ N·C^{-1}

Worked example 7: Two-dimensional electric fields

QUESTION

Two point charges form a right-angled triangle with the point A at the origin. Their charges are $Q_2 = 6 \times 10^{-9}$ C $=$ 6 nC and $Q_3 = -3 \times 10^{-9}$ C $= -3$ nC. The distance between A and Q_2 is 5×10^{-2} m and the distance between A and Q_3 is 3×10^{-2} m. What is the net electric field measured at A from the two charges if they are arranged as shown?



SOLUTION

Step 1: Determine what is required

We are required to calculate the net electric field at A. This field is the sum of the two electric fields - the field from Q_2 at A and from Q_3 at A.

Step 2: Determine how to approach the problem

- We need to calculate the two fields at A, using $E = k\frac{Q}{r^2}$ for the magnitude and determining the direction from the charge signs.
- We then need to add up the two fields using our rules for adding vector quantities, because the electric field is a vector quantity.

Step 3: Determine what is given

We are given all the charges and the distances.

Step 4: Calculate the magnitude of the fields.

The magnitude of the field from Q_2 at A, which we will call E_2, is:

$$
\begin{aligned}
E_2 &= k\frac{Q_2}{r^2} \\
&= (9{,}0 \times 10^9)\frac{(6 \times 10^{-9})}{(5 \times 10^{-2})^2} \\
&= (9{,}0 \times 10^9)\frac{(6 \times 10^{-9})}{(25 \times 10^{-4})} \\
&= 2{,}158 \times 10^4 \text{ N·C}^{-1}
\end{aligned}
$$

The magnitude of the electric field from Q_3 at A, which we will call E_3, is:

$$
\begin{aligned}
E_3 &= k\frac{Q_3}{r^2} \\
&= (9{,}0 \times 10^9)\frac{(3 \times 10^{-9})}{(3 \times 10^{-2})^2} \\
&= (9{,}0 \times 10^9)\frac{(3 \times 10^{-9})}{(9 \times 10^{-4})} \\
&= 2{,}997 \times 10^4 \text{ N·C}^{-1}
\end{aligned}
$$

Step 5: Vector addition of electric fields

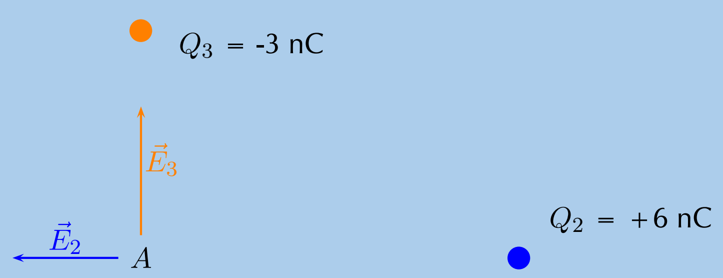
We will use precisely the **same procedure** as before. Determine the vectors on the Cartesian plane, break them into components in the x- and y-directions, sum components in each direction to get the components of the resultant.

We choose the positive directions to be to the right (the positive x-direction) and up (the positive y-direction). We know the electric field magnitudes but we need to use the charges to determine the direction. Then we can use the diagram to determine the directions.

The force between a positive test charge and Q_2 is repulsive (like charges). This means that the electric field is to the left, or in the negative x-direction.

The force between a positive test charge and Q_3 is attractive (unlike charges) and the electric field will be in the positive y-direction.

We can redraw the diagram illustrating the fields to make sure we can visualise the situation:

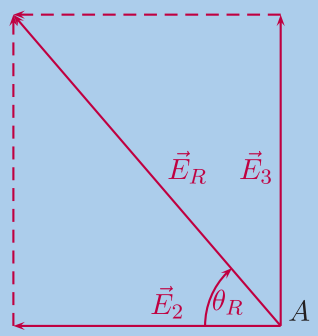


Step 6: Resultant force

The magnitude of the resultant force acting on Q_1 can be calculated from the forces using Pythagoras' theorem because there are only two forces and they act in the x- and y-directions:

$$E_R^2 = E_2^2 + E_3^2 \text{ Pythagoras' theorem}$$
$$E_R = \sqrt{(2{,}158 \times 10^4)^2 + (2{,}997 \times 10^4)^2}$$
$$E_R = 3{,}693 \times 10^4 \text{ N·C}^{-1}$$

and the angle, θ_R made with the x-axis can be found using trigonometry.

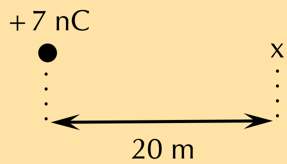


$$\tan(\theta_R) = \frac{\text{y-component}}{\text{x-component}}$$
$$\tan(\theta_R) = \frac{2{,}997 \times 10^4}{2{,}158 \times 10^4}$$
$$\theta_R = \tan^{-1}(\frac{2{,}997 \times 10^4}{2{,}158 \times 10^4})$$
$$\theta_R = 54{,}24°$$

The final resultant electric field acting at A is $3{,}693 \times 10^4$ N·C^{-1} acting at 54,24° to the negative x-axis or 125,76°to the positive x-axis.

Exercise 9 – 2: Electric fields

1. Calculate the electric field strength 20 m from a 7 nC charge.



2. Two charges of $Q_1 = -6$ pC and $Q_2 = -8$ pC are separated by a distance of 3 km. What is the electric field strength at a point that is 2 km from Q_1 and 1 km from Q_2? The point lies between Q_1 and Q_2.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23Z8 2. 23Z9

www.everythingscience.co.za m.everythingscience.co.za

9.4 Chapter summary ESBPQ

See presentation: 23ZB at www.everythingscience.co.za

- Objects can be **positively**, **negatively** charged or **neutral**.
- Coulomb's law describes the electrostatic force between two point charges and can be stated as: the magnitude of the electrostatic force between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.
  $F = \frac{kQ_1Q_2}{r^2}$
- An electric field is a region of space in which an electric charge will experience a force. The direction of the field at a point in space is the direction in which a positive test charge would moved if placed at that point.
- We can represent the electric field using field lines. By convention electric field lines point away from positive charges (like charges repel) and towards negative charges (unlike charges attract).
- The magnitude of the electric field, E, at a point can be quantified as the force per unit charge We can write this as: $E = \frac{F}{q}$ where F is the Coulomb force exerted by a charge on a test charge q. The units of the electric field are newtons per coulomb: N·C^{-1} .
- The electric field due to a point charge Q is defined as the force per unit charge:
  $E = \frac{F}{q} = \frac{kQ}{r^2}$
- The electrostatic force is attractive for unlike charges and repulsive for like charges.

Physical Quantities		
Quantity	**Unit name**	**Unit symbol**
Charge (q)	coulomb	C
Distance (d)	metre	m
Electric field (E)	newtons per coulomb	N·C^{-1}
Force (F)	newton	N

Exercise 9 – 3:

1. Two charges of $+3$ nC and -5 nC are separated by a distance of 40 cm. What is the electrostatic force between the two charges?
2. Two conducting metal spheres carrying charges of $+6$ nC and -10 nC are separated by a distance of 20 mm.
   a) What is the electrostatic force between the spheres?
   b) The two spheres are touched and then separated by a distance of 60 mm. What are the new charges on the spheres?
   c) What is new electrostatic force between the spheres at this distance?

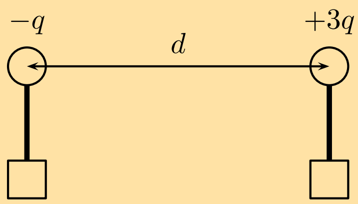
3. The electrostatic force between two charged spheres of +3 nC and +4 nC respectively is 0,4 N. What is the distance between the spheres?

4. Draw the electric field pattern lines between:

    a) two equal positive point charges.

    b) two equal negative point charges.

5. Two small identical metal spheres, on insulated stands, carry charges $-q$ and $+3q$ respectively. When the centres of the spheres are separated by a distance d the one exerts an electrostatic force of magnitude F on the other.

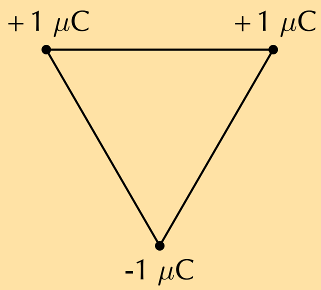


The spheres are now made to touch each other and are then brought back to the same distance d apart. What will be the magnitude of the electrostatic force which one sphere now exerts on the other?

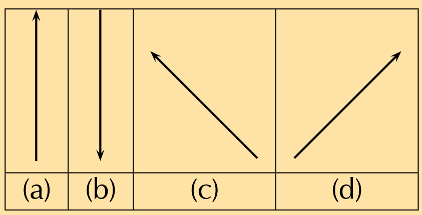
    a) $\frac{1}{4}F$
    b) $\frac{1}{3}F$
    c) $\frac{1}{2}F$
    d) $3F$

[SC 2003/11]

6. Three point charges of magnitude +1 C, +1 C and −1 C respectively are placed on the three corners of an equilateral triangle as shown.



Which vector best represents the direction of the resultant force acting on the −1 C charge as a result of the forces exerted by the other two charges?



[SC 2003/11]

7. a) Write a statement of Coulomb's law.
   b) Calculate the magnitude of the force exerted by a point charge of $+2$ nC on another point charge of -3 nC separated by a distance of 60 mm.
   c) Sketch the electric field between two point charges of $+2$ nC and -3 nC, respectively, placed 60 mm apart from each other.

   [IEB 2003/11 HG1 - Force Fields]

8. The electric field strength at a distance x from a point charge is E. What is the magnitude of the electric field strength at a distance $2x$ away from the point charge?

   a) $\frac{1}{4}E$
   b) $\frac{1}{2}E$
   c) $2E$
   d) $4E$

   [SC 2002/03 HG1]

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23ZC 2. 23ZD 3. 23ZF 4a. 23ZG 4b. 23ZH 5. 23ZJ
6. 23ZK 7. 23ZM 8. 23ZN

www.everythingscience.co.za m.everythingscience.co.za

CHAPTER 10

Electromagnetism

10.1 Introduction ESBPR

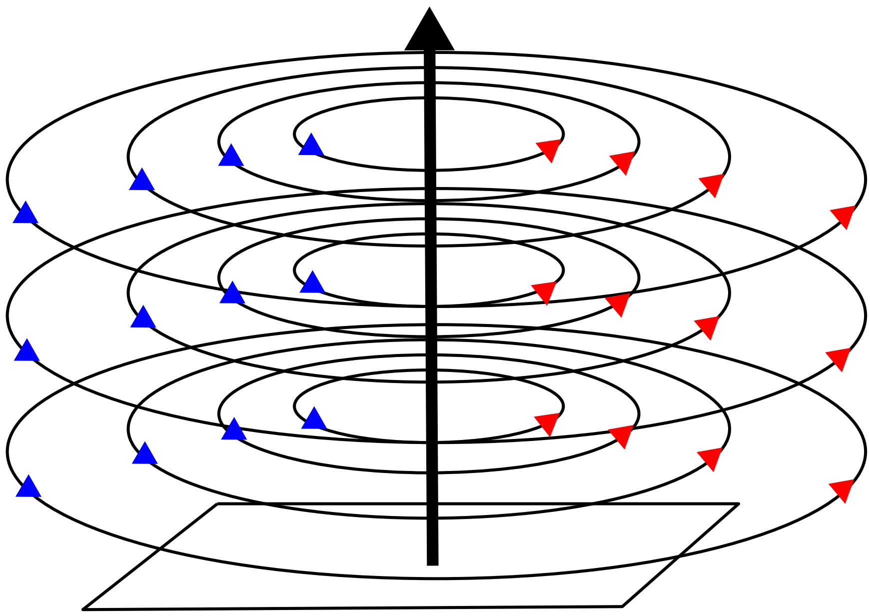
Electromagnetism describes the interaction between charges, currents and the electric and magnetic fields to which they give rise. An electric current creates a magnetic field and a changing magnetic field will create a flow of charge. This relationship between electricity and magnetism has been studied extensively. This has resulted in the invention of many devices which are useful to humans, for example cellular telephones, microwave ovens, radios, televisions and many more.

10.2 Magnetic field associated with a current ESBPS

If you hold a compass near a wire through which current is flowing, the needle on the compass will be deflected.

Since compasses work by pointing along magnetic field lines, this means that there must be a magnetic field near the wire through which the current is flowing.

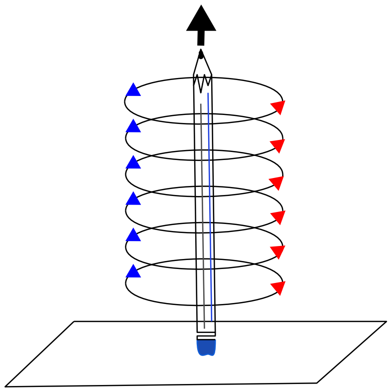
The magnetic field produced by an electric current is always oriented perpendicular to the direction of the current flow. Below is a sketch of what the magnetic field around a wire looks like when the wire has a current flowing in it. We use $\vec{B}$ to denote a magnetic field and arrows on field lines to show the direction of the magnetic field. **Note** that if there is no current there will be no magnetic field.



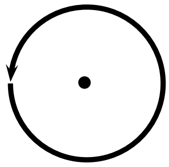
The direction of the current in the conductor (wire) is shown by the central arrow. The circles are field lines and they also have a direction indicated by the arrows on the lines. Similar to the situation with electric field lines, the greater the number of lines (or the closer they are together) in an area the stronger the magnetic field.

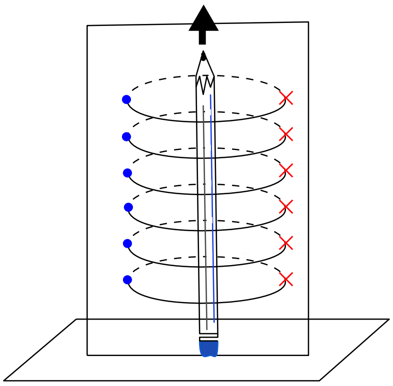
Important: all of our discussion regarding field directions assumes that we are dealing with **conventional current**.

To help you visualise this situation, stand a pen or pencil straight up on a desk. The circles are centred around the pencil or pen and would be drawn parallel to the surface of the desk. The tip of the pen or pencil would point in the direction of the current flow.



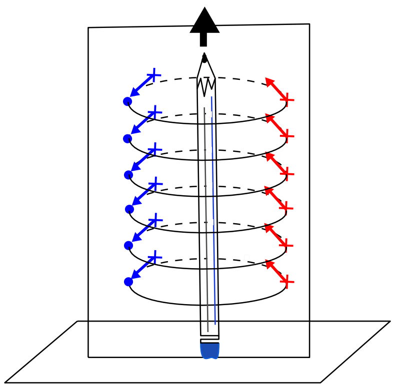
You can look at the pencil or pen from above and the pencil or pen will be a dot in the centre of the circles. The direction of the magnetic field lines is counter-clockwise for this situation.
To make it easier to see what is happening we are only going to draw one set of circular fields lines but note that this is just for the illustration.





If you put a piece of paper behind the pencil and look at it from the side, then you would be seeing the circular field lines side on and it is hard to know that they are circular. They go through the paper. Remember that field lines have a direction, so when you are looking at the piece of paper from the side it means that the circles go into the paper on one side of the pencil and come out of the paper on the other side.

When we are drawing directions of magnetic fields and currents, we use the symbols $\odot$ and $\otimes$. The symbol $\odot$ represents an arrow that is coming out of the page and the symbol $\otimes$ represents an arrow that is going into the page.
It is easy to remember the meanings of the symbols if you think of an arrow with a sharp tip at the head and a tail with feathers in the shape of a cross.



We will now look at three examples of current carrying wires. For each example we will determine the magnetic field and draw the magnetic field lines around the conductor.

FACT
The Danish physicist, Hans Christian Oersted, was lecturing one day in 1820 on the possibility of electricity and magnetism being related to one another, and in the process demonstrated it conclusively with an experiment in front of his whole class. By passing an electric current through a metal wire suspended above a magnetic compass, Oersted was able to produce a definite motion of the compass needle in response to the current. What began as a guess at the start of the class session was confirmed as fact at the end. Needless to say, Oersted had to revise his lecture notes for future classes. His discovery paved the way for a whole new branch of science - electromagnetism.

The direction of the magnetic field around the current carrying conductor is shown in Figure 10.1.

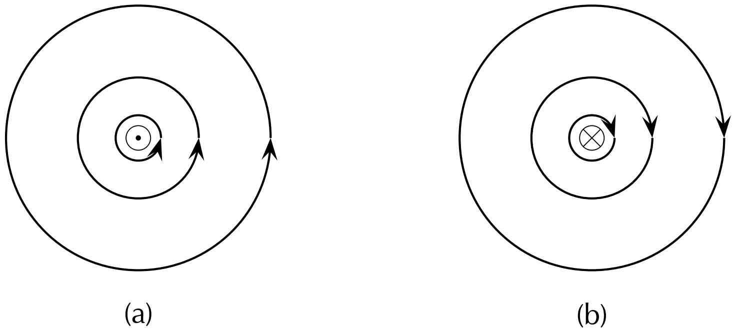


Figure 10.1:

Magnetic field around a conductor when you look at the conductor from one end. (a) Current flows out of the page and the magnetic field is counter-clockwise. (b) Current flows into the page and the magnetic field is clockwise.

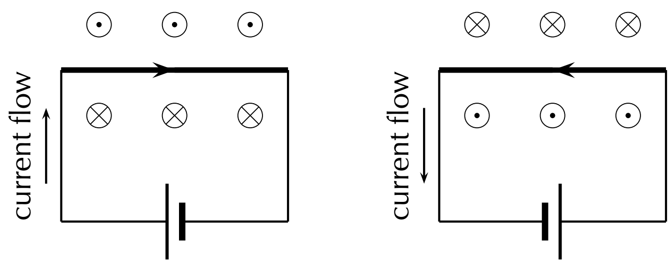


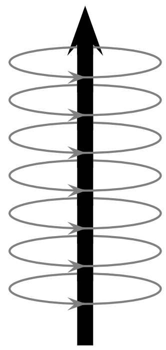
Figure 10.2:

Magnetic fields around a conductor looking down on the conductor. (a) Current flows clockwise. (b) current flows counter-clockwise.

Activity: Direction of a magnetic field

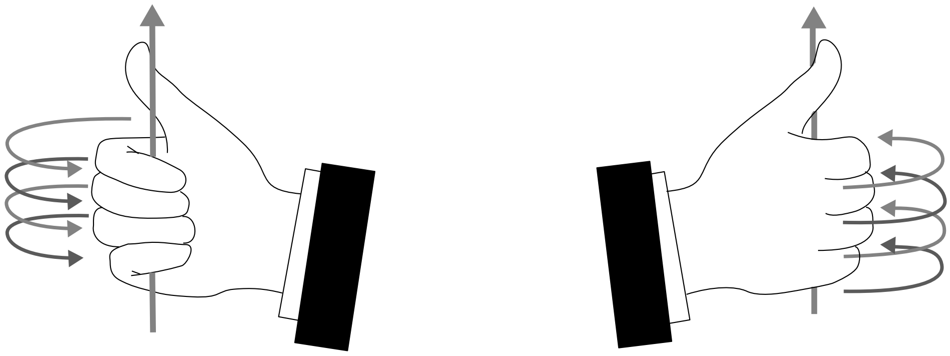
Using the directions given in Figure 10.1 and Figure 10.2 try to find a rule that easily tells you the direction of the magnetic field.

Hint: Use your fingers. Hold the wire in your hands and try to find a link between the direction of your thumb and the direction in which your fingers curl.



The magnetic field around a current carrying conductor.

There is a simple method of finding the relationship between the direction of the current flowing in a conductor and the direction of the magnetic field around the same conductor. The method is called the *Right Hand Rule*. Simply stated, the Right Hand Rule says that the magnetic field lines produced by a current-carrying wire will be oriented in the same direction as the curled fingers of a person's right hand (in the "hitchhiking" position), with the thumb pointing in the direction of the current flow.

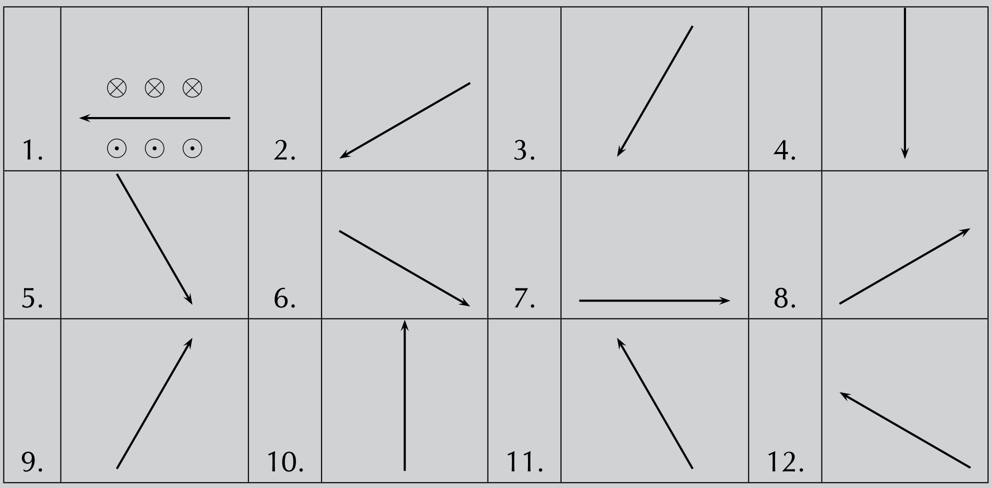


IMPORTANT!

Your right hand and left hand are unique in the sense that you cannot rotate one of them to be in the same position as the other. This means that the right hand part of the rule is essential. You will always get the wrong answer if you use the wrong hand.

Activity: The Right Hand Rule

Use the Right Hand Rule to draw in the directions of the magnetic fields for the following conductors with the currents flowing in the directions shown by the arrows. The first problem has been completed for you.



Activity: Magnetic field around a current carrying conductor

Apparatus:

1. one 9 V battery with holder
2. two hookup wires with alligator clips
3. compass
4. stop watch

Method:

1. Connect your wires to the battery leaving one end of each wire unconnected so that the circuit is not closed.
2. Be sure to limit the current flow to 10 seconds at a time (Why you might ask, the wire has very little resistance on its own so the battery will go flat very quickly). This is to preserve battery life as well as to prevent overheating of the wires and battery contacts.
3. Place the compass close to the wire.
4. Close the circuit and observe what happens to the compass.
5. Reverse the polarity of the battery and close the circuit. Observe what happens to the compass.

Conclusions:

Use your observations to answer the following questions:

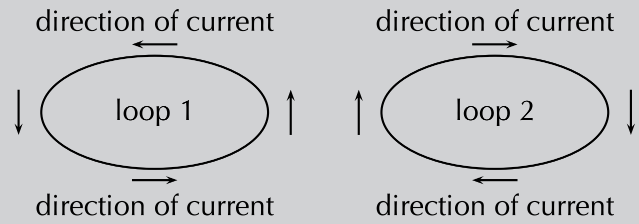
1. Does a current flowing in a wire generate a magnetic field?
2. Is the magnetic field present when the current is not flowing?
3. Does the direction of the magnetic field produced by a current in a wire depend on the direction of the current flow?
4. How does the direction of the current affect the magnetic field?

Magnetic field around a current carrying loop ESBPV

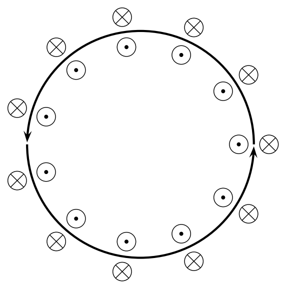
So far we have only looked at straight wires carrying a current and the magnetic fields around them. We are going to study the magnetic field set up by circular loops of wire carrying a current because the field has very useful properties, for example you will see that we can set up a uniform magnetic field.

Activity: Magnetic field around a loop of conductor

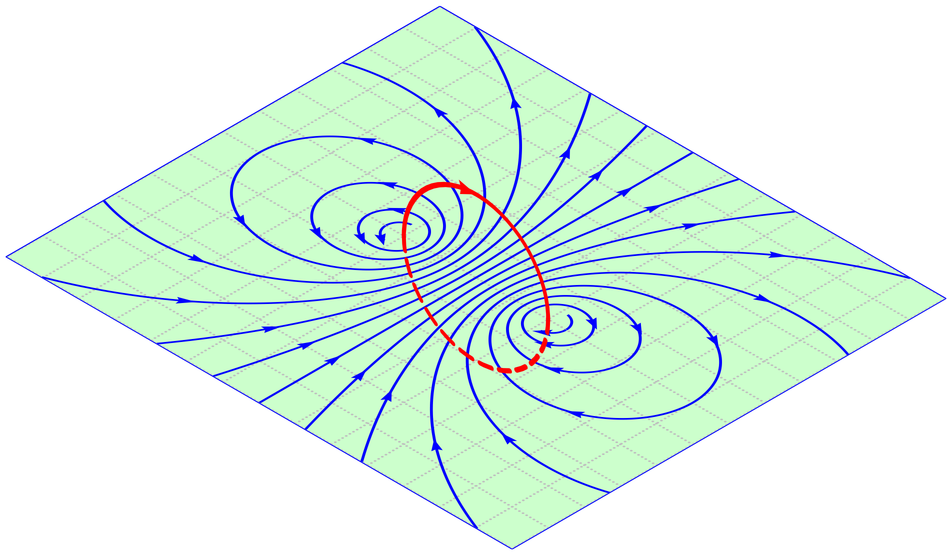
Imagine two loops made from wire which carry currents (in opposite directions) and are parallel to the page of your book. By using the Right Hand Rule, draw what you think the magnetic field would look like at different points around each of the two loops. Loop 1 has the current flowing in a counter-clockwise direction, while loop 2 has the current flowing in a clockwise direction.



If you make a loop of current carrying conductor, then the direction of the magnetic field is obtained by applying the Right Hand Rule to different points in the loop.



The directions of the magnetic field around a loop of current carrying conductor with the current flowing in a counter-clockwise direction is shown.



Notice that there is a variation on the Right Hand Rule. If you make the fingers of your right hand follow the direction of the current in the loop, your thumb will point in the direction where the field lines emerge. This is similar to the north pole (where the field lines emerge from a bar magnet) and shows you which side of the loop would attract a bar magnet's north pole.

Magnetic field around a solenoid ESBPW

If we now add another loop with the current in the same direction, then the magnetic field around each loop can be added together to create a stronger magnetic field. A coil of many such loops is called a *solenoid*. A solenoid is a cylindrical coil of wire acting as a magnet when an electric current flows through the wire. The magnetic field pattern around a solenoid is similar to the magnetic field pattern around the bar magnet that you studied in Grade 10, which had a definite north and south pole as shown in Figure 10.3.

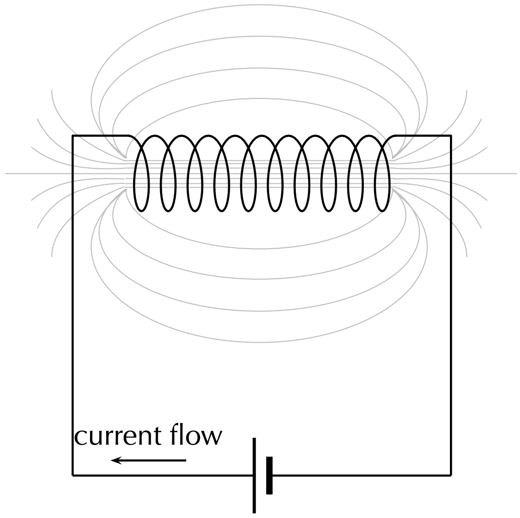
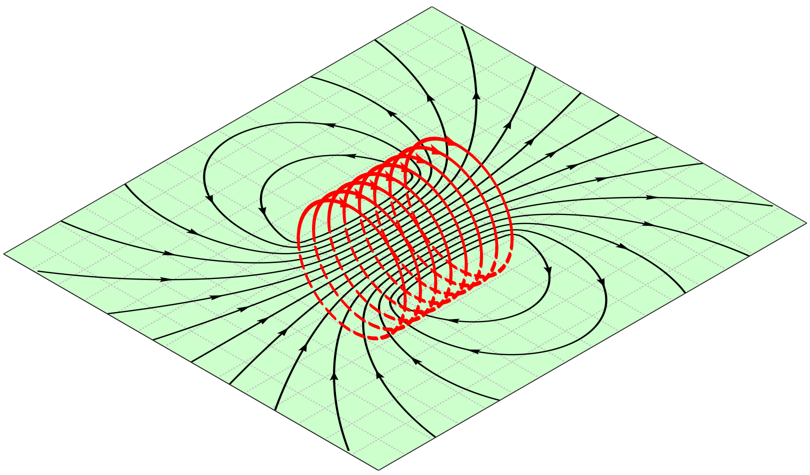


Figure 10.3:
Magnetic field around a solenoid.



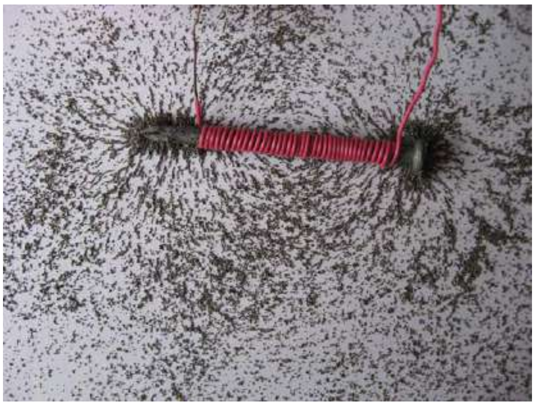
Real-world applications ESBPX

Electromagnets

An *electromagnet* is a piece of wire intended to generate a magnetic field with the passage of electric current through it. Though all current-carrying conductors produce magnetic fields, an electromagnet is usually constructed in such a way as to maximise the strength of the magnetic field it produces for a special purpose. Electromagnets are commonly used in research, industry, medical, and consumer products. An example

of a commonly used electromagnet is in security doors, e.g. on shop doors which open automatically.

As an electrically-controllable magnet, electromagnets form part of a wide variety of "electromechanical" devices: machines that produce a mechanical force or motion through electrical power. Perhaps the most obvious example of such a machine is the *electric motor* which will be described in detail in Grade 12. Other examples of the use of electromagnets are electric bells, relays, loudspeakers and scrapyard cranes.



See video: 23ZP at www.everythingscience.co.za

General experiment: Electromagnets

Aim:

A magnetic field is created when an electric current flows through a wire. A single wire does not produce a strong magnetic field, but a wire coiled around an iron core does. We will investigate this behaviour.

Apparatus:

1. a battery and holder
2. a length of wire
3. a compass
4. a few nails

Method:

1. If you have not done the previous experiment in this chapter do it now.
2. Bend the wire into a series of coils before attaching it to the battery. Observe what happens to the deflection of the needle on the compass. Has the deflection of the compass grown stronger?
3. Repeat the experiment by changing the number and size of the coils in the wire. Observe what happens to the deflection on the compass.

4. Coil the wire around an iron nail and then attach the coil to the battery. Observe what happens to the deflection of the compass needle.

Conclusions:

1. Does the number of coils affect the strength of the magnetic field?
2. Does the iron nail increase or decrease the strength of the magnetic field?

Case study: Overhead power lines and the environment

Physical impact:

Power lines are a common sight all across our country. These lines bring power from the power stations to our homes and offices. But these power lines can have negative impacts on the environment. One hazard that they pose is to birds which fly into them. Conservationist Jessica Shaw has spent the last few years looking at this threat. In fact, power lines pose the primary threat to the blue crane, South Africa's national bird, in the Karoo.

"We are lucky in South Africa to have a wide range of bird species, including many large birds like cranes, storks and bustards. Unfortunately, there are also a lot of power lines, which can impact on birds in two ways. They can be electrocuted when they perch on some types of pylons, and can also be killed by colliding with the line if they fly into it, either from the impact with the line or from hitting the ground afterwards.

These collisions often happen to large birds, which are too heavy to avoid a power line if they only see it at the last minute. Other reasons that birds might collide include bad weather, flying in flocks and the lack of experience of younger birds.

Over the past few years we have been researching the serious impact that power line collisions have on Blue Cranes and Ludwig's Bustards. These are two of our endemic species, which means they are only found in southern Africa. They are both big birds that have long lifespans and breed slowly, so the populations might not recover from high mortality rates. We have walked and driven under power lines across the Overberg and the Karoo to count dead birds. The data show that thousands of these birds

are killed by collisions every year, and Ludwig's Bustard is now listed as an Endangered species because of this high level of unnatural mortality. We are also looking for ways to reduce this problem, and have been working with Eskom to test different line marking devices. When markers are hung on power lines, birds might be able to see the power line from further away, which will give them enough time to avoid a collision."

Impact of fields:

The fact that a field is created around the power lines means that they can potentially have an impact at a distance. This has been studied and continues to be a topic of significant debate. At the time of writing, the World Health Organisation guidelines for human exposure to electric and magnetic fields indicate that there is no clear link between exposure to the magnetic and electric fields that the general public encounters from power lines, because these are extremely low frequency fields.

Power line noise can interfere with radio communications and broadcasting. Essentially, the power lines or associated hardware improperly generate unwanted radio signals that override or compete with desired radio signals. Power line noise can impact the quality of radio and television reception. Disruption of radio communications, such as amateur radio, can also occur. Loss of critical communications, such as police, fire, military and other similar users of the radio spectrum, can result in even more serious consequences.

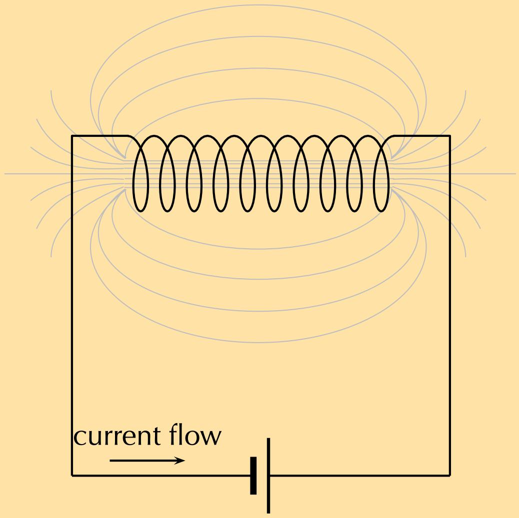
Group discussion:

- Discuss the above information.
- Discuss other ways that power lines affect the environment.

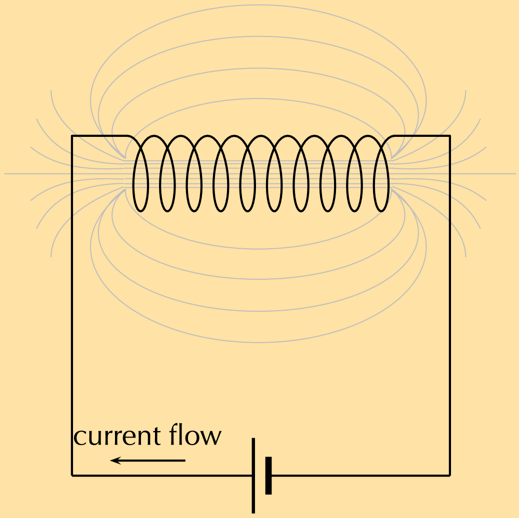
FACT
When lightning strikes a ship or an aeroplane, it can damage or otherwise change its magnetic compass. There have been recorded instances of a lightning strike changing the polarity of the compass so the needle points south instead of north.

Exercise 10 – 1: Magnetic Fields

1. Give evidence for the existence of a magnetic field near a current carrying wire.
2. Describe how you would use your right hand to determine the direction of a magnetic field around a current carrying conductor.
3. Use the Right Hand Rule to determine the direction of the magnetic field for the following situations:

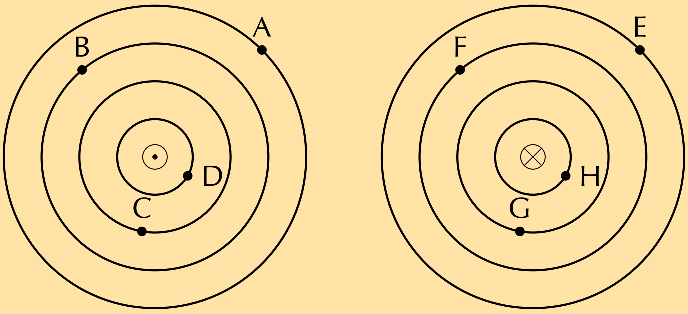


a)



b)

4. Use the Right Hand Rule to find the direction of the magnetic fields at each of the points labelled A - H in the following diagrams.



Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23ZQ 2. 23ZR 3a. 23ZS 3b. 23ZT 4. 23ZV

www.everythingscience.co.za **m.everythingscience.co.za**

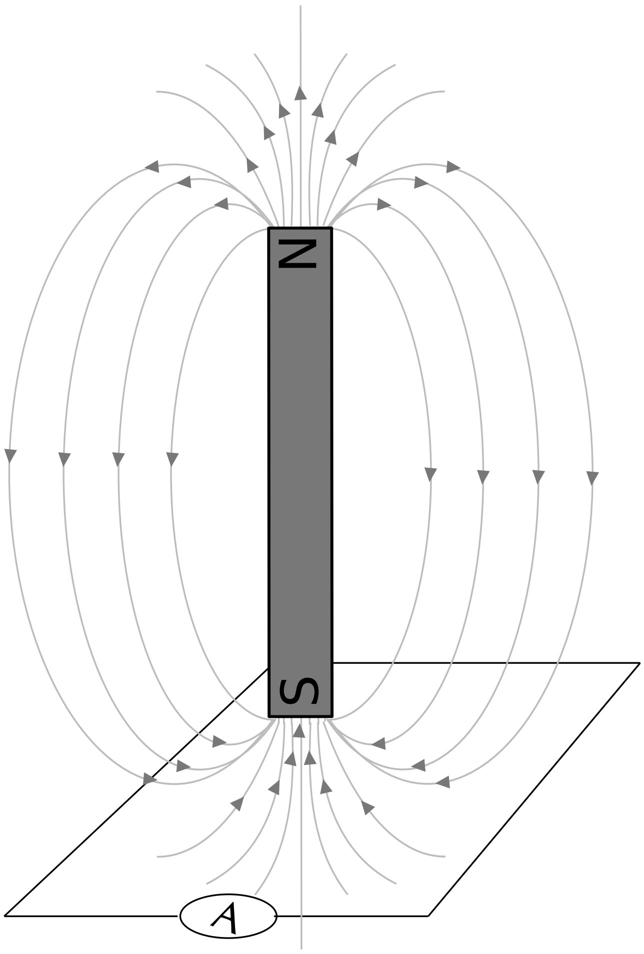
10.3 Faraday's law of electromagnetic induction ESBPY

Current induced by a changing magnetic field ESBPZ

While Oersted's surprising discovery of electromagnetism paved the way for more practical applications of electricity, it was Michael Faraday who gave us the key to the practical generation of electricity: **electromagnetic induction**.

Faraday discovered that when he moved a magnet near a wire a voltage was generated across it. If the magnet was held stationary no voltage was generated, the voltage only existed while the magnet was moving. We call this voltage the induced emf ($\mathcal{E}$).

A circuit loop connected to a sensitive ammeter will register a current if it is set up as in this figure and the magnet is moved up and down:



Magnetic flux

Before we move onto the definition of Faraday's law of electromagnetic induction and examples, we first need to spend some time looking at the magnetic flux. For a loop of area A in the presence of a uniform magnetic field, $\vec{B}$, the magnetic flux (ϕ) is defined as:

$$\phi = BA\cos\theta$$

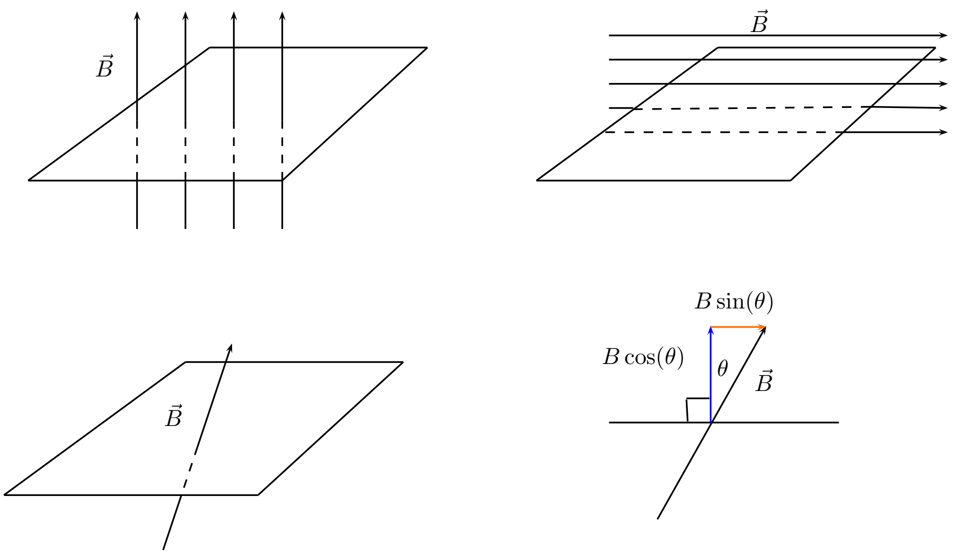
Where:

$\theta =$ the angle between the magnetic field, B, and the normal to the loop of area A
$A =$ the area of the loop
$B =$ the magnetic field

The S.I. unit of magnetic flux is the weber (Wb).

You might ask yourself why the angle θ is included. The flux depends on the magnetic field that passes through surface. We know that a field parallel to the surface can't induce a current because it doesn't pass through the surface. If the magnetic field is not perpendicular to the surface then there is a component which is perpendicular and a component which is parallel to the surface. The parallel component can't contribute to the flux, only the vertical component can.

In this diagram we show that a magnetic field at an angle other than perpendicular can be broken into components. The component perpendicular to the surface has the magnitude $B\cos(\theta)$ where θ is the angle between the normal and the magnetic field.



DEFINITION: *Faraday's Law of electromagnetic induction*

The emf, $\mathcal{E}$, produced around a loop of conductor is proportional to the rate of change of the magnetic flux, ϕ, through the area, A, of the loop. This can be stated mathematically as:

$$\mathcal{E} = -N\frac{\Delta\phi}{\Delta t}$$

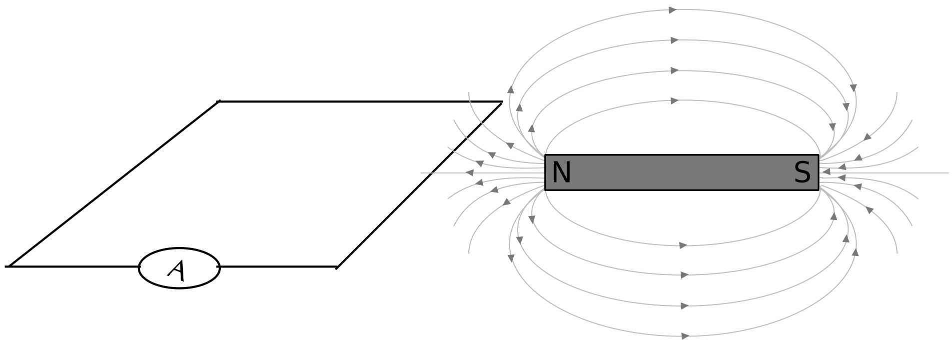
where $\phi = B \cdot A$ and B is the strength of the magnetic field. N is the number of circuit loops. A magnetic field is measured in units of teslas (T). The minus sign indicates direction and that the induced emf tends to oppose the change in the magnetic flux. The minus sign can be ignored when calculating magnitudes.

Faraday's Law relates induced emf to the rate of change of flux, which is the product of the magnetic field and the cross-sectional area through which the field lines pass.

IMPORTANT!

It is not the area of the wire itself but the area that the wire encloses. This means that if you bend the wire into a circle, the area we would use in a flux calculation is the surface area of the circle, not the wire.

In this illustration, where the magnet is in the same plane as the circuit loop, there would be no current even if the magnet were moved closer and further away. This is because the magnetic field lines do not pass through the enclosed area but are parallel to it. The magnetic field lines must pass through the area enclosed by the circuit loop for an emf to be induced.

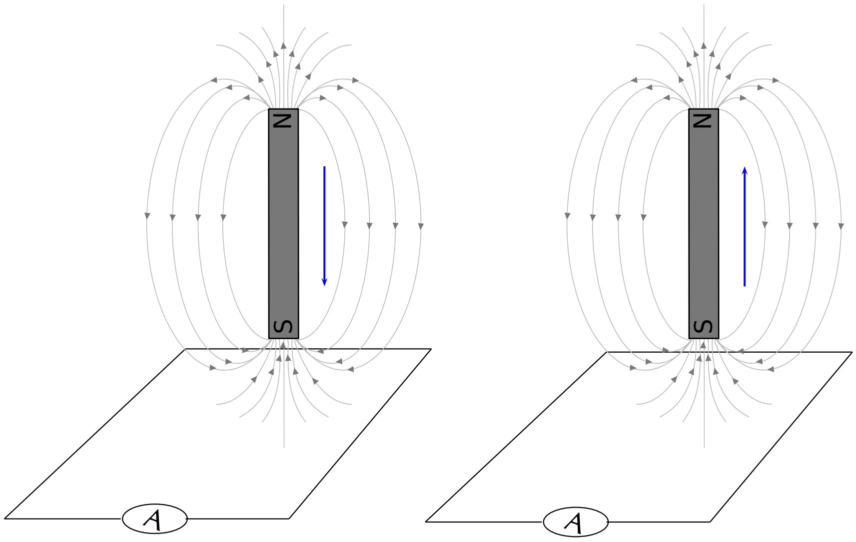


Direction of induced current ESBQ2

The most important thing to remember is that the induced current opposes whatever change is taking place.

In the first picture (left) the circuit loop has the south pole of a magnet moving closer. The magnitude of the field from the magnet is getting larger. The response from the induced emf will be to try to resist the field towards the pole getting stronger. The field is a vector so the current will flow in a direction so that the fields due to the current tend to cancel those from the magnet, keeping the resultant field the same.

To resist the change from an approaching south pole from above, the current must result in field lines that move away from the approaching pole. The induced magnetic field must therefore have field lines that go down on the inside of the loop. The current direction indicated by the arrows on the circuit loop will achieve this. Test this by using the Right Hand Rule. Put your right thumb in the direction of one of the arrows and notice what the field curls downwards into the area enclosed by the loop.



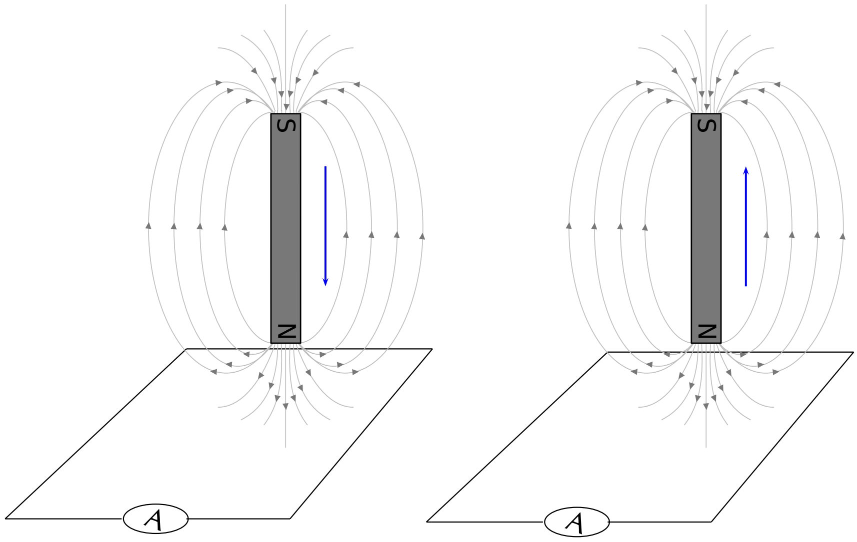
In the second diagram the south pole is moving away. This means that the field from the magnet will be getting weaker. The response from the induced current will be to set up a magnetic field that adds to the existing one from the magnetic to resist it decreasing in strength.

Another way to think of the same feature is just using poles. To resist an approaching south pole the current that is induced creates a field that looks like another south pole on the side of the approaching south pole. Like poles repel, you can think of the current setting up a south pole to repel the approaching south pole. In the second panel, the current sets up a north pole to attract the south pole to stop it moving away.

We can also use the variation of the Right Hand Rule, putting your fingers in the direction of the current to get your thumb to point in the direction of the field lines (or the north pole).

We can test all of these on the cases of a north pole moving closer or further away from the circuit. For the first case of the north pole approaching, the current will resist the change by setting up a field in the opposite direction to the field from the magnet that is getting stronger. Use the Right Hand Rule to confirm that the arrows create a field with field lines that curl upwards in the enclosed area cancelling out those curling downwards from the north pole of the magnet.

Like poles repel, alternatively test that putting the fingers of your right hand in the direction of the current leaves your thumb pointing upwards indicating a north pole.



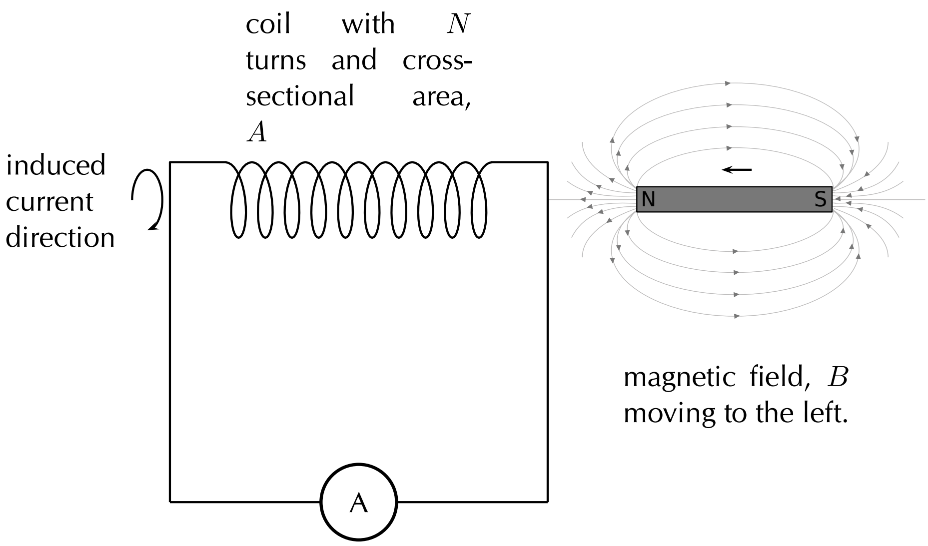
For the second figure where the north pole is moving away the situation is reversed.

Direction of induced current in a solenoid ESBQ3

The approach for looking at the direction of current in a solenoid is the same the approach described above. The only difference being that in a solenoid there are a number of loops of wire so the magnitude of the induced emf will be different. The flux would be calculated using the surface area of the solenoid multiplied by the number of loops.

Remember: the directions of currents and associated magnetic fields can all be found using only the Right Hand Rule. When the fingers of the right hand are pointed in the direction of the magnetic field, the thumb points in the direction of the current. When the thumb is pointed in the direction of the magnetic field, the fingers point in the direction of the current.

The direction of the current will be such as to oppose the change. We would use a setup as in this sketch to do the test:

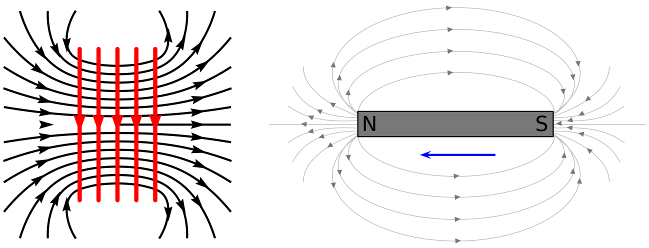


In the case where a north pole is brought towards the solenoid the current will flow so

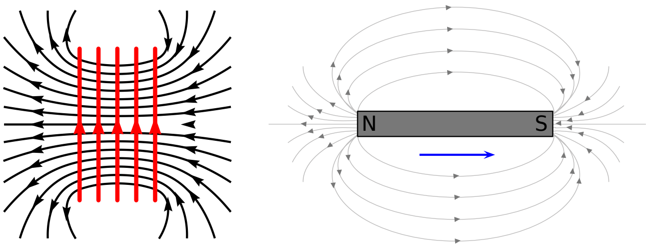
TIP
An easy way to create a magnetic field of changing intensity is to move a permanent magnet next to a wire or coil of wire. The magnetic field must increase or decrease in intensity *perpendicular* to the wire (so that the magnetic field lines "cut across" the conductor), or else no voltage will be induced.

TIP
The induced current generates a magnetic field. The induced magnetic field is in a direction that tends to cancel out the change in the magnetic field in the loop of wire. So, you can use the Right Hand Rule to find the direction of the induced current by remembering that the induced magnetic field is opposite in direction to the change in the magnetic field.

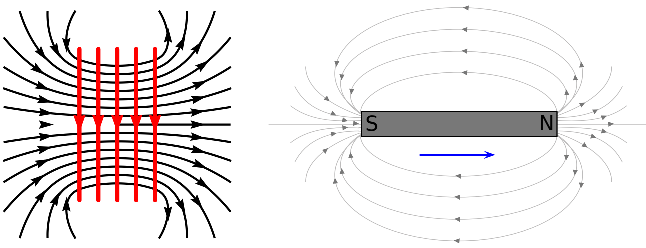
that a north pole is established at the end of the solenoid closest to the approaching magnet to repel it (verify using the Right Hand Rule):



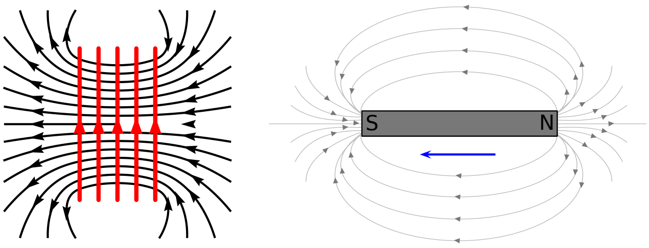
In the case where a north pole is moving away from the solenoid the current will flow so that a south pole is established at the end of the solenoid closest to the receding magnet to attract it:



In the case where a south pole is moving away from the solenoid the current will flow so that a north pole is established at the end of the solenoid closest to the receding magnet to attract it:



In the case where a south pole is brought towards the solenoid the current will flow so that a south pole is established at the end of the solenoid closest to the approaching magnet to repel it:



Induction

Electromagnetic induction is put into practical use in the construction of electrical generators which use mechanical power to move a magnetic field past coils of wire to generate voltage. However, this is by no means the only practical use for this principle.

If we recall, the magnetic field produced by a current-carrying wire is always perpendicular to the wire, and that the flux intensity of this magnetic field varies with the amount of current which passes through it. We can therefore see that a wire is capable of inducing a voltage *along its own length* if the current is changing. This effect is called *self-induction*. Self-induction is when a changing magnetic field is produced by changes in current through a wire, inducing a voltage along the length of that same wire.

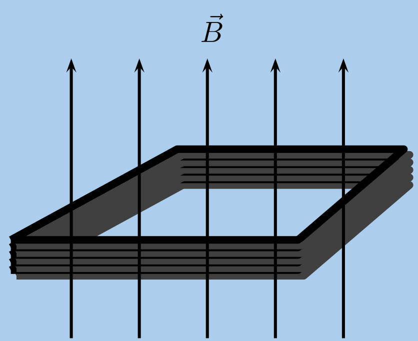
If the magnetic flux is enhanced by bending the wire into the shape of a coil, and/or wrapping that coil around a material of high permeability, this effect of self-induced voltage will be more intense. A device constructed to take advantage of this effect is called an *inductor*.

Remember that the induced current will create a magnetic field that opposes the change in the magnetic flux. This is known as Lenz's law.

Worked example 1: Faraday's law

QUESTION

Consider a flat square coil with 5 turns. The coil is 0,50 m on each side and has a magnetic field of 0,5 T passing through it. The plane of the coil is perpendicular to the magnetic field: the field points out of the page. Use Faraday's Law to calculate the induced emf, if the magnetic field is increases uniformly from 0,5 T to 1 T in 10 s. Determine the direction of the induced current.



SOLUTION

Step 1: Identify what is required

We are required to use Faraday's Law to calculate the induced emf.

Step 2: Write Faraday's Law

$$\mathcal{E} = -N\frac{\Delta\phi}{\Delta t}$$

We know that the magnetic field is at right angles to the surface and so aligned with the normal. This means we do not need to worry about the angle that the field makes with the normal and $\phi = BA$. The starting or initial magnetic field, B_i, is given as is the final field magnitude, B_f. We want to determine the magnitude of the emf so we can ignore the minus sign.

The area, A, is the area of square coil.

Step 3: Solve Problem

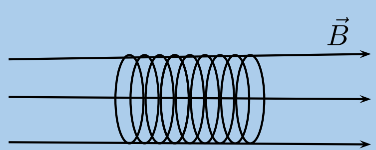
$$\begin{aligned}
\mathcal{E} &= N\frac{\Delta\phi}{\Delta t} \\
&= N\frac{\phi_f - \phi_i}{\Delta t} \\
&= N\frac{B_f A - B_i A}{\Delta t} \\
&= N\frac{A(B_f - B_i)}{\Delta t} \\
&= (5)\frac{(0{,}50)^2(1 - 0{,}50)}{10} \\
&= (5)\frac{(0{,}50)^2(1 - 0{,}50)}{10} \\
&= 0{,}0625 \text{ V}
\end{aligned}$$

The induced current is anti-clockwise as viewed from the direction of the increasing magnetic field.

Worked example 2: Faraday's law

QUESTION

Consider a solenoid of 9 turns with unknown radius, r. The solenoid is subjected to a magnetic field of 0,12 T. The axis of the solenoid is parallel to the magnetic field. When the field is uniformly switched to 12 T over a period of 2 minutes an emf with a magnitude of $-0{,}3$ V is induced. Determine the radius of the solenoid.



SOLUTION

Step 1: Identify what is required

We are required to determine the radius of the solenoid. We know that the relationship between the induced emf and the field is governed by Faraday's law which includes the geometry of the solenoid. We can use this relationship to find the radius.

Step 2: Write Faraday's Law

$$\mathcal{E} = -N\frac{\Delta\phi}{\Delta t}$$

We know that the magnetic field is at right angles to the surface and so aligned with the normal. This means we do not need to worry about the angle the field makes with the normal and $\phi = BA$. The starting or initial magnetic field, B_i, is given as is the final field magnitude, B_f. We can drop the minus sign because we are working with the magnitude of the emf only.

The area, A, is the surface area of the solenoid which is πr^2.

Step 3: Solve Problem

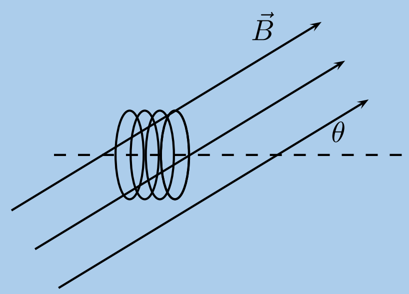
$$\begin{aligned}
\mathcal{E} &= N\frac{\Delta\phi}{\Delta t} \\
&= N\frac{\phi_f - \phi_i}{\Delta t} \\
&= N\frac{B_f A - B_i A}{\Delta t} \\
&= N\frac{A(B_f - B_i)}{\Delta t} \\
(0{,}30) &= (9)\frac{(\pi r^2)(12 - 0{,}12)}{120} \\
r^2 &= \frac{(0{,}30)(120)}{(9)\pi(12 - 0{,}12)} \\
r^2 &= 0{,}107175 \\
r &= 0{,}32 \text{ m}
\end{aligned}$$

The solenoid has a radius of 0,32 m.

Worked example 3: Faraday's law

QUESTION

Consider a circular coil of 4 turns with radius 3×10^{-2} m. The solenoid is subjected to a varying magnetic field that changes uniformly from 0,4 T to 3,4 T in an interval of 27 s. The axis of the solenoid makes an angle of 35° to the magnetic field. Find the induced emf.



SOLUTION

Step 1: Identify what is required

We are required to use Faraday's Law to calculate the induced emf.

Step 2: Write Faraday's Law

$$\mathcal{E} = -N\frac{\Delta\phi}{\Delta t}$$

We know that the magnetic field is at an angle to the surface normal. This means we must account for the angle that the field makes with the normal and $\phi = BA\cos(\theta)$. The starting or initial magnetic field, B_i, is given as is the final field magnitude, B_f. We want to determine the magnitude of the emf so we can ignore the minus sign.

The area, A, will be πr^2.

Step 3: Solve Problem

$$\begin{aligned}
\mathcal{E} &= N\frac{\Delta\phi}{\Delta t} \\
&= N\frac{\phi_f - \phi_i}{\Delta t} \\
&= N\frac{B_f A\cos(\theta) - B_i A\cos(\theta)}{\Delta t} \\
&= N\frac{A\cos(\theta)(B_f - B_i)}{\Delta t} \\
&= (4)\frac{(\pi(0{,}03)^2\cos(35))(3{,}4 - 0{,}4)}{27} \\
&= 1{,}03 \times 10^{-3}\ \text{V}
\end{aligned}$$

The induced current is anti-clockwise as viewed from the direction of the increasing magnetic field.

See simulation: 23ZW at www.everythingscience.co.za

Real-life applications

The following devices use Faraday's Law in their operation.

- induction stoves
- tape players
- metal detectors
- transformers

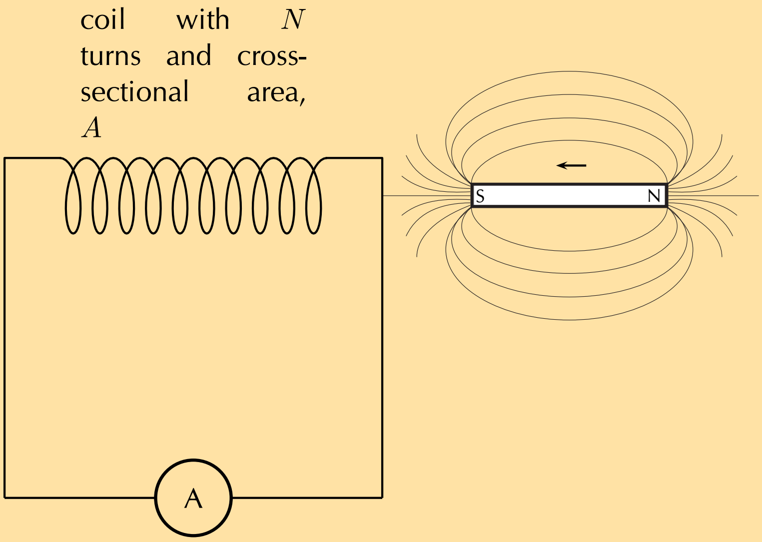
Project: Real-life applications of Faraday's Law

Choose one of the following devices and do some research on the internet, or in a library, how your device works. You will need to refer to Faraday's Law in your explanation.

- induction stoves
- tape players
- metal detectors
- transformers

Exercise 10 – 2: Faraday's Law

1. State Faraday's Law of electromagnetic induction in words and write down a mathematical relationship.
2. Describe what happens when a bar magnet is pushed into or pulled out of a solenoid connected to an ammeter. Draw pictures to support your description.
3. Explain how it is possible for the magnetic flux to be zero when the magnetic field is not zero.
4. Use the Right Hand Rule to determine the direction of the induced current in the solenoid below.



5. Consider a circular coil of 5 turns with radius 1,73 m. The coil is subjected to a varying magnetic field that changes uniformly from 2,18 T to 12,7 T in an interval of 3 minutes. The axis of the solenoid makes an angle of 27° to the magnetic field. Find the induced emf.

6. Consider a solenoid coil of 11 turns with radius $13,8 \times 10^{-2}$ m. The solenoid is subjected to a varying magnetic field that changes uniformly from 5,34 T to 2,7 T in an interval of 12 s. The axis of the solenoid makes an angle of 13° to the magnetic field.

    a) Find the induced emf.

    b) If the angle is changed to 67,4°, what would the radius need to be for the emf to remain the same?

7. Consider a solenoid with 5 turns and a radius of 11×10^{-2} m. The axis of the solenoid makes an angle of 23° to the magnetic field.

    a) Find the change in flux if the emf is 12 V over a period of 12 s.

    b) If the angle is changed to 45°, what would the time interval need to change to for the induced emf to remain the same?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 23ZX 2. 23ZY 3. 23ZZ 4. 2422 5. 2423 6. 2424
7. 2425

www.everythingscience.co.za **m.everythingscience.co.za**

10.4 Chapter summary ESBQ4

See presentation: 2426 at www.everythingscience.co.za

- Electromagnetism is the study of the properties and relationship between electric currents and magnetism.
- A current-carrying conductor will produce a magnetic field around the conductor.
- The direction of the magnetic field is found by using the Right Hand Rule.
- Electromagnets are temporary magnets formed by current-carrying conductors.
- The magnetic flux through a surface is the product of the component of the magnetic field normal to the surface and the surface area, $\phi = BA\cos(\theta)$.
- Electromagnetic induction occurs when a changing magnetic field induces a voltage in a current-carrying conductor.
- The magnitude of the induced emf is given by Faraday's law of electromagnetic induction: $\mathcal{E} = -N\frac{\Delta\phi}{\Delta t}$

Physical Quantities		
Quantity	**Unit name**	**Unit symbol**
Induced emf ($\mathcal{E}$)	volt	V
Magnetic field (B)	tesla	T
Magnetic flux (ϕ)	weber	Wb
Time (t)	seconds	s

Exercise 10 – 3:

1. What did Hans Oersted discover about the relationship between electricity and magnetism?
2. List two uses of electromagnetism.
3. a) A uniform magnetic field of 0,35 T in the vertical direction exists. A piece of cardboard, of surface area 0,35 m^2 is placed flat on a horizontal surface inside the field. What is the magnetic flux through the cardboard?
   b) The one edge is then lifted so that the cardboard is now inclined at 17° to the positive x-direction. What is the magnetic flux through the cardboard? What will the induced emf be if the field drops to zero in the space of 3 s? Why?
4. A uniform magnetic field of 5 T in the vertical direction exists. What is the magnetic flux through a horizontal surface of area 0,68 m^2? What is the flux if the magnetic field changes to being in the positive x-direction?
5. A uniform magnetic field of 5 T in the vertical direction exists. What is the magnetic flux through a horizontal circle of radius 0,68 m?

6. Consider a square coil of 3 turns with a side length of 1,56 m. The coil is subjected to a varying magnetic field that changes uniformly from 4,38 T to 0,35 T in an interval of 3 minutes. The axis of the solenoid makes an angle of 197° to the magnetic field. Find the induced emf.

7. Consider a solenoid coil of 13 turns with radius $6,8 \times 10^{-2}$ m. The solenoid is subjected to a varying magnetic field that changes uniformly from -5 T to 1,8 T in an interval of 18 s. The axis of the solenoid makes an angle of 88° to the magnetic field.

   a) Find the induced emf.

   b) If the angle is changed to 39°, what would the radius need to be for the emf to remain the same?

8. Consider a solenoid with 5 turns and a radius of $4,3 \times 10^{-1}$ mm. The axis of the solenoid makes an angle of 11° to the magnetic field.

   Find the change in flux if the emf is 0,12 V over a period of 0,5 s.

9. Consider a rectangular coil of area 1,73 m^2. The coil is subjected to a varying magnetic field that changes uniformly from 2 T to 10 T in an interval of 3 ms. The axis of the solenoid makes an angle of 55° to the magnetic field. Find the induced emf.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 2427 2. 2428 3. 242B 4. 242C 5. 242D 6. 242F
7. 242G 8. 242H 9. 242J

www.everythingscience.co.za m.everythingscience.co.za

CHAPTER 11

Electric circuits

11.1 Introduction ESBQ5

The study of electrical circuits is essential to understand the technology that uses electricity in the real-world. We depend on electricity and electrical appliances to make many things possible in our daily lives. This becomes very clear when there is a power failure and we can't use the kettle to boil water for tea or coffee, can't use the stove or oven to cook dinner, can't charge our cellphone batteries, watch TV, or use electric lights.

Key Mathematics Concepts

- Units and unit conversions — Physical Sciences, Grade 10, Science skills
- Equations — Mathematics, Grade 10, Equations and inequalities
- Graphs — Mathematics, Grade 10, Functions and graphs

11.2 Ohm's Law ESBQ6

Three quantities which are fundamental to electric circuits are **current, voltage (potential difference)** and **resistance**. To recap:

1. Electrical current, I, is defined as the rate of flow of charge through a circuit.
2. Potential difference or voltage, V, is the amount of energy per unit charge needed to move that charge between two points in a circuit.
3. Resistance, R, is a measure of how 'hard' it is to push current through a circuit element.

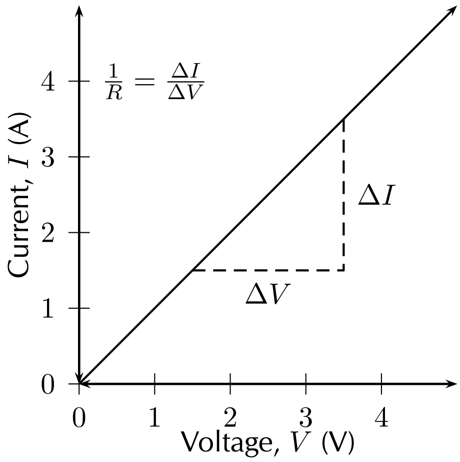
We will now look at how these three quantities are related to each other in electric circuits.

An important relationship between the current, voltage and resistance in a circuit was discovered by Georg Simon Ohm and it is called **Ohm's Law**.

DEFINITION: *Ohm's Law*

The amount of electric current through a metal conductor, at a constant temperature, in a circuit is proportional to the voltage across the conductor and can be described by
$I = \frac{V}{R}$
where I is the current through the conductor, V is the voltage across the conductor and R is the resistance of the conductor. In other words, at constant temperature, the resistance of the conductor is constant, independent of the voltage applied across it or current passed through it.

Ohm's Law tells us that if a conductor is at a constant temperature, the current flowing through the conductor is directly proportional to the voltage across it. This means that if we plot voltage on the x-axis of a graph and current on the y-axis of the graph, we will get a straight-line.



The gradient of the straight-line graph is related to the resistance of the conductor as

$$\frac{I}{V} = \frac{1}{R}.$$

This can be rearranged in terms of the constant resistance as:

$$R = \frac{V}{I}.$$

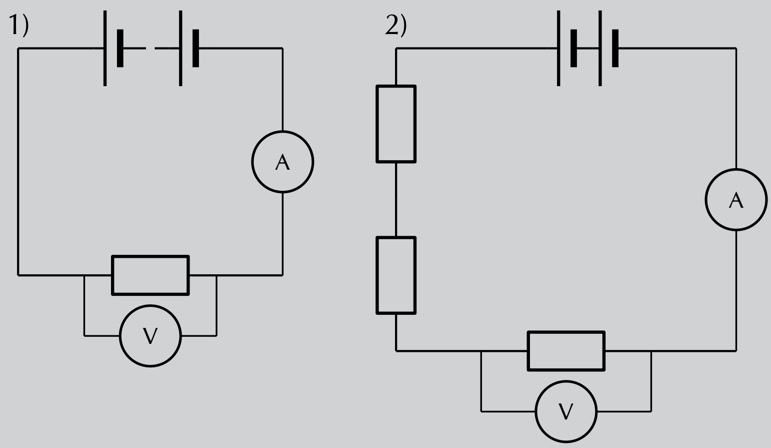
General experiment: Ohm's Law

Aim:

To determine the relationship between the current going through a resistor and the potential difference (voltage) across the same resistor.

Apparatus:

4 cells, 4 resistors, an ammeter, a voltmeter, connecting wires



Method:

This experiment has two parts. In the first part we will vary the applied voltage across the resistor and measure the resulting current through the circuit. In the second part we will vary the current in the circuit and measure the resulting voltage across the resistor. After obtaining both sets of measurements, we will examine the relationship between the current and the voltage across the resistor.

1. Varying the voltage:
   a) Set up the circuit according to circuit diagram 1), starting with just one cell.
   b) Draw the following table in your lab book.

Number of cells	Voltage, V (V)	Current, I (A)
1		
2		
3		
4		

   c) Get your teacher to check the circuit before turning the power on.
   d) Measure the voltage across the resistor using the voltmeter, and the current in the circuit using the ammeter.
   e) Add one more 1,5 V cell to the circuit and repeat your measurements.
   f) Repeat until you have four cells and you have completed your table.

2. Varying the current:
   a) Set up the circuit according to circuit diagram 2), starting with only 1 resistor in the circuit.
   b) Draw the following table in your lab book.

Voltage, V (V)	Current, I (A)

   c) Get your teacher to check your circuit before turning the power on.
   d) Measure the current and measure the voltage across the single resistor.
   e) Now add another resistor in series in the circuit and measure the current and the voltage across only the original resistor again. Continue adding resistors until you have four in series, but remember to only measure the voltage across the original resistor each time. Enter the values you measure into the table.

Analysis and results:

1. Using the data you recorded in the first table, draw a graph of current versus voltage. Since the voltage is the variable which we are directly varying, it is the independent variable and will be plotted on the x-axis. The current is the dependent variable and must be plotted on the y-axis.

2. Using the data you recorded in the second table, draw a graph of voltage vs. current. In this case the independent variable is the current which must be plotted on the x-axis, and the voltage is the dependent variable and must be plotted on the y-axis.

Conclusions:

1. Examine the graph you made from the first table. What happens to the current through the resistor when the voltage across it is increased? i.e. Does it increase or decrease?
2. Examine the graph you made from the second table. What happens to the voltage across the resistor when the current increases through the resistor? i.e. Does it increase or decrease?
3. Do your experimental results verify Ohm's Law? Explain.

Questions and discussion:

1. For each of your graphs, calculate the gradient and from this determine the resistance of the original resistor. Do you get the same value when you calculate it for each of your graphs?
2. How would you go about finding the resistance of an unknown resistor using only a power supply, a voltmeter and a known resistor R_0?

See simulation: 242K at www.everythingscience.co.za

Exercise 11 – 1: Ohm's Law

1. Use the data in the table below to answer the following questions.

Voltage, V (V)	Current, I (A)
3,0	0,4
6,0	0,8
9,0	1,2
12,0	1,6

a) Plot a graph of voltage (on the x-axis) and current (on the y-axis).

b) What type of graph do you obtain (straight-line, parabola, other curve)

c) Calculate the gradient of the graph.

d) Do your experimental results verify Ohm's Law? Explain.

e) How would you go about finding the resistance of an unknown resistor using only a power supply, a voltmeter and a known resistor R_0?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 242M

www.everythingscience.co.za m.everythingscience.co.za

Ohmic and non-ohmic conductors ESBQ7

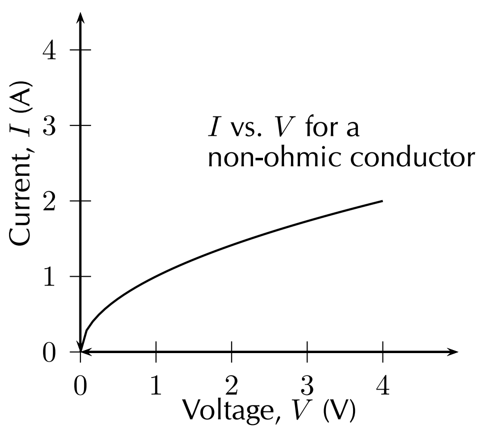
Conductors which obey Ohm's Law have a constant resistance when the voltage is varied across them or the current through them is increased. These conductors are called *ohmic* conductors. A graph of the current vs. the voltage across these conductors will be a straight-line. Some examples of ohmic conductors are circuit resistors and nichrome wire.

As you have seen, there is a mention of *constant temperature* when we talk about Ohm's Law. This is because the resistance of some conductors changes as their temperature changes. These types of conductors are called *non-ohmic* conductors, because they do not obey Ohm's Law. A light bulb is a common example of a non-ohmic conductor. Other examples of non-ohmic conductors are diodes and transistors.

In a light bulb, the resistance of the filament wire will increase dramatically as it warms from room temperature to operating temperature. If we increase the supply voltage in a real lamp circuit, the resulting increase in current causes the filament to increase in temperature, which increases its resistance. This effectively limits the increase in current. In this case, voltage and current do not obey Ohm's Law.

The phenomenon of resistance changing with variations in temperature is one shared by almost all metals, of which most wires are made. For most applications, these changes in resistance are small enough to be ignored. In the application of metal lamp filaments, which increase a lot in temperature (up to about 1000 °C, and starting from room temperature) the change is quite large.

In general, for non-ohmic conductors, a graph of voltage against current will not be a straight-line, indicating that the resistance is not constant over all values of voltage and current.



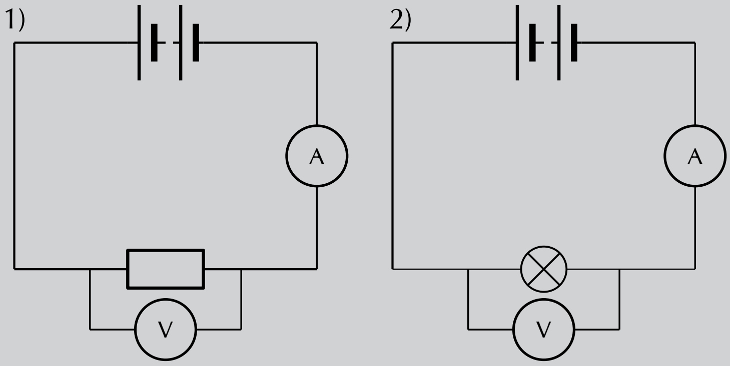
Informal experiment:Ohmic and non-ohmic conductors

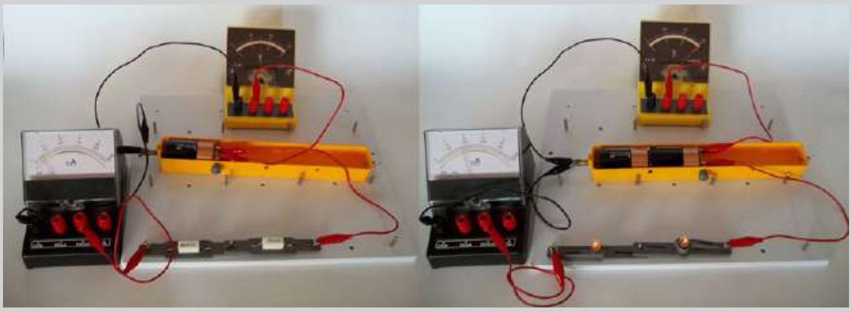
Aim:

To determine whether two circuit elements (a resistor and a lightbulb) obey Ohm's Law

Apparatus:

4 cells, a resistor, a lightbulb, connecting wires, a voltmeter, an ammeter





Method:

The two circuits shown in the diagrams above are the same, except in the first there is a resistor and in the second there is a lightbulb. Set up both the circuits above, starting with 1 cell. For each circuit:

1. Measure the voltage across the circuit element (either the resistor or lightbulb) using the voltmeter.
2. Measure the current in the circuit using the ammeter.
3. Add another cell and repeat your measurements until you have 4 cells in your circuit.

Results:

Draw two tables which look like the following in your book. You should have one table for the first circuit measurements with the resistor and another table for the second circuit measurements with the lightbulb.

Number of cells	Voltage, V (V)	Current, I (A)
1		
2		
3		
4		

Analysis:

Using the data in your tables, draw two graphs of I (y-axis) vs. V (x-axis), one for the resistor and one for the lightbulb.

Questions and Discussion:

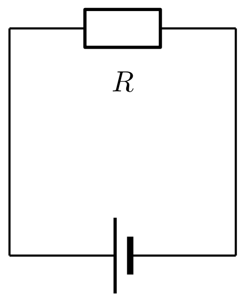
Examine your graphs closely and answer the following questions:

1. What should the graph of I vs. V look like for a conductor which obeys Ohm's Law?
2. Do either or both your graphs look like this?
3. What can you conclude about whether or not the resistor and/or the lightbulb obey Ohm's Law?
4. Is the lightbulb an ohmic or non-ohmic conductor?

Using Ohm's Law ESBQ8

We are now ready to see how Ohm's Law is used to analyse circuits.

Consider a circuit with a cell and an ohmic resistor, R. If the resistor has a resistance of 5 Ω and voltage across the resistor is 5 V, then we can use Ohm's Law to calculate the current flowing through the resistor. Our first task is to draw the circuit diagram. When solving any problem with electric circuits it is very important to make a diagram of the circuit before doing any calculations. The circuit diagram for this problem is shown alongside.



The equation for Ohm's Law is:

$$R = \frac{V}{I}$$

which can be rearranged to:

$$I = \frac{V}{R}$$

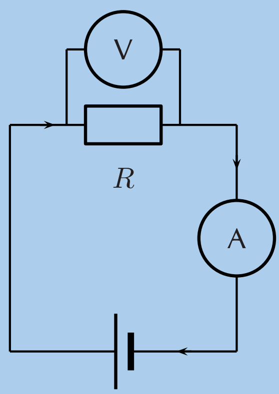
The current flowing through the resistor is:

$$
\begin{aligned}
I &= \frac{V}{R} \\
&= \frac{5\text{ V}}{5\Omega} \\
&= 1\text{ A}
\end{aligned}
$$

Worked example 1: Ohm's Law

QUESTION

Study the circuit diagram below:



The resistance of the resistor is 10 Ω and the current going through the resistor is 4 A. What is the potential difference (voltage) across the resistor?

SOLUTION

Step 1: Determine how to approach the problem

We are given the resistance of the resistor and the current passing through it and are asked to calculate the voltage across it. We can apply Ohm's Law to this problem using:

$$R = \frac{V}{I}.$$

Step 2: Solve the problem

Rearrange the equation above and substitute the known values for R and I to solve for V.

$$
\begin{aligned}
R &= \frac{V}{I} \\
R \times I &= \frac{V}{I} \times I \\
V &= I \times R \\
&= 10 \times 4 \\
&= 40\text{ V}
\end{aligned}
$$

Step 3: Write the final answer

The voltage across the resistor is 40 V.

Exercise 11 – 2: Ohm's Law

1. Calculate the resistance of a resistor that has a potential difference of 8 V across it when a current of 2 A flows through it. Draw the circuit diagram before doing the calculation.
2. What current will flow through a resistor of 6 Ω when there is a potential difference of 18 V across its ends? Draw the circuit diagram before doing the calculation.
3. What is the voltage across a 10 Ω resistor when a current of 1,5 A flows though it? Draw the circuit diagram before doing the calculation.

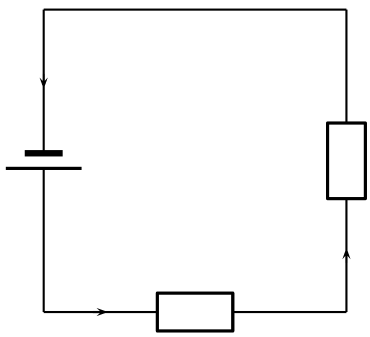
Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 242N 2. 242P 3. 242Q

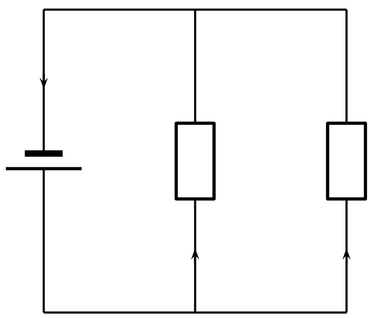
www.everythingscience.co.za m.everythingscience.co.za

Recap of resistors in series and parallel ESBQ9

In Grade 10, you learnt about resistors and were introduced to circuits where resistors were connected in series and in parallel. In a series circuit there is one path along which current flows. In a parallel circuit there are multiple paths along which current flows.



series circuit
one current path



parallel circuit
multiple current paths

When there is more than one resistor in a circuit, we are usually able to calculate the total combined resistance of all the resistors. This is known as the *equivalent resistance*.

Equivalent series resistance

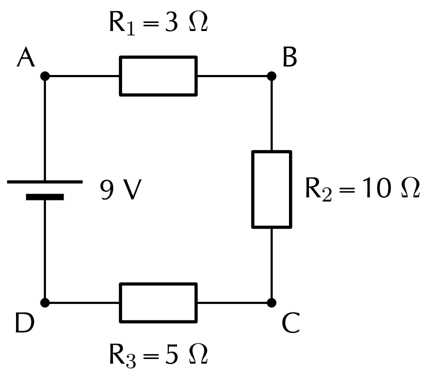
In a circuit where the resistors are connected in series, the equivalent resistance is just the sum of the resistances of all the resistors.

DEFINITION: *Equivalent resistance in a series circuit,*

For n resistors in series the equivalent resistance is:

$$R_s = R_1 + R_2 + R_3 + \ldots + R_n$$

Let us apply this to the following circuit.



The resistors are in series, therefore:

$$\begin{aligned} R_s &= R_1 + R_2 + R_3 \\ &= 3\ \Omega + 10\ \Omega + 5\ \Omega \\ &= 18\ \Omega \end{aligned}$$

See simulation: 242R at www.everythingscience.co.za

See video: 242S at www.everythingscience.co.za

Equivalent parallel resistance

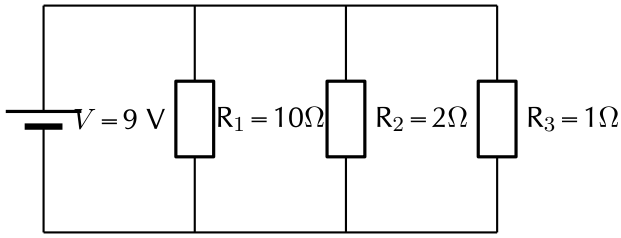
In a circuit where the resistors are connected in parallel, the equivalent resistance is given by the following definition.

DEFINITION: *Equivalent resistance in a parallel circuit*

For n resistors in parallel, the equivalent resistance is:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$$

Let us apply this formula to the following circuit.



What is the total (equivalent) resistance in the circuit?

$$\begin{aligned}
\frac{1}{R_p} &= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \\
&= \left(\frac{1}{10\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{1\ \Omega}\right) \\
&= \left(\frac{1\ \Omega + 5\ \Omega + 10\ \Omega}{10\ \Omega}\right) \\
&= \left(\frac{16\ \Omega}{10\ \Omega}\right) \\
R_p &= 0{,}625\ \Omega
\end{aligned}$$

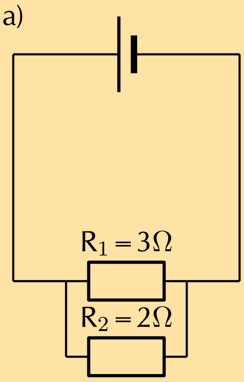
See video: 242T at www.everythingscience.co.za

See video: 242V at www.everythingscience.co.za

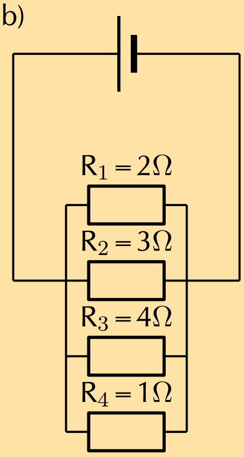
Exercise 11 – 3: Series and parallel resistance

1. Two 10 kΩ resistors are connected in series. Calculate the equivalent resistance.
2. Two resistors are connected in series. The equivalent resistance is 100 Ω. If one resistor is 10 Ω, calculate the value of the second resistor.
3. Two 10 kΩ resistors are connected in parallel. Calculate the equivalent resistance.
4. Two resistors are connected in parallel. The equivalent resistance is 3,75 Ω. If one resistor has a resistance of 10 Ω, what is the resistance of the second resistor?
5. Calculate the equivalent resistance in each of the following circuits:

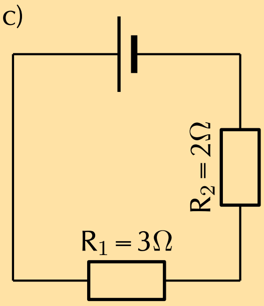
a)



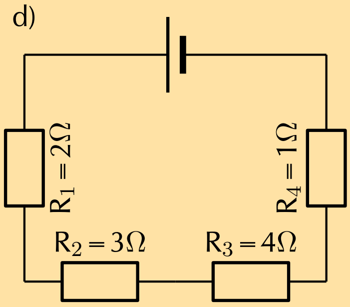
b)



c)



d)



Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 242W 2. 242X 3. 242Y 4. 242Z 5. 2432

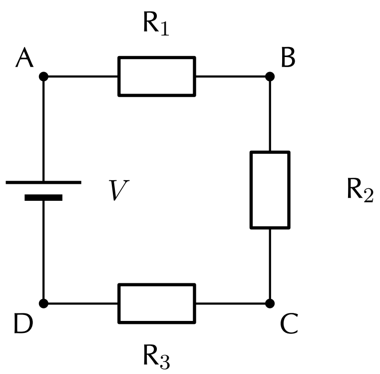
www.everythingscience.co.za m.everythingscience.co.za

Use of Ohm's Law in series and parallel circuits ESBQB

Using the definitions for equivalent resistance for resistors in series or in parallel, we can analyse some circuits with these setups.

Series circuits

Consider a circuit consisting of three resistors and a single cell connected in series.



The first principle to understand about series circuits is that the amount of current is the same through any component in the circuit. This is because there is only one path for electrons to flow in a series circuit. From the way that the battery is connected, we can tell in which direction the current will flow. We know that current flows from positive to negative by convention. Conventional current in this circuit will flow in a clockwise direction, from point A to B to C to D and back to A.

We know that in a series circuit the current has to be the same in all components. So we can write:

$$I = I_1 = I_2 = I_3.$$

We also know that total voltage of the circuit has to be equal to the sum of the voltages over all three resistors. So we can write:

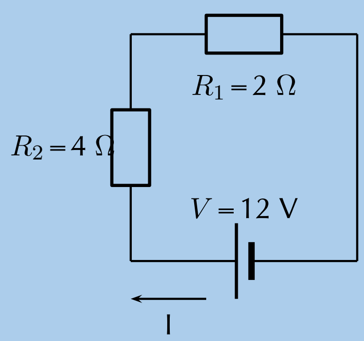
$$V = V_1 + V_2 + V_3$$

Using this information and what we know about calculating the equivalent resistance of resistors in series, we can approach some circuit problems.

Worked example 2: Ohm's Law, series circuit

QUESTION

Calculate the current (I) in this circuit if the resistors are both ohmic in nature.



SOLUTION

Step 1: Determine what is required

We are required to calculate the current flowing in the circuit.

Step 2: Determine how to approach the problem

Since the resistors are ohmic in nature, we can use Ohm's Law. There are however two resistors in the circuit and we need to find the total resistance.

Step 3: Find total resistance in circuit

Since the resistors are connected in series, the total (equivalent) resistance R is:

$$R = R_1 + R_2$$

Therefore,

$$\begin{aligned} R &= 2 + 4 \\ &= 6\ \Omega \end{aligned}$$

Step 4: Apply Ohm's Law

$$\begin{aligned} R &= \frac{V}{I} \\ R \times \frac{I}{R} &= \frac{V}{I} \times \frac{I}{R} \\ I &= \frac{V}{R} \\ &= \frac{12}{6} \\ &= 2 \text{ A} \end{aligned}$$

Step 5: Write the final answer

A current of 2 A is flowing in the circuit.

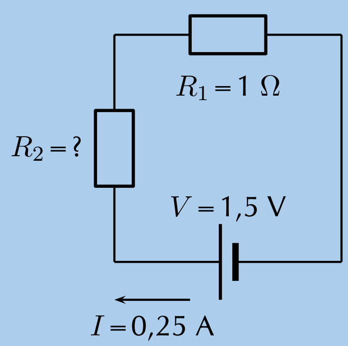
Worked example 3: Ohm's Law, series circuit

QUESTION

Two ohmic resistors (R_1 and R_2) are connected in series with a cell. Find the resistance of R_2, given that the current flowing through R_1 and R_2 is 0,25 A and that the voltage across the cell is 1,5 V. $R_1 = 1\ \Omega$.

SOLUTION

Step 1: Draw the circuit and fill in all known values.



Step 2: Determine how to approach the problem.

We can use Ohm's Law to find the total resistance R in the circuit, and then calculate the unknown resistance using:

$$R = R_1 + R_2$$

because it is in a series circuit.

Step 3: Find the total resistance

$$
\begin{aligned}
R &= \frac{V}{I} \\
&= \frac{1{,}5}{0{,}25} \\
&= 6\ \Omega
\end{aligned}
$$

Step 4: Find the unknown resistance

We know that:

$$R = 6\ \Omega$$

and that

$$R_1 = 1\ \Omega$$

Since

$$R = R_1 + R_2$$

$$R_2 = R - R_1$$

Therefore,

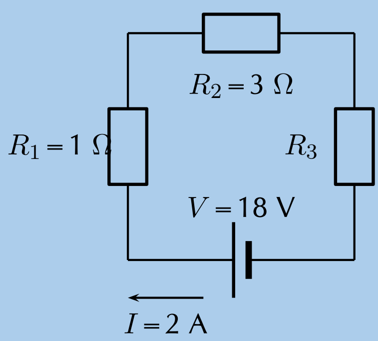
$$R_1 = 5\ \Omega$$

Worked example 4: Ohm's Law, series circuit

QUESTION

For the following circuit, calculate:

1. the voltage drops V_1, V_2 and V_3 across the resistors R_1, R_2, and R_3
2. the resistance of R_3.



SOLUTION

Step 1: Determine how to approach the problem

We are given the voltage across the cell and the current in the circuit, as well as the resistances of two of the three resistors. We can use Ohm's Law to calculate the voltage drop across the known resistors. Since the resistors are in a series circuit the voltage is $V = V_1 + V_2 + V_3$ and we can calculate V_3. Now we can use this information to find the voltage across the unknown resistor R_3.

Step 2: Calculate voltage drop across R_1

Using Ohm's Law:

$$\begin{aligned} R_1 &= \frac{V_1}{I} \\ I \cdot R_1 &= I \cdot \frac{V_1}{I} \\ V_1 &= I \cdot R_1 \\ &= 2 \cdot 1 \\ V_1 &= 2\text{ V} \end{aligned}$$

Step 3: Calculate voltage drop across R_2

Again using Ohm's Law:

$$\begin{aligned} R_2 &= \frac{V_2}{I} \\ I \cdot R_2 &= I \cdot \frac{V_2}{I} \\ V_2 &= I \cdot R_2 \\ &= 2 \cdot 3 \\ V_2 &= 6\text{ V} \end{aligned}$$

Step 4: Calculate voltage drop across R_3

Since the voltage drop across all the resistors combined must be the same as the voltage drop across the cell in a series circuit, we can find V_3 using:

$$\begin{aligned} V &= V_1 + V_2 + V_3 \\ V_3 &= V - V_1 - V_2 \\ &= 18 - 2 - 6 \\ V_3 &= 10\text{ V} \end{aligned}$$

Step 5: Find the resistance of R_3

We know the voltage across R_3 and the current through it, so we can use Ohm's Law to calculate the value for the resistance:

$$\begin{aligned} R_3 &= \frac{V_3}{I} \\ &= \frac{10}{2} \\ R_3 &= 5\Omega \end{aligned}$$

Step 6: Write the final answer

$V_1 = 2$ V

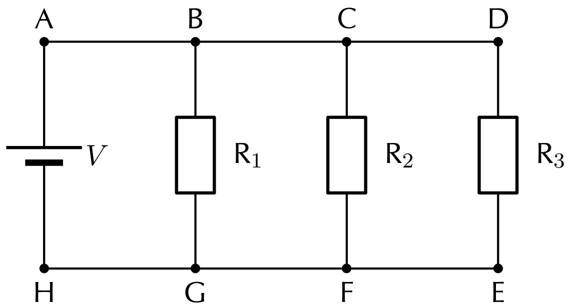
$V_2 = 6$ V

$V_3 = 10$ V

$R_1 = 5\Omega$

Parallel circuits

Consider a circuit consisting of a single cell and three resistors that are connected in parallel.



The first principle to understand about parallel circuits is that the voltage is equal across all components in the circuit. This is because there are only two sets of electrically common points in a parallel circuit, and voltage measured between sets of common points must always be the same at any given time. So, for the circuit shown, the following is true:

$$V = V_1 = V_2 = V_3.$$

The second principle for a parallel circuit is that all the currents through each resistor must add up to the total current in the circuit:

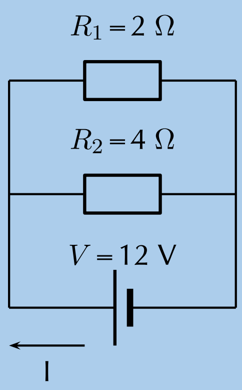
$$I = I_1 + I_2 + I_3.$$

Using these principles and our knowledge of how to calculate the equivalent resistance of parallel resistors, we can now approach some circuit problems involving parallel resistors.

Worked example 5: Ohm's Law, parallel circuit

QUESTION

Calculate the current (I) in this circuit if the resistors are both ohmic in nature.



SOLUTION

Step 1: Determine what is required

We are required to calculate the current flowing in the circuit.

Step 2: Determine how to approach the problem

Since the resistors are ohmic in nature, we can use Ohm's Law. There are however two resistors in the circuit and we need to find the total resistance.

Step 3: Find the equivalent resistance in circuit

Since the resistors are connected in parallel, the total (equivalent) resistance R is:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

$$\begin{aligned}\frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{1}{2} + \frac{1}{4} \\ &= \frac{2+1}{4} \\ &= \frac{3}{4} \\ \text{Therefore, } R &= 1{,}33\Omega\end{aligned}$$

Step 4: Apply Ohm's Law

$$\begin{aligned} R &= \frac{V}{I} \\ R \cdot \frac{I}{R} &= \frac{V}{I} \cdot \frac{I}{R} \\ I &= \frac{V}{R} \\ I &= V \cdot \frac{1}{R} \\ &= (12)\left(\frac{3}{4}\right) \\ &= 9 \text{ A} \end{aligned}$$

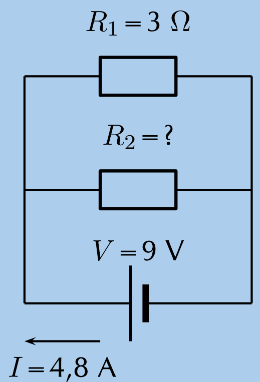
Step 5: Write the final answer

The current flowing in the circuit is 9 A.

Worked example 6: Ohm's Law, parallel circuit

QUESTION

Two ohmic resistors (R_1 and R_2) are connected in parallel with a cell. Find the resistance of R_2, given that the current flowing through the cell is 4,8 A and that the voltage across the cell is 9 V.



SOLUTION

Step 1: Determine what is required

We need to calculate the resistance R_2.

Step 2: Determine how to approach the problem

Since the resistors are ohmic and we are given the voltage across the cell and the current through the cell, we can use Ohm's Law to find the equivalent resistance in

the circuit.

$$\begin{aligned} R &= \frac{V}{I} \\ &= \frac{9}{4{,}8} \\ &= 1{,}875\ \Omega \end{aligned}$$

Step 3: Calculate the value for R_2

Since we know the equivalent resistance and the resistance of R_1, we can use the formula for resistors in parallel to find the resistance of R_2.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Rearranging to solve for R_2:

$$\begin{aligned} \frac{1}{R_2} &= \frac{1}{R} - \frac{1}{R_1} \\ &= \frac{1}{1{,}875} - \frac{1}{3} \\ &= 0{,}2 \\ R_2 &= \frac{1}{0{,}2} \\ &= 5\ \Omega \end{aligned}$$

Step 4: Write the final answer

The resistance R_2 is 5 Ω

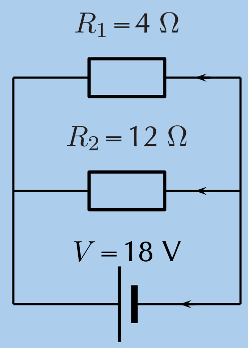
Worked example 7: Ohm's Law, parallel circuit

QUESTION

An 18 volt cell is connected to two parallel resistors of 4 Ω and 12 Ω respectively. Calculate the current through the cell and through each of the resistors.

SOLUTION

Step 1: First draw the circuit before doing any calculations



Step 2: Determine how to approach the problem

We need to determine the current through the cell and each of the parallel resistors. We have been given the potential difference across the cell and the resistances of the resistors, so we can use Ohm's Law to calculate the current.

Step 3: Calculate the current through the cell

To calculate the current through the cell we first need to determine the equivalent resistance of the rest of the circuit. The resistors are in parallel and therefore:

$$\begin{aligned}\frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2}\\ &= \frac{1}{4} + \frac{1}{12}\\ &= \frac{3+1}{12}\\ &= \frac{4}{12}\\ R &= \frac{12}{4} = 3\ \Omega\end{aligned}$$

Now using Ohm's Law to find the current through the cell:

$$\begin{aligned}R &= \frac{V}{I}\\ I &= \frac{V}{R}\\ &= \frac{18}{3}\\ I &= 6\text{ A}\end{aligned}$$

Step 4: Now determine the current through one of the parallel resistors

We know that for a purely parallel circuit, the voltage across the cell is the same as the voltage across each of the parallel resistors. For this circuit:

$$V = V_1 = V_2 = 18\text{ V}$$

Let's start with calculating the current through R_1 using Ohm's Law:

$$
\begin{aligned}
R_1 &= \frac{V_1}{I_1} \\
I_1 &= \frac{V_1}{R_1} \\
&= \frac{18}{4} \\
I_1 &= 4{,}5 \text{ A}
\end{aligned}
$$

Step 5: Calculate the current through the other parallel resistor

We can use Ohm's Law again to find the current in R_2:

$$
\begin{aligned}
R_2 &= \frac{V_2}{I_2} \\
I_2 &= \frac{V_2}{R_2} \\
&= \frac{18}{12} \\
I_2 &= 1{,}5 \text{ A}
\end{aligned}
$$

An alternative method of calculating I_2 would have been to use the fact that the currents through each of the parallel resistors must add up to the total current through the cell:

$$
\begin{aligned}
I &= I_1 + I_2 \\
I_2 &= I - I_1 \\
&= 6 - 4.5 \\
I_2 &= 1{,}5 \text{ A}
\end{aligned}
$$

Step 6: Write the final answer

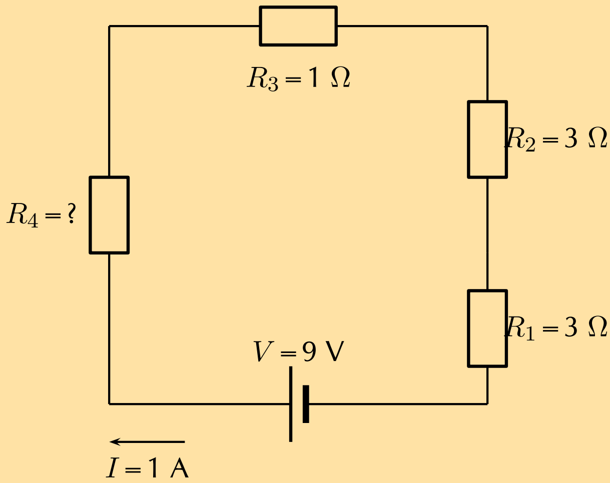
The current through the cell is 6 A.

The current through the 4 Ω resistor is 4,5 A.

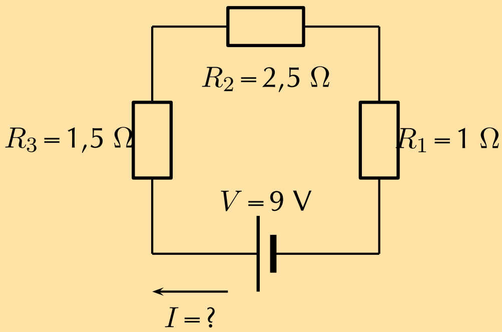
The current through the 12 Ω resistor is 1,5 A.

Exercise 11 – 4: Ohm's Law in series and parallel circuits

1. Calculate the value of the unknown resistor in the circuit:

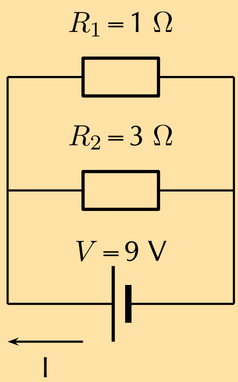


2. Calculate the value of the current in the following circuit:

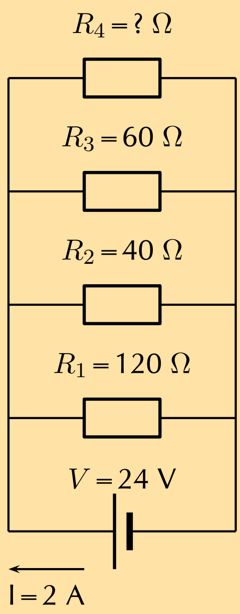


3. Three resistors with resistance 1 Ω, 5 Ω and 10 Ω respectively, are connected in series with a 12 V battery. Calculate the value of the current in the circuit.

4. Calculate the current through the cell if the resistors are both ohmic in nature.



5. Calculate the value of the unknown resistor R_4 in the circuit:



6. Three resistors with resistance 1 Ω, 5 Ω and 10 Ω respectively, are connected in parallel with a 20 V battery. All the resistors are ohmic in nature. Calculate:
   a) the value of the current through the battery
   b) the value of the current in each of the three resistors.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 2433 2. 2434 3. 2435 4. 2436 5. 2437 6. 2438

www.everythingscience.co.za m.everythingscience.co.za

Series and parallel networks of resistors ESBQC

Now that you know how to handle simple series and parallel circuits, you are ready to tackle circuits which combine these two setups such as the following circuit:

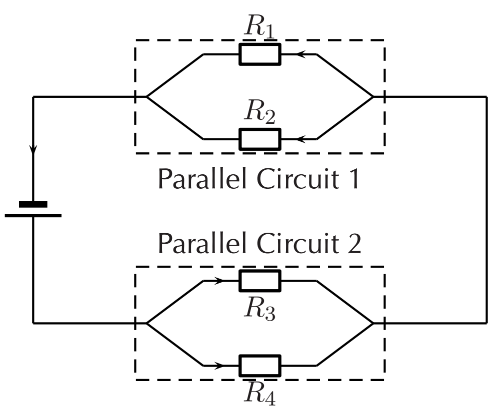
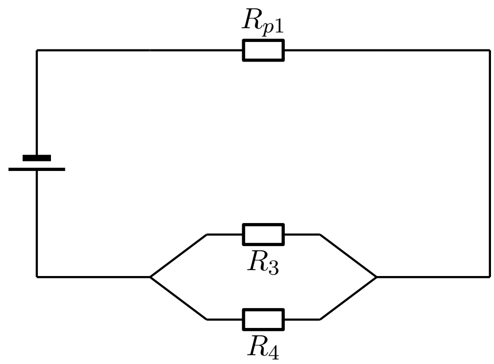


Figure 11.1: An example of a series-parallel network. The dashed boxes indicate parallel sections of the circuit.

It is relatively easy to work out these kind of circuits because you use everything you have already learnt about series and parallel circuits. The only difference is that you do it in stages. In figure 11.1, the circuit consists of 2 parallel portions that are then in series with a cell. To work out the equivalent resistance for the circuit, you start by calculating the total resistance of each of the parallel portions and then add up these resistances in series. If all the resistors in figure 11.1 had resistances of 10 Ω, we can calculate the equivalent resistance of the entire circuit.

We start by calculating the total resistance of *Parallel Circuit 1*.



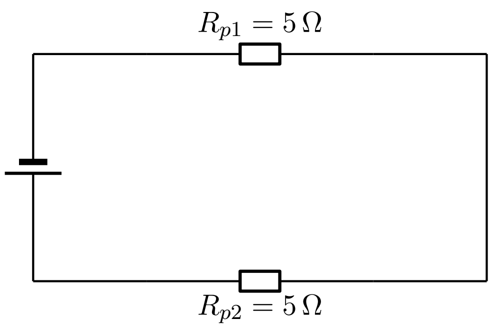
The value of R_{p1} is:

$$
\begin{aligned}
\frac{1}{R_{p1}} &= \frac{1}{R_1} + \frac{1}{R_2} \\
R_{p1} &= \left(\frac{1}{10} + \frac{1}{10}\right)^{-1} \\
&= \left(\frac{1+1}{10}\right)^{-1} \\
&= \left(\frac{2}{10}\right)^{-1} \\
&= 5\,\Omega
\end{aligned}
$$

We can similarly calculate the total resistance of *Parallel Circuit 2*:

$$
\begin{aligned}
\frac{1}{R_{p2}} &= \frac{1}{R_3} + \frac{1}{R_4} \\
R_{p2} &= \left(\frac{1}{10} + \frac{1}{10}\right)^{-1} \\
&= \left(\frac{1+1}{10}\right)^{-1} \\
&= \left(\frac{2}{10}\right)^{-1} \\
&= 5\,\Omega
\end{aligned}
$$

You can now treat the circuit like a simple series circuit as follows:



Therefore the equivalent resistance is:

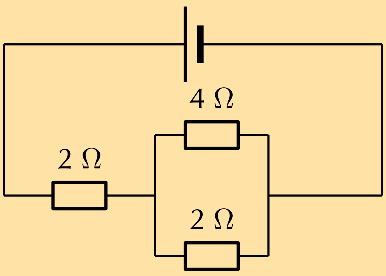
$$\begin{aligned} R &= R_{p1} + R_{p2} \\ &= 5 + 5 \\ &= 10\,\Omega \end{aligned}$$

The equivalent resistance of the circuit in figure 11.1 is 10 Ω.

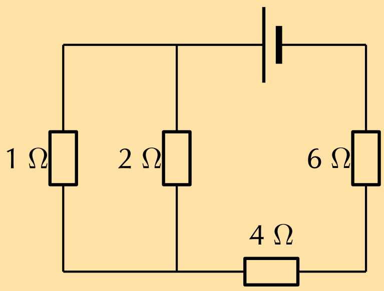
Exercise 11 – 5: Series and parallel networks

1. Determine the equivalent resistance of the following circuits:

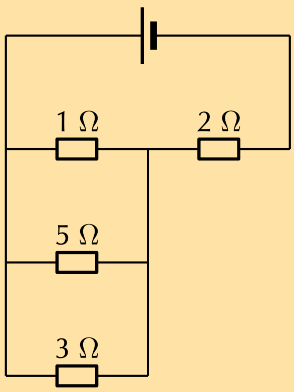
a)



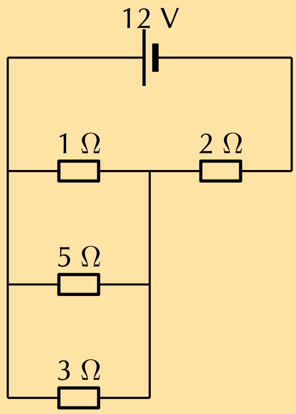
b)



c)



2. Examine the circuit below:

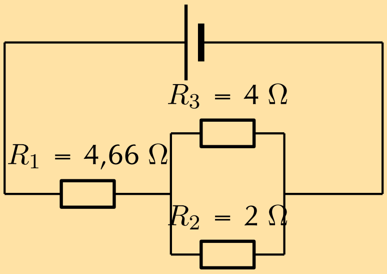


If the potential difference across the cell is 12 V, calculate:

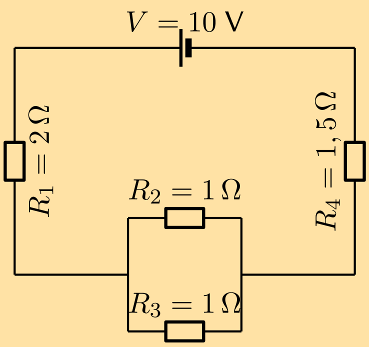
a) the current I through the cell.

b) the current through the 5 Ω resistor.

3. If current flowing through the cell is 2 A, and all the resistors are ohmic, calculate the voltage across the cell and each of the resistors, R_1, R_2, and R_3 respectively.



4. For the following circuit, calculate:



a) the current through the cell

b) the voltage drop across R_4

c) the current through R_2

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 2439 1b. 243B 1c. 243C 2. 243D 3. 243F 4. 243G

www.everythingscience.co.za m.everythingscience.co.za

11.3 Power and energy ESBQD

Electrical power ESBQF

A source of energy is required to drive current round a complete circuit. This is provided by batteries in the circuits you have been looking at. The batteries convert chemical potential energy into electrical energy. The energy is used to do work on the electrons in the circuit.

Power is a measure of how rapidly *work* is done. Power is the **rate** at which the work is done, work done per unit time. Work is measured in joules (J) and time in seconds (s) so power will be $\frac{\text{J}}{\text{s}}$ which we call a watt (W).

In electric circuits, power is a function of both voltage and current and we talk about the power *dissipated* in a circuit element:

DEFINITION: *Electrical Power*

Electrical power is the rate at which electrical energy is converted in an electric circuit. It calculated as:

$$P = I \cdot V$$

Power (P) is exactly equal to current (I) multiplied by voltage (V), there is no extra constant of proportionality. The unit of measurement for power is the watt (abbreviated W).

FACT
It was James Prescott Joule, not Georg Simon Ohm, who first discovered the mathematical relationship between power dissipation and current through a resistance. This discovery, published in 1841, followed the form of the equation: $P = I^2R$ and is properly known as Joule's Law. However, these power equations are so commonly associated with the Ohm's Law equations relating voltage, current, and resistance that they are frequently credited to Ohm.

Equivalent forms

We can use Ohm's Law to show that $P = VI$ is equivalent to $P = I^2R$ and $P = \frac{V^2}{R}$.

Using $V = I \cdot R$ allows us to show:

$$\begin{aligned} P &= V \cdot I \\ &= (I \cdot R) \cdot I \text{ Ohm's Law} \\ &= I^2R \end{aligned}$$

Using $I = \frac{V}{R}$ allows us to show:

$$\begin{aligned} P &= V \cdot I \\ &= V \cdot \frac{V}{R} \text{ Ohm's Law} \\ &= \frac{V^2}{R} \end{aligned}$$

Worked example 8: Electrical power

QUESTION

Given a circuit component that has a voltage of 5 V and a resistance of 2 Ω what is the power dissipated?

SOLUTION

Step 1: Write down what you are given and what you need to find

$$V = 5 \text{ V}$$
$$R = 2\ \Omega$$
$$P = ?$$

Step 2: Write down an equation for power

The equation for power is:

$$P = V^2 R$$

Step 3: Solve the problem

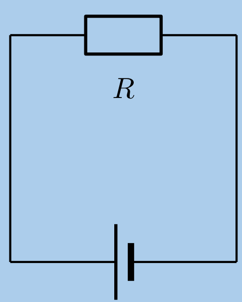
$$\begin{aligned} P &= \frac{V^2}{R} \\ &= \frac{(5)^2}{(2)} \\ &= 12{,}5 \text{ W} \end{aligned}$$

The power is 12,5 W.

Worked example 9: Electrical power

QUESTION

Study the circuit diagram below:



The resistance of the resistor is 15 Ω and the current going through the resistor is 4 A. What is the power for the resistor?

SOLUTION

Step 1: Determine how to approach the problem

We are given the resistance of the resistor and the current passing through it and are asked to calculate the power. We can have verified that:

$$P = I^2R$$

Step 2: Solve the problem

We can simply substitute the known values for R and I to solve for P.

$$\begin{aligned} P &= I^2R \\ &= (4)^2 \times 15 \\ &= 240 \text{ W} \end{aligned}$$

Step 3: Write the final answer

The power for the resistor is 240 W.

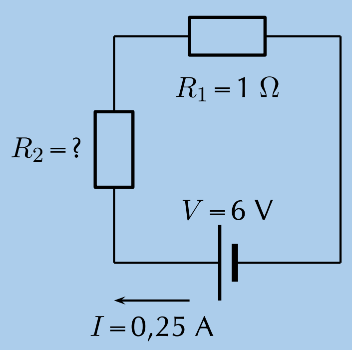
Worked example 10: Power in series circuit

QUESTION

Two ohmic resistors (R_1 and R_2) are connected in series with a cell. Find the resistance and power of R_2, given that the current flowing through R_1 and R_2 is 0,25 A and that the voltage across the cell is 6 V. $R_1 = 1\ \Omega$.

SOLUTION

Step 1: Draw the circuit and fill in all known values.



Step 2: Determine how to approach the problem.

TIP
Notice that we use the same circuits in examples as we extend our knowledge of electric circuits. This is to emphasise that you can always combine all of the principles you have learnt when dealing with any circuit.

We can use Ohm's Law to find the total resistance R in the circuit, and then calculate the unknown resistance using:

$$R = R_1 + R_2$$

because it is in a series circuit.

Step 3: Find the total resistance

$$\begin{aligned} V &= R \cdot I \\ R &= \frac{V}{I} \\ &= \frac{6}{0,25} \\ &= 24\ \Omega \end{aligned}$$

Step 4: Find the unknown resistance

We know that:

$$R = 24\ \Omega$$

and that

$$R_1 = 1\ \Omega$$

Since

$$\begin{aligned} R &= R_1 + R_2 \\ R_2 &= R - R_1 \end{aligned}$$

Therefore,

$$R = 23\ \Omega$$

Step 5: Solve the problem

Now that the resistance is known and the current, we can determine the power:

$$\begin{aligned} P &= I^2 R \\ &= (0,25)^2(23) \\ &= 1,44\ \text{W} \end{aligned}$$

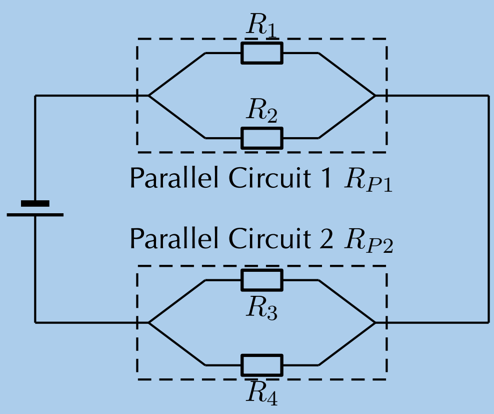
Step 6: Write the final answer

The power for the resistor R_2 is 1,44 W.

Worked example 11: Power in series and parallel networks of resistors

QUESTION

Given the following circuit:



The current leaving the battery is 1,07 A, the total power dissipated in the circuit is 6,42 W, the ratio of the total resistances of the two parallel networks $R_{P1} : R_{P2}$ is 1:2, the ratio $R_1 : R_2$ is 3:5 and $R_3 = 7\ \Omega$.

Determine the:

1. voltage of the battery,
2. the power dissipated in R_{P1} and R_{P2}, and
3. the value of each resistor and the power dissipated in each of them.

SOLUTION

Step 1: What is required

In this question you are given various pieces of information and asked to determine the power dissipated in each resistor and each combination of resistors. Notice that the information given is mostly for the overall circuit. This is a clue that you should start with the overall circuit and work downwards to more specific circuit elements.

Step 2: Calculating the voltage of the battery

Firstly we focus on the battery. We are given the power for the overall circuit as well as the current leaving the battery. We know that the voltage across the terminals of the battery is the voltage across the circuit as a whole.

We can use the relationship $P = VI$ for the entire circuit because the voltage is the

same as the voltage across the terminals of the battery:

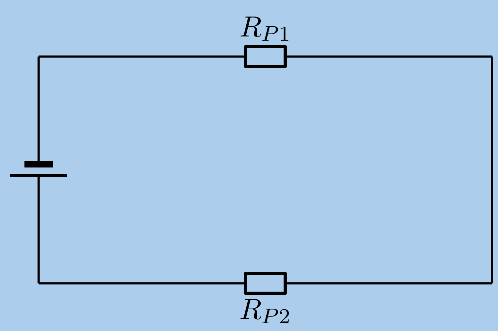
$$
\begin{aligned}
P &= VI \\
V &= \frac{P}{I} \\
&= \frac{6{,}42}{1{,}07} \\
&= 6{,}00 \text{ V}
\end{aligned}
$$

The voltage across the battery is 6,00 V.

Step 3: Power dissipated in R_{P1} and R_{P2}

Remember that we are working from the overall circuit details down towards those for individual elements, this is opposite to how you treated this circuit earlier.

We can treat the parallel networks like the equivalent resistors so the circuit we are currently dealing with looks like:



We know that the current through the two circuit elements will be the same because it is a series circuit and that the resistance for the total circuit must be: $R_T = R_{P1} + R_{P2}$. We can determine the total resistance from Ohm's Law for the circuit as a whole:

$$
\begin{aligned}
V_{battery} &= IR_T \\
R_T &= \frac{V_{battery}}{I} \\
&= \frac{6{,}00}{1{,}07} \\
&= 5{,}61\ \Omega
\end{aligned}
$$

We know that the ratio between $R_{P1} : R_{P2}$ is 1:2 which means that we know:

$$\begin{aligned}
R_{P1} &= \frac{1}{2}R_{P2} \text{ and} \\
R_T &= R_{P1} + R_{P2} \\
&= \frac{1}{2}R_{P2} + R_{P2} \\
&= \frac{3}{2}R_{P2} \\
(5{,}61) &= \frac{3}{2}R_{P2} \\
R_{P2} &= \frac{2}{3}(5{,}61) \\
R_{P2} &= 3{,}74\ \Omega
\end{aligned}$$

and therefore:

$$\begin{aligned}
R_{P1} &= \frac{1}{2}R_{P2} \\
&= \frac{1}{2}(3.74) \\
&= 1{,}87\ \Omega
\end{aligned}$$

Now that we know the total resistance of each of the parallel networks we can calculate the power dissipated in each:

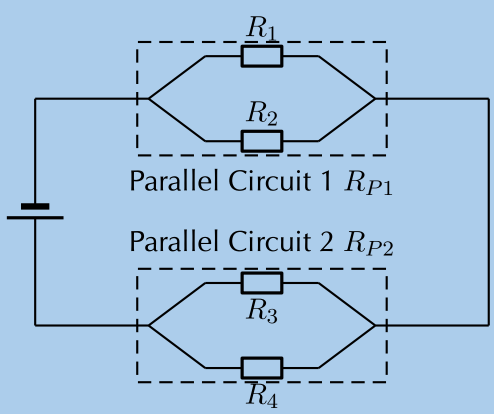
$$\begin{aligned}
P_{P1} &= I^2 R_{P1} \\
&= (1{,}07)^2(1{,}87) \\
&= 2{,}14\ \text{W}
\end{aligned}$$

and

$$\begin{aligned}
P_{P2} &= I^2 R_{P2} \\
&= (1{,}07)^2(3{,}74) \\
&= 4{,}28\ \text{W}
\end{aligned}$$

Step 4: Parallel network 1 calculations

Now we can begin to do the detailed calculation for the first set of parallel resistors.



We know that the ratio between $R_1 : R_2$ is 3:5 which means that we know $R_1 = \frac{3}{5}R_2$. We also know the total resistance for the two parallel resistors in this network is 1,87 Ω. We can use the relationship between the values of the two resistors as well as the formula for the total resistance ($\frac{1}{R_PT} = \frac{1}{R_1} + \frac{1}{R_2}$)to find the resistor values:

$$
\begin{aligned}
\frac{1}{R_{P1}} &= \frac{1}{R_1} + \frac{1}{R_2} \\
\frac{1}{R_{P1}} &= \frac{5}{3R_2} + \frac{1}{R_2} \\
\frac{1}{R_{P1}} &= \frac{1}{R_2}(\frac{5}{3} + 1) \\
\frac{1}{R_{P1}} &= \frac{1}{R_2}(\frac{5}{3} + \frac{3}{3}) \\
\frac{1}{R_{P1}} &= \frac{1}{R_2}\frac{8}{3} \\
R_2 &= R_{P1}\frac{8}{3} \\
&= (1{,}87)\frac{8}{3} \\
&= 4{,}99\ \Omega
\end{aligned}
$$

We can also calculate R_1:

$$
\begin{aligned}
R_1 &= \frac{3}{5}R_2 \\
&= \frac{3}{5}(4{,}99) \\
&= 2{,}99\ \Omega
\end{aligned}
$$

To determine the power we need the resistance which we have calculated and either the voltage or current. The two resistors are in parallel so the voltage across them is the same as well as the same as the voltage across the parallel network. We can use Ohm's Law to determine the voltage across the network of parallel resistors as we know the total resistance and we know the current:

$$
\begin{aligned}
V &= IR \\
&= (1{,}07)(1{,}87) \\
&= 2{,}00\ \text{V}
\end{aligned}
$$

We now have the information we need to determine the power through each resistor:

$$
\begin{aligned}
P_1 &= \frac{V^2}{R_1} \\
&= \frac{(2{,}00)^2}{2{,}99} \\
&= 1{,}34\ \text{W}
\end{aligned}
$$

$$
\begin{aligned}
P_2 &= \frac{V^2}{R_2} \\
&= \frac{(2{,}00)^2}{4{,}99} \\
&= 0{,}80\ \text{W}
\end{aligned}
$$

Step 5: Parallel network 2 calculations

Now we can begin to do the detailed calculation for the second set of parallel resistors.

We are given $R_3 = 7{,}00\ \Omega$ and we know R_{P2} so we can calculate R_4 from:

$$\begin{aligned}\frac{1}{R_{P2}} &= \frac{1}{R_3} + \frac{1}{R_4}\\ \frac{1}{3{,}74} &= \frac{1}{7{,}00} + \frac{1}{R_4}\\ R_4 &= 8{,}03\ \Omega\end{aligned}$$

We can calculate the voltage across the second parallel network by subtracting the voltage of the first parallel network from the battery voltage, $V_{P2} = 6{,}00 - 2{,}00 = 4{,}00$ V.

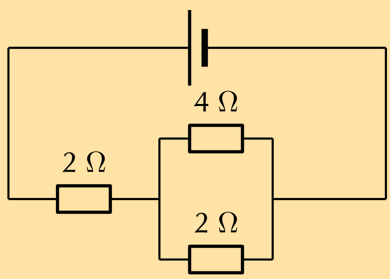
We can now determine the power dissipated in each resistor:

$$\begin{aligned}P_3 &= \frac{V^2}{R_3}\\ &= \frac{(4{,}00)^2}{7{,}00}\\ &= 2{,}29\ \text{W}\end{aligned}$$

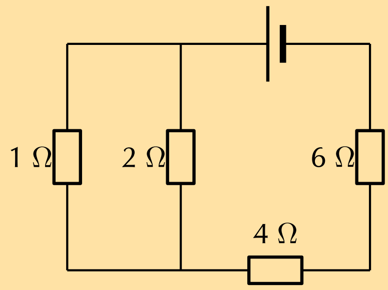
$$\begin{aligned}P_4 &= \frac{V^2}{R_2}\\ &= \frac{(4{,}00)^2}{8{,}03}\\ &= 1{,}99\ \text{W}\end{aligned}$$

Exercise 11 – 6:

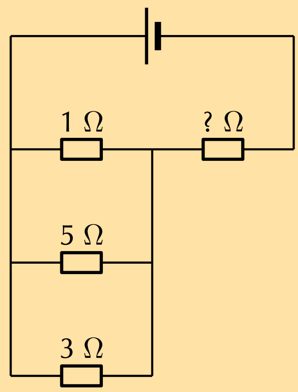
1. What is the power of a $1{,}00 \times 10^8$ V lightning bolt having a current of $2{,}00 \times 10^4$ A?
2. How many watts does a torch that has $6{,}00 \times 10^2$ C pass through it in 0,50 h use if its voltage is 3,00 V?
3. Find the power dissipated in each of these extension cords:
   a) an extension cord having a 0,06 Ω resistance and through which 5,00 A is flowing
   b) a cheaper cord utilising (using) thinner wire and with a resistance of 0,30 Ω, through which 5,00 A is flowing
4. Determine the power dissipated by each the resistors in the following circuits, if the batteries are 6 V:
   a)



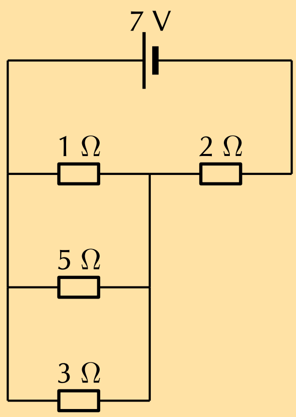
   b)



c) Also determine the value of the unknown resistor if the total power dissipated is 9,8 W



5. Examine the circuit below:



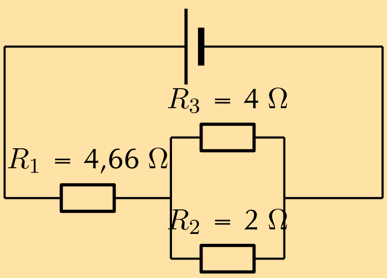
If the potential difference across the cell is 7 V, calculate:

a) the current I through the cell.

b) the current through the 5 Ω resistor

c) the power dissipated in the 5 Ω resistor

6. If current flowing through the cell is 2 A, and all the resistors are ohmic, calculate the power dissipated in each of the resistors, R_1, R_2, and R_3 respectively.



Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 243H 2. 243J 3. 243K 4a. 243M 4b. 243N 4c. 243P
5. 243Q 6. 243R

www.everythingscience.co.za m.everythingscience.co.za

Electrical energy ESBQG

When power is dissipated in a device there is a transfer of energy from one kind to another. For example, a resistor may get very hot which indicates that the energy is being dissipated as heat. Power was the rate at which work was done, the rate at which energy is transferred. If we want to calculate the total amount of energy we need to multiply the rate of energy transfer by the time over which that energy transfer took place.

Electrical energy is simply power times time. Mathematically we write:

$$E = P \times t$$

Energy is measured in joules (J) and time in seconds (s).

Worked example 12: Electrical energy

QUESTION

A 30 W light bulb is left on for 8 hours overnight, how much energy was wasted?

SOLUTION

Step 1: What is required

We need to determine the total amount of electrical energy dissipated by the light bulb. We know the relationship between the power and energy and we are given the time. Time is not given in the correct units so we first need to convert to S.I. units:

$$\begin{aligned} 8\text{ hr} &= 8 \times 3600\text{ s} \\ &= 28\ 800\text{ s} \end{aligned}$$

Step 2: Calculate the energy

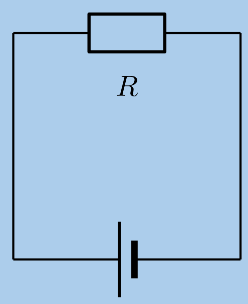
We know that:

$$\begin{aligned} E &= Pt \\ &= (30)(28\ 800) \\ &= 864\ 000\text{ J} \end{aligned}$$

Worked example 13: Electrical energy

QUESTION

Study the circuit diagram below:



The resistance of the resistor is 27 Ω and the current going through the resistor is 3,3 A. What is the power for the resistor and how much energy is dissipated in 35 s?

SOLUTION

Step 1: Determine how to approach the problem

We are given the resistance of the resistor and the current passing through it and are asked to calculate the power. We have verified that:

$$P = I^2 R$$

and we know that

$$E = Pt$$

Step 2: Solve the problem

We can simply substitute the known values for R and I to solve for P.

$$\begin{aligned} P &= I^2 R \\ &= (3{,}3)^2 \times 27 \\ &= 294{,}03 \text{ W} \end{aligned}$$

Now that we have determined the power we can calculate the energy:

$$\begin{aligned} E &= Pt \\ &= (294{,}03)(35) \\ &= 10\ 291{,}05 \text{ J} \end{aligned}$$

Step 3: Write the final answer

The power for the resistor is 294,03 W and 10 291,05 J are dissipated.

Electricity is sold in units which are one kilowatt hour (kWh). A kilowatt hour is simply the use of 1 kW for 1 hr. Using this you can work out exactly how much electricity different appliances will use and how much this will cost you.

Worked example 14: Cost of electricity

QUESTION

How much does it cost to run a 900 W microwave oven for 2,5 minutes if the cost of electricity is 61,6 c per kWh?

SOLUTION

Step 1: What is required

We are given the details for a device that uses electrical energy and the price of electricity. Given a certain amount of time for use we need to determine how much energy was used and what the cost of that would be.

The various quantities provided are in different units. We need to use consistent units to get an answer that makes sense.

The microwave is given in W but we can convert to kW:900 W = 0,9 kW.

Time is given in minutes but when working with household electricity it is normal to work in hours. 2,5 minutes $= \frac{2,5}{60} = 4,17 \times 10^{-2}$ h.

Step 2: Calculate usage

The electrical power is:

$$\begin{aligned} E &= Pt \\ &= (0,9)(4,17 \times 10^{-2}) \\ &= 3,75 \times 10^{-2} \text{ kWh} \end{aligned}$$

Step 3: Calculate cost (C) of electricity

The cost for the electrical power is:

$$\begin{aligned} C &= E \times \text{price} \\ &= (3,75 \times 10^{-2})(61,6) \\ &= 3,75 \times 10^{-2} \text{ kWh} \\ &= 2,31 \text{ c} \end{aligned}$$

Activity: Using electricity

The following table gives the cost of electricity for users who consume less than 450 kWh on average per month.

Units (kWh)	Cost per unit (c)
0–150	61,60
150–350	81,04
350–600	107,43
$>$ 600	118,06

You are given the following appliances with their power ratings.

Appliance	Power rating
Stove	3600 W
Microwave	1200 W
Washing machine	2200 W
Kettle	2200 W
Fridge	230 W
Toaster	750 W
Energy saver globe	40 W
Light bulb	120 W
Vacuum cleaner	1600 W

You have R 150,00 to spend on electricity each month.

1. Which usage class do you fall into?
2. Complete the following table.

Appliance	Cost to run for 1 hour
Stove	
Microwave	
Washing machine	
Kettle	
Fridge	
Toaster	
Energy saver globe	
Vacuum cleaner	

3. For how long can you use each appliance to ensure that you spend less than R 150,00 per month? Assume you are using a maximum of 20 energy saver globes around your home.

See presentation: 243S at www.everythingscience.co.za

- Ohm's Law states that the amount of current through a conductor, at constant temperature, is proportional to the voltage across the resistor. Mathematically we write $I = \frac{V}{R}$
- Conductors that obey Ohm's Law are called ohmic conductors; those that do not are called non-ohmic conductors.
- We use Ohm's Law to calculate the resistance of a resistor. $R = \frac{V}{I}$
- The equivalent resistance of resistors in series (R_s) can be calculated as follows: $R_s = R_1 + R_2 + R_3 + \ldots + R_n$
- The equivalent resistance of resistors in parallel (R_p) can be calculated as follows: $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$
- Electrical power is the rate at which electrical energy is converted in an electric circuit.
- The electrical power dissipated in a circuit element or device is $P = VI$ and can also be written as $P = I^2R$ or $P = \frac{V^2}{R}$ and is measured in joules (J).
- The electrical energy dissipated is $E = Pt$ and is measured in joules (J).
- One kilowatt hour refers to the use of one kilowatt of power for one hour.

Physical Quantities		
Quantity	**Unit name**	**Unit symbol**
Current (I)	ampere	A
Electrical energy (E)	joule	J
Power (P)	watt	W
Resistance (R)	ohm	Ω
Voltage (V)	volt	V

Exercise 11 – 7:

1. Give one word or term for each of the following definitions:
   a) The amount of energy per unit charge needed to move that charge between two points in a circuit.
   b) The rate at which electrical energy is converted in an electric circuit.
   c) A law that states that the amount of current through a conductor, at constant temperature, is proportional to the voltage across the resistor.
2. A 10 Ω has a voltage of 5 V across it. What is the current through the resistor?
   a) 50 A
   b) 5 A

c) 0,5 A

d) 7 A

3. Three resistors are connected in series. The resistances of the three resistors are: 10 Ω, 4 Ω and 3 Ω. What is the equivalent series resistance?

a) 1,5 Ω

b) 17 Ω

c) 0,68 Ω

d) 8 Ω

4. Three resistors are connected in parallel. The resistances of the three resistors are: 5 Ω, 4 Ω and 2 Ω. What is the equivalent parallel resistance?

a) 1,05 Ω

b) 11 Ω

c) 0,95 Ω

d) 3 Ω

5. A circuit consists of a 6 Ω resistor. The voltage across the resistor is 12 V. How much power is dissipated in the circuit?

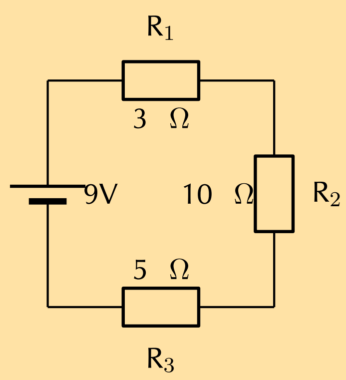
a) 864 W

b) 3 W

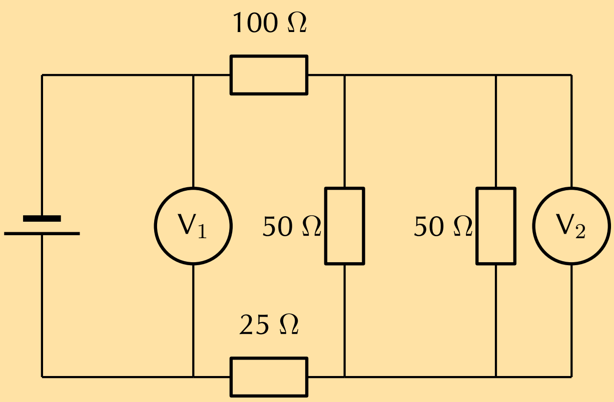
c) 2 W

d) 24 W

6. Calculate the current in the following circuit and then use the current to calculate the voltage drops across each resistor.



7. A battery is connected to this arrangement of resistors. The power dissipated in the 100 Ω resistor is 0,81 W. The resistances of voltmeters V_1 and V_2 are so high that they do not affect the current in the circuit.



a) Calculate the current in the 100 Ω resistor.
b) Calculate the reading on voltmeter V_2.
c) Calculate the reading on voltmeter V_1.

8. A kettle is marked 240 V; 1500 W.

   a) Calculate the resistance of the kettle when operating according to the above specifications.
   b) If the kettle takes 3 minutes to boil some water, calculate the amount of electrical energy transferred to the kettle.

   [SC 2003/11]

9. **Electric Eels**

   Electric eels have a series of cells from head to tail. When the cells are activated by a nerve impulse, a potential difference is created from head to tail. A healthy electric eel can produce a potential difference of 600 V.

   a) What is meant by "a potential difference of 600 V"?
   b) How much energy is transferred when an electron is moved through a potential difference of 600 V?

   [IEB 2001/11 HG1]

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 243T 1b. 243V 1c. 243W 2. 243X 3. 243Y 4. 243Z
5. 2442 6. 2443 7. 2444 8. 2445 9. 2446

www.everythingscience.co.za **m.everythingscience.co.za**

CHAPTER 12

Energy and chemical change

You have probably seen a fire burning or burnt fuel for warmth or cooking or light. A fire burning is one of the most noticeable examples of a chemical reaction that produces a lot of energy.

All chemical reactions involve energy changes. In some reactions, we are able to observe these energy changes as either an increase or a decrease in the overall energy of the system. In some reactions we see this as a change in the temperature. In other reactions we can observe this change when a reaction starts to give off light or when a reaction will only work after light is shone on it.

The study of energy changes (particularly heat) in chemical reactions is known as chemical thermodynamics. This is also sometimes called thermochemistry.

See video: 2447 at www.everythingscience.co.za

Key Mathematics Concepts

- Graphs — Mathematics, Grade 10, Functions and graphs
- Equations — Mathematics, Grade 10, Equations and inequalities
- Units and unit conversions — Physical Sciences, Grade 10, Science skills

12.1 Energy changes in chemical reactions ESBQJ

What causes the energy changes in chemical reactions? ESBQK

When a chemical reaction occurs, bonds in the reactants *break*, while new bonds *form* in the product. The following example explains this. Hydrogen reacts with oxygen to form water, according to the following equation:

$$2H_2(g) + O_2(g) \rightarrow 2H_2O(g)$$

In this reaction, the bond between the two hydrogen atoms in the H_2 molecule will *break*, as will the bond between the oxygen atoms in the O_2 molecule. New bonds will *form* between the two hydrogen atoms and the single oxygen atom in the water molecule that is formed as the product.

For bonds to *break*, energy must be *absorbed*. When new bonds *form*, energy is *released*. The energy that is needed to break a bond is called the **bond energy** or **bond dissociation energy**. Bond energies are measured in units of kJ·mol^{-1}.

DEFINITION: *Bond energy*

Bond energy is a measure of bond strength in a chemical bond. It is the amount of energy (in kJ·mol^{-1}) that is needed to break the chemical bond between two atoms.

Remember when we discussed bonding (chapter 3) we used the following energy diagram:

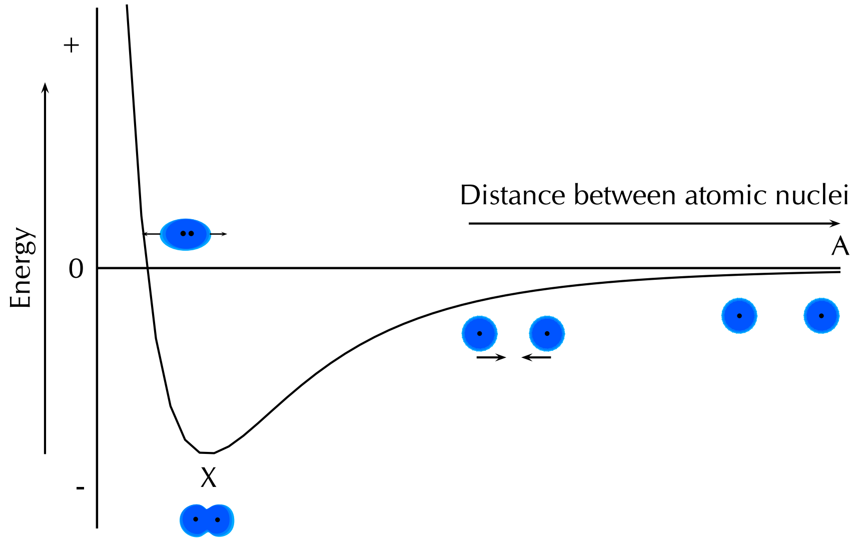


Figure 12.1: Graph showing the changes in energy that take place as the distance between two atoms changes.

TIP
A chemical system is a closed system that contains only the reactants and products involved in the reaction.

We can use this diagram to understand why bond breaking requires energy and bond making releases energy. Point X on the diagram is at the lowest energy. When a bond breaks, the atoms move apart and the distance between them increases (i.e. the atom moves to the right on the x-axis or from point X to point A). Looking at the diagram we see that when this happens, the energy increases (i.e. the energy at point A is greater than the energy at point X). So when a bond breaks energy is needed.

When a bond forms the atoms move closer together and the distance between them decreases (i.e. the atom moves to the left on the x-axis or from point A to point X). Looking at the diagram we see that when this happens, the energy decreases (i.e. the energy at point X is less than the energy at point A). So when a bond forms energy is released.

Looking at the example of hydrogen reacting with oxygen to form water:

$$2\text{H}_2(\text{g}) + \text{O}_2(\text{g}) \rightarrow 2\text{H}_2\text{O}(\text{g})$$

We see that energy is needed to break the bonds in the hydrogen molecule and to break the bonds in the oxygen molecule). And we also see that energy is released when hydrogen and oxygen bond to form water). When we look at the entire reaction and consider both bond breaking and bond forming we need to look at the **enthalpy** of the system.

DEFINITION: *Enthalpy*

Enthalpy is a measure of the total energy of a chemical system for a given pressure, and is given the symbol H.

As we learn about exothermic and endothermic reactions we will see more on the concept of enthalpy.

TIP
Δ is read as delta and means a change in. You may recall this symbol from physics.

Exothermic and endothermic reactions ESBQM

In some reactions, the energy that must be *absorbed* to break the bonds in the reactants, is less than the energy that is *released* when the new bonds of the products are formed. This means that in the overall reaction, energy is *released* as either heat or light. This type of reaction is called an **exothermic** reaction.

DEFINITION: *Exothermic reaction*

An exothermic reaction is one that releases energy in the form of heat or light.

Another way of describing an exothermic reaction is that it is one in which the energy of the products is less than the energy of the reactants, because energy has been released during the reaction. We can represent this using the following general formula:

$$\text{Reactants} \rightarrow \text{Products} + \text{Energy}$$

In other reactions, the energy that must be *absorbed* to break the bonds in the reactants, is more than the energy that is *released* when the new bonds in the products are formed. This means that in the overall reaction, energy must be *absorbed* from the surroundings. This type of reaction is known as an **endothermic** reaction.

DEFINITION: *Endothermic reaction*

An endothermic reaction is one that absorbs energy in the form of heat or light.

Another way of describing an endothermic reaction is that it is one in which the energy of the products is greater than the energy of the reactants, because energy has been absorbed during the reaction. This can be represented by the following general formula:

$$\text{Reactants} + \text{Energy} \rightarrow \text{Products}$$

The difference in energy (E) between the reactants and the products is known as the **heat of the reaction**. It is also sometimes referred to as the **enthalpy change** of the system. This is represented using ΔH

Formal experiment: Endothermic and exothermic reactions - part 1

Apparatus and materials:

You will need:

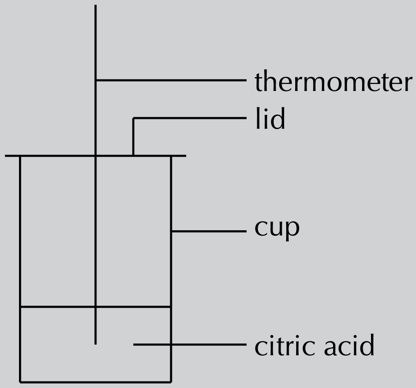
- citric acid
- sodium bicarbonate
- a polystyrene cup
- a lid for the cup

- thermometer
- glass stirring rod
- scissors

Note that citric acid is found in citrus fruits such as lemons. Sodium bicarbonate is actually bicarbonate of soda (baking soda), the baking ingredient that helps cakes to rise.

Method:

1. If your lid does not have a hole for a straw, then cut a small hole into the lid.
2. Pour some citric acid ($C_6H_8O_7$) into the polystyrene cup, cover the cup with its lid and record the temperature of the solution.
3. Stir in the sodium bicarbonate ($NaHCO_3$), then cover the cup again.
4. Immediately record the temperature, and then take a temperature reading every two minutes after that. Record your results.



The equation for the reaction that takes place is:

$$C_6H_8O_7(aq) + 3NaHCO_3(s) \rightarrow 3CO_2(g) + 3H_2O(\ell) + Na_3C_6H_5O_7(aq)$$

Results:

Time (mins)	0	2	4	6
Temperature (°C)				

Plot your temperature results on a graph of time (x-axis) against temperature (y-axis).

Discussion and conclusion:

- What happens to the temperature during this reaction?
- Is this an exothermic or an endothermic reaction? (Was energy taken in or given out? Did the temperature increase or decrease?)
- Why was it important to keep the cup covered with a lid?

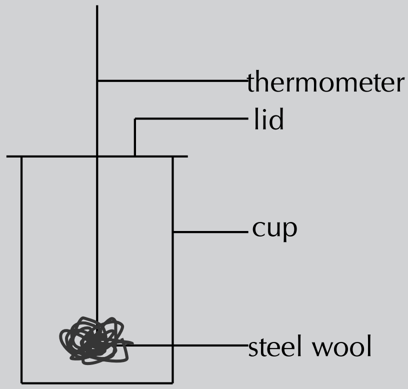
Formal experiment: Endothermic and exothermic reactions - part 2

Apparatus and materials:

- Vinegar
- steel wool
- thermometer
- polystyrene cup and plastic lid (from previous experiment)

Method:

1. Put the thermometer through the plastic lid, cover the cup and record the temperature in the empty cup. You will need to leave the thermometer in the cup for about 5 minutes in order to get an accurate reading.
2. Soak a piece of steel wool in vinegar for about a minute. The vinegar removes the protective coating from the steel wool so that the metal is exposed to oxygen.
3. Take the thermometer out of the cup. Keep the thermometer through the hole of the lid.



4. After the steel wool has been in the vinegar, remove it and squeeze out any vinegar that is still on the wool. Wrap the steel wool around the thermometer and place it (still wrapped round the thermometer) back into the cup. The cup is automatically sealed when you do this because the thermometer is through the top of the lid.
5. Leave the steel wool in the cup for about 5 minutes and then record the temperature. Record your observations.

Results:

You should notice that the temperature *increases* when the steel wool is wrapped around the thermometer.

Conclusion:

The reaction between oxygen and the exposed metal in the steel wool is **exothermic**, which means that energy is released and the temperature increases.

Examples of endothermic and exothermic reactions ESBQN

There are many examples of endothermic and exothermic reactions that occur around us all the time. The following are just a few examples.

1. **Endothermic reactions**

   - **Photosynthesis**
     Photosynthesis is the chemical reaction that takes place in green plants, which uses energy from the sun to change carbon dioxide and water into food that the plant needs to survive, and which other organisms (such as humans and other animals) can eat so that they too can survive. The equation for this reaction is:

     $$6CO_2(g) + 6H_2O\ (l) + \text{energy} \rightarrow C_6H_{12}O_6(s) + 6O_2(g)$$

     Photosynthesis is an endothermic reaction. Energy in the form of sunlight is absorbed during the reaction.
   - **The thermal decomposition of limestone**
     In industry, the breakdown of limestone into quicklime and carbon dioxide is very important. Quicklime can be used to make steel from iron and also to neutralise soils that are too acid. However, the limestone must be heated in a kiln (oven) at a temperature of over 900 °C before the decomposition reaction will take place. The equation for the reaction is shown below:

     $$CaCO_3(s) \rightarrow CaO\ (s) + CO_2(g)$$

2. **Exothermic reactions**

   - **Combustion reactions**
     The burning of fuel is an example of a combustion reaction, and we as humans rely heavily on this process for our energy requirements. The following equations describe the combustion of a hydrocarbon such as *petrol* (C_8H_{18}):
     fuel + oxygen → heat + water + carbon dioxide

     $$2C_8H_{18}(l) + 25O_2(g) \rightarrow 16CO_2(g) + 18H_2O\ (g) + \text{heat}$$

     This is why we burn fuels (such as paraffin, coal, propane and butane) for energy, because the chemical changes that take place during the reaction release huge amounts of energy, which we then use for things like power and electricity. You should also note that *carbon dioxide* is produced during this reaction. The chemical reaction that takes place when fuels burn has both positive and negative consequences. Although we benefit from heat, power and electricity the carbon dioxide that is produced has a negative impact on the environment.
   - **Respiration**
     Respiration is the chemical reaction that happens in our bodies to produce energy for our cells. The equation below describes what happens during this reaction:

     $$C_6H_{12}O_6(s) + 6O_2(g) \rightarrow 6CO_2(g) + 6H_2O(l) + \text{energy}$$

     In the reaction above, glucose (a type of carbohydrate in the food we eat) reacts with oxygen from the air that we breathe in, to form carbon dioxide

TIP
Note that we are only discussing chemical changes (recall from grade 10 about physical and chemical changes). Physical changes can also be classified as exothermic or endothermic. When we are referring to physical change then we talk about exothermic or endothermic processes. Evaporation is an endothermic process while condensation is an exothermic process.

(which we breathe out), water and energy. The energy that is produced allows the cell to carry out its functions efficiently. Can you see now why you must eat food to get energy? It is not the food itself that provides you with energy, but the exothermic reaction that takes place when compounds within the food react with the oxygen you have breathed in!

FACT

Lightsticks or glowsticks are used by divers, campers, and for decoration and fun. A lightstick is a plastic tube with a glass vial inside it. To activate a lightstick, you bend the plastic stick, which breaks the glass vial. This allows the chemicals that are inside the glass to mix with the chemicals in the plastic tube. These two chemicals react and release energy. Another part of a lightstick is a fluorescent dye which changes this energy into light, causing the lightstick to glow! This is known as phosphorescence or chemiluminescence.

See video: 2448 at www.everythingscience.co.za

See video: 2449 at www.everythingscience.co.za

Exercise 12 – 1: Exothermic and endothermic reactions 1

1. State whether energy is taken in or released in each of the following situations:
   a) The bond between hydrogen and chlorine in a molecule of hydrogen chloride breaks.
   b) A bond is formed between hydrogen and fluorine to form a molecule of hydrogen fluoride.
   c) A molecule of nitrogen (N_2) is formed.
   d) A molecule of carbon monoxide breaks apart.
2. State whether the following descriptions are used to describe an endothermic or an exothermic reaction:
   a) Reactants react to give products and energy.
   b) The energy that must be absorbed to break the bonds in the reactants is greater than the energy that is released when the products form.
   c) The energy of the products is found to be greater than the energy of the reactants for this type of reaction.
   d) Heat or light must be absorbed from the surroundings before this type of reaction takes place.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 244B 1b. 244C 1c. 244D 1d. 244F 2a. 244G 2b. 244H
2c. 244J 2d. 244K

www.everythingscience.co.za **m.everythingscience.co.za**

12.2 Exothermic and endothermic reactions ESBQP

The heat of reaction ESBQQ

The **heat of the reaction** is represented by the symbol ΔH, where:

$\Delta\text{H} = E_{\text{prod}} - E_{\text{react}}$

- In an *exothermic* reaction, ΔH is less than zero because the energy of the reactants is greater than the energy of the products. Energy is released in the reaction. For example:

$$\text{H}_2(\text{g}) + \text{Cl}_2(\text{g}) \rightarrow 2\text{HCl (g)} \qquad \Delta\text{H} < 0$$

- In an *endothermic* reaction, ΔH is greater than zero because the energy of the reactants is less than the energy of the products. Energy is absorbed in the reaction. For example:

$$\text{C (s)} + \text{H}_2\text{O (l)} \rightarrow \text{CO (g)} + \text{H}_2(\text{g}) \qquad \Delta\text{H} > 0$$

Some of the information relating to exothermic and endothermic reactions is summarised in Table 12.1.

Type of reaction	Exothermic	Endothermic
Energy absorbed or released	Released	Absorbed
Relative energy of reactants and products	Energy of reactants greater than energy of product	Energy of reactants less than energy of product
Sign of ΔH	Negative (i.e. < 0)	Positive (i.e. > 0)

Table 12.1: A comparison of exothermic and endothermic reactions.

TIP
ΔH has been calculated for many different reactions and so instead of saying that ΔH is positive or negative, we can look up the value of ΔH for the reaction and use that value instead.

Writing equations using ΔH

There are two ways to write the heat of the reaction in an equation. For the exothermic reaction $\text{C(s)} + \text{O}_2(\text{g}) \rightarrow \text{CO}_2(\text{g})$, we can write:

$\text{C(s)} + \text{O}_2(\text{g}) \rightarrow \text{CO}_2(\text{g}) \qquad \Delta\text{H} = -393\ \text{kJ}\cdot\text{mol}^{-1}$

or

$\text{C(s)} + \text{O}_2(\text{g}) \rightarrow \text{CO}_2(\text{g}) + 393\ \text{kJ}\cdot\text{mol}^{-1}$

For the endothermic reaction, $\text{C(s)} + \text{H}_2\text{O(g)} \rightarrow \text{H}_2(\text{g}) + \text{CO(g)}$, we can write:

$\text{C(s)} + \text{H}_2\text{O(g)} \rightarrow \text{H}_2(\text{g}) + \text{CO(g)} \qquad \Delta\text{H} = +131\ \text{kJ}\cdot\text{mol}^{-1}$

or

$\text{C(s)} + \text{H}_2\text{O(g)} + +131\ \text{kJ}\cdot\text{mol}^{-1} \rightarrow \text{H}_2(\text{g}) + \text{CO(g)}$

The **units** for ΔH are kJ·mol^{-1}. In other words, the ΔH value gives the amount of energy that is absorbed or released per mole of product that is formed. Units can also be written as kJ, which then gives the total amount of energy that is released or absorbed when the product forms.

The energy changes during exothermic and endothermic reactions can be plotted on a graph:

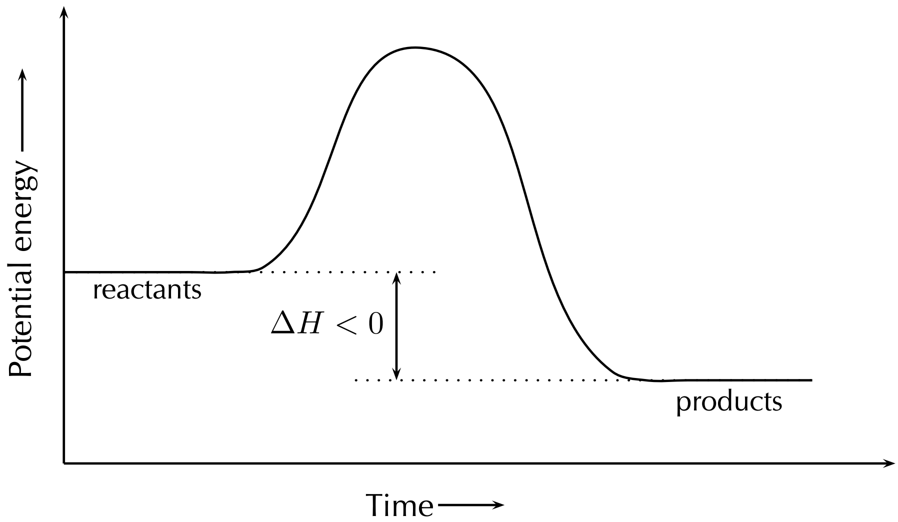


Figure 12.2: The energy changes that take place during an exothermic reaction.

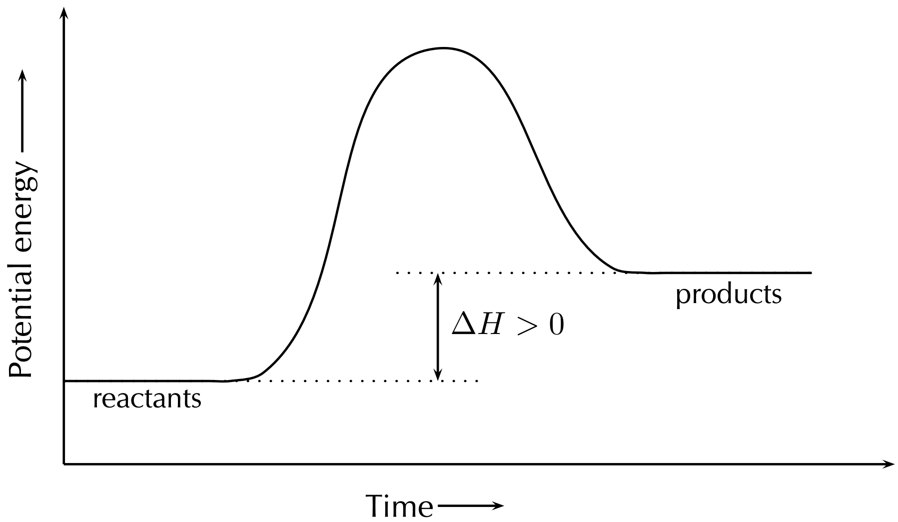


Figure 12.3: The energy changes that take place during an endothermic reaction.

We will explain shortly why we draw these graphs with a curve rather than simply drawing a straight line from the reactants energy to the products energy.

Investigation: Endothermic and exothermic reactions

Aim:

To investigate exothermic and endothermic reactions.

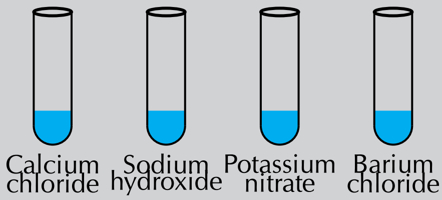
Apparatus and materials:

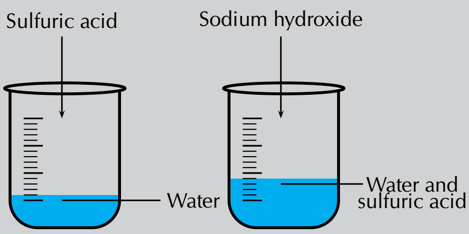
- Approximately 2 g of calcium chloride ($CaCl_2$)
- Approximately 2 g of sodium hydroxide ($NaOH$)
- Approximately 2 g of potassium nitrate (KNO_3)
- Approximately 2 g of barium chloride ($BaCl_2$)
- concentrated sulphuric acid (H_2SO_4) (Be careful, this can cause serious burns)
- 5 test tubes
- thermometer

WARNING!

When working with concentrated sulfuric acid always wear gloves and safety glasses. Always work in a well ventilated room or in a fume cupboard.

Method:





1. Dissolve about 1 g of each of the following substances in 5-10 cm^3 of water in a test tube: $CaCl_2$, $NaOH$, KNO_3 and $BaCl_2$.
2. Observe whether the reaction is endothermic or exothermic, either by feeling whether the side of the test tube gets hot or cold, or using a thermometer.
3. Dilute 3 cm^3 of concentrated H_2SO_4 in 10 cm^3 of water in the fifth test tube and observe whether the temperature changes.

   WARNING!

   Remember to always add the acid to the water.
4. Wait a few minutes and then carefully add $NaOH$ to the diluted H_2SO_4. Observe any temperature (energy) changes.

Results:

Record which of the above reactions are endothermic and which are exothermic.

Exothermic reactions	Endothermic reactions

- When $BaCl_2$ and KNO_3 dissolve in water, they take in heat from the surroundings. The dissolution of these salts is **endothermic**.
- When $CaCl_2$ and NaOH dissolve in water, heat is released. The process is **exothermic**.
- The reaction of H_2SO_4 and NaOH is also **exothermic**.

Exercise 12 – 2: Endothermic and exothermic reactions

1. In each of the following reactions, say whether the reaction is endothermic or exothermic, and give a reason for your answer. Draw the resulting energy graph for each reaction.

    a) $H_2(g) + I_2(g) \rightarrow 2HI\ (g) + 21$ kJ·mol^{-1}
    b) $CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O\ (g)$ $\Delta H = -802$ kJ·mol^{-1}
    c) The following reaction takes place in a flask:
    $Ba(OH)_2.8H_2O\ (s) + 2NH_4NO_3(aq) \rightarrow Ba(NO_3)_2(aq) + 2NH_3(aq) + 10H_2O\ (l)$
    Within a few minutes, the temperature of the flask drops by approximately 20°C.
    d) $2Na\ (aq) + Cl_2(aq) \rightarrow 2NaCl\ (aq)$ $\Delta H = -411$ kJ·mol^{-1}
    e) $C\ (s) + O_2(g) \rightarrow CO_2(g)$

2. For each of the following descriptions, say whether the process is endothermic or exothermic and give a reason for your answer.

    a) evaporation
    b) the combustion reaction in a car engine
    c) bomb explosions
    d) melting ice
    e) digestion of food
    f) condensation

3. When you add water to acid the resulting solution splashes up. The beaker also gets very hot. Explain why.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 244M 1b. 244N 1c. 244P 1d. 244Q 1e. 244R 2a. 244S
2b. 244T 2c. 244V 2d. 244W 2e. 244X 2f. 244Y 3. 244Z

www.everythingscience.co.za m.everythingscience.co.za

TIP
It is important to realise that even though exothermic reactions release energy they still need a small amount of energy to start the reaction.

12.3 Activation energy and the activated complex ESBQR

If you take a match and just hold it or wave it around in the air, the match will not light. You have to strike the match against the side of the box. All chemical reactions need something that makes them start going.

Chemical reactions will not take place until the system has some minimum amount of energy added to it. This energy is called the **activation energy**.

DEFINITION: *Activation energy*

Activation energy is the minimum amount of energy that is needed to start a chemical reaction.

Recall from earlier that we drew graphs for the energy changes in exothermic and endothermic reactions. We can now add some information to these graphs. This will also explain why we draw these graphs with a curve rather than using a straight line from the reactants energy to the products energy.

We will start by looking at exothermic reactions. We will use:

$$H_2(g) + F_2(g) \rightarrow 2HF\ (g)$$

as an example of an exothermic reaction.

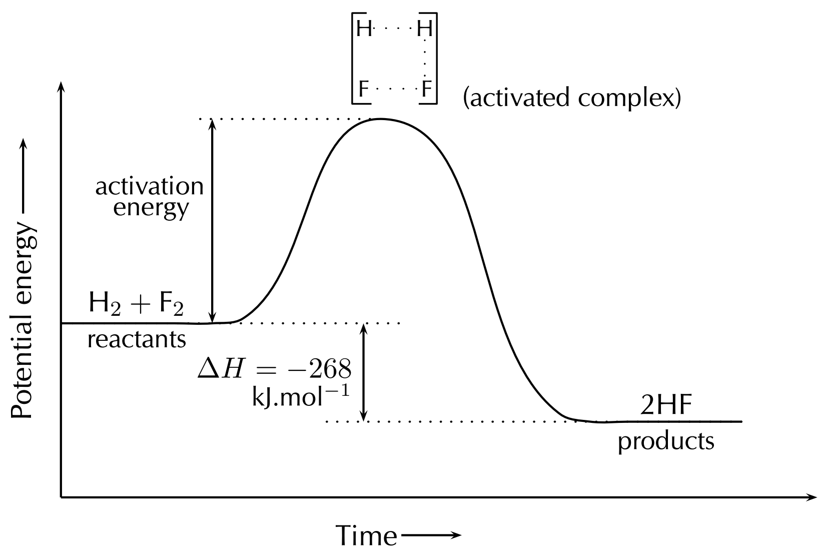


Figure 12.4: The energy changes that take place during an exothermic reaction.

TIP

The activation energy is the difference between the energy of the **reactants** and the maximum energy (i.e. the energy of the activated complex).

FACT

The reaction between H_2 and F_2 was considered by NASA (National Aeronautics and Space Administration) as a fuel system for rocket boosters because of the energy that is released during this exothermic reaction.

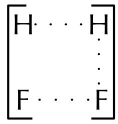
TIP

Enzymes and activation energy

An enzyme is a catalyst that helps to speed up the rate of a reaction by lowering the activation energy of a reaction. There are many enzymes in the human body, without which lots of important reactions would never take place. Cellular respiration is one example of a reaction that is catalysed by enzymes. You will learn more about catalysts in Grade 12.

The reaction between $H_2(g)$ and $F_2(g)$ (Figure 12.4) needs energy in order to proceed, and this is the activation energy. To form the product the bond between H and H in H_2 must break. The bond between F and F in F_2 must also break. A new bond between H and F must also form to make HF. The reactant bonds break at the same time that the product bonds form.

We can show this as:



This is called the **activated complex** or transition state. The activated complex lasts for only a *very short time*. After this short time one of two things will happen: the original bonds will reform, or the bonds are broken and a new product forms. In this example, the final product is HF and it has a lower energy than the reactants. The reaction is exothermic and ΔH is negative.

The activated complex is the complex that exists as the bonds in the products are forming and the bonds in the reactants are breaking. This complex exists for a very short period of time and is found when the energy of the system is at its maximum.

In endothermic reactions, the final products have a higher energy than the reactants. An energy diagram is shown below (Figure 12.5) for the endothermic reaction:

$$O_2(g) + N_2(g) \rightarrow 2NO\ (g)$$

Notice that the activation energy for the endothermic reaction is much greater than for the exothermic reaction.

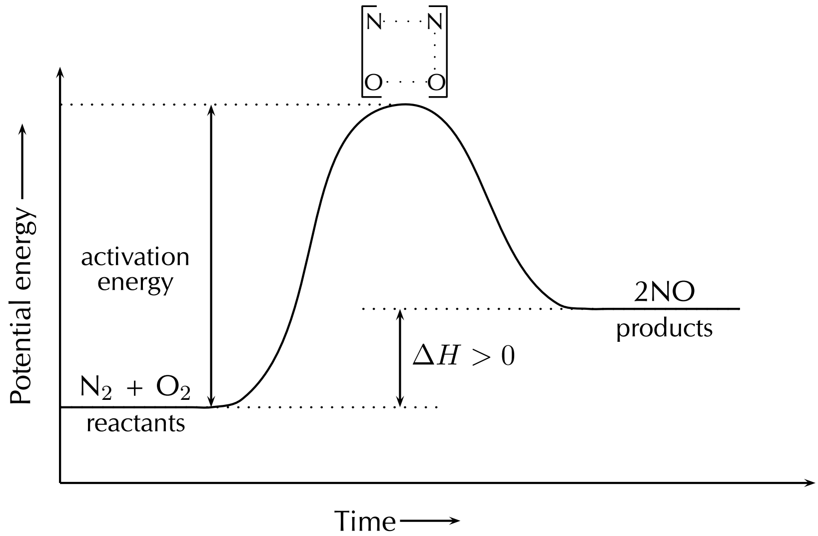


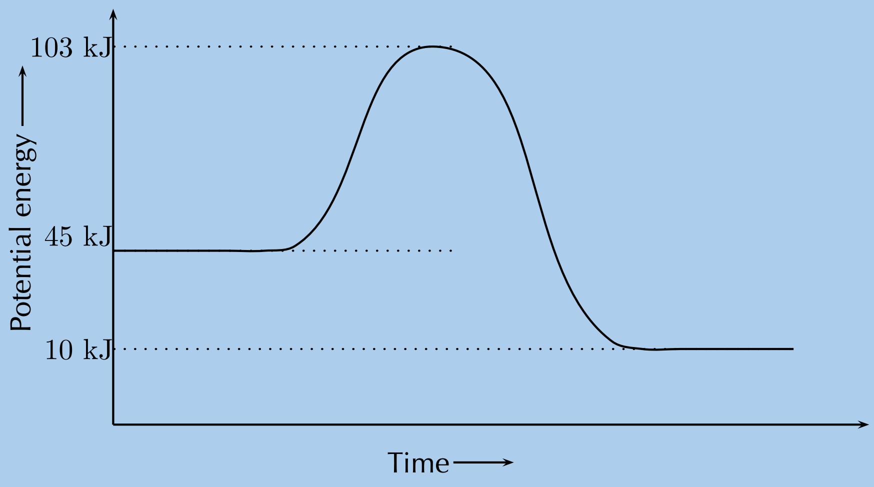
Figure 12.5: The energy changes that take place during an endothermic reaction.

It is because of this activation energy that we first need to show an increase in energy from the reactant to the activated complex and then a decrease in energy from the activated complex to the product. We show this on the energy graphs by drawing a curve from the energy of the reactants to the energy of the products.

Worked example 1: Activation energy

QUESTION

Refer to the graph below and then answer the questions that follow:



1. Calculate ΔH.
2. Is the reaction endothermic or exothermic and why?
3. Calculate the activation energy for this reaction.

SOLUTION

Step 1: Calculate ΔH

ΔH is found by subtracting the energy of the **reactants** from the energy of the **products**. We find the energy of the reactants and the products from the graph.

$$\begin{aligned}\Delta \mathrm{H} &= \text{energy of products} - \text{energy of reactants}\\ &= 10\text{ kJ} - 45\text{ kJ}\\ &= -35\text{ kJ}\end{aligned}$$

Step 2: Determine if this is exothermic or endothermic.

The reaction is exothermic since $\Delta \mathrm{H} < 0$. We also note that the energy of the reactants is greater than the energy of the products.

Step 3: Calculate the activation energy

The activation energy is found by subtracting the energy of the reactants from the energy of the activated complex. Again we can read the energy of the reactants and activated complex off the graph.

$$\begin{aligned}\text{activation energy} &= \text{energy of activated complex} - \text{energy of reactants}\\ &= 103\text{ kJ} - 45\text{ kJ}\\ &= 58\text{ kJ}\end{aligned}$$

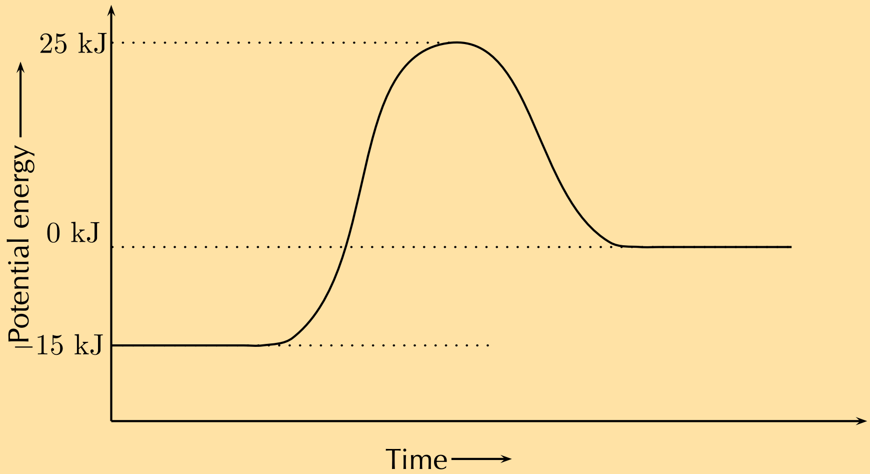
Exercise 12 – 3: Energy and reactions

1. Carbon reacts with water according to the following equation:

   $C\ (s) + H_2O\ (g) \rightarrow CO\ (g) + H_2(g) \qquad \Delta H > 0$

   Is this reaction endothermic or exothermic? Give a reason for your answer.

2. Refer to the graph below and then answer the questions that follow:



   a) What is the energy of the reactants?
   b) What is the energy of the products?
   c) Calculate ΔH.
   d) What is the activation energy for this reaction?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 2452 2. 2453

www.everythingscience.co.za m.everythingscience.co.za

12.4 Chapter summary ESBQS

See presentation: 2454 at www.everythingscience.co.za

- When a reaction occurs, bonds in the reactants break and new bonds in the products form. These changes involve **energy**.
- When bonds break, energy is **absorbed** and when new bonds form, energy is **released**.
- The **bond energy** is a measure of bond strength in a chemical bond. It is the amount of energy (in kJ·mol^{-1}) that is needed to break the chemical bond between two atoms.
- **Enthalpy** is a measure of the total energy of a chemical system for a given pressure and is given the symbol H.

- If the energy that is needed to *break* the bonds is less than the energy that is released when new bonds *form*, then the reaction is **exothermic**. The energy of the products is less than the energy of the reactants.
- An **exothermic** reaction is one that **releases energy** in the form of heat or light.
- If the energy that is needed to *break* the bonds is more than the energy that is released when new bonds *form*, then the reaction is **endothermic**. The energy of the products is greater than the energy of the reactants.
- An **endothermic** reaction is one that **absorbs energy** in the form of heat or light.
- Photosynthesis and the thermal decomposition of limestone are both examples of endothermic reactions.
- Combustion reactions and respiration are both examples of exothermic reactions.
- The difference in energy between the reactants and the product is called the **heat of reaction** and has the symbol ΔH.
- ΔH is calculated using: $\Delta H = E_{\text{prod}} - E_{\text{react}}$
- In an endothermic reaction, **ΔH is a positive number** (greater than 0). In an exothermic reaction, **ΔH will be negative** (less than 0).
- Chemical reactions will not take place until the system has some minimum amount of energy added to it.
- The **activation energy** is the minimum amount of energy that is needed to start a chemical reaction.
- The **activated complex** (or transition state) is the complex that exists as the bonds in the products are forming and the bonds in the reactants are breaking. This complex exists for a very short period of time and is found when the energy of the system is at its maximum.

Exercise 12 – 4:

1. For each of the following, give one word or term for the description.
   a) The minimum amount of energy that is needed for a reaction to proceed.
   b) A measure of the bond strength in a chemical bond.
   c) A type of reaction where ΔH is less than zero.
   d) A type of reaction that requires heat or light to proceed.
2. For the following reaction:

$$\text{HCl (aq)} + \text{NaOH (aq)} \rightarrow \text{NaCl (aq)} + \text{H}_2\text{O (l)}$$

   choose the correct statement from the list below.
   a) Energy is taken in when the new bonds in NaCl are formed.
   b) Energy is released when the bonds in HCl break.

c) Energy is released when the bonds in H_2O form.
d) Energy is released when the bonds in NaOH break.

3. For the following reaction:

$$\mathrm{A + B \rightarrow AB} \qquad \Delta\mathrm{H} = -129\ \mathrm{kJ{\cdot}mol^{-1}}$$

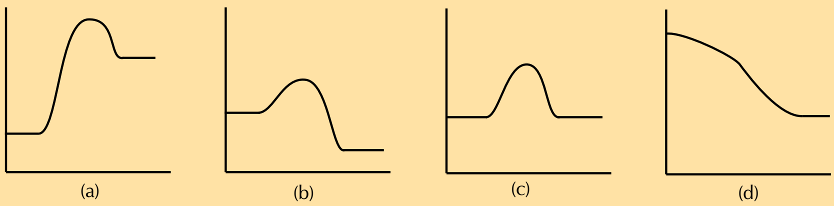
choose the correct statement from the list below.

a) The energy of the reactants is less than the energy of the product.
b) The energy of the product is less than the energy of the reactants.
c) The reaction needs energy to occur.
d) The overall energy of the system increases during the reaction.

4. Consider the following chemical reaction:

$$\mathrm{2NO_2(g) \rightarrow N_2O_4(g)} \qquad \Delta\mathrm{H} < 0$$

Which one of the following graphs best represents the changes in potential energy that take place during the production of N_2O_4?



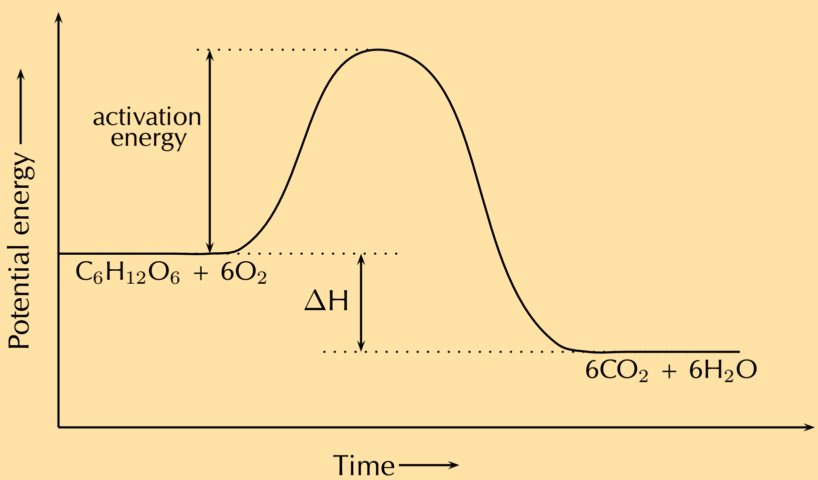
5. In each of the following reactions, say whether the reaction is endothermic or exothermic, and give a reason for your answer. Draw the resulting energy graph for each reaction.

a) $\mathrm{Fe_2O_3(s) + 2Al\ (s) \rightarrow 2Fe\ (s) + Al_2O_3(s) +\ heat}$
b) $\mathrm{NH_4Cl\ (s) + heat \rightarrow NH_3(g) + HCl\ (g)}$

6. The cellular respiration reaction is catalysed by enzymes. The equation for the reaction is:

$$\mathrm{C_6H_{12}O_6(s) + 6O_2(g) \rightarrow 6CO_2(g) + 6H_2O\ (l)}$$

The change in potential energy during this reaction is shown below:



a) Will the value of ΔH be positive or negative? Give a reason for your answer.
b) Explain what is meant by activation energy.
c) Glucose is one of the reactants in cellular respiration. What important chemical reaction produces glucose?
d) Is the reaction in your answer above an endothermic or an exothermic one? Explain your answer.
e) Draw the energy graph for the reaction that produces glucose.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 2455 1b. 2456 1c. 2457 1d. 2458 2. 2459 3. 245B
4. 245C 5a. 245D 5b. 245F 6. 245G

www.everythingscience.co.za m.everythingscience.co.za

CHAPTER 13

Types of reactions

All around you there are chemical reactions taking place. Green plants are photosynthesising, car engines are relying on the reaction between petrol and air and your body is performing many complex reactions. In this chapter we will look at two common types of reactions that can occur in the world around you and in the chemistry laboratory. These two types of reactions are acid-base reactions and redox reactions.

13.1 Acids and bases ESBQT

What are acids and bases? ESBQV

Activity: Household acids and bases

Look around your home and school and find examples of acids and bases. Remember that foods can also be acidic or basic.

Make a list of all the items you find. Why do you think they are acids or bases?

Some common acids and bases, and their chemical formulae, are shown in Table 13.1.

Acid	Formula	Base	Formula
Hydrochloric acid	HCl	Sodium hydroxide	$NaOH$
Sulfuric acid	H_2SO_4	Potassium hydroxide	KOH
Sulfurous acid	H_2SO_3	Sodium carbonate	Na_2CO_3
Acetic (ethanoic) acid	CH_3COOH	Calcium hydroxide	$Ca(OH)_2$
Carbonic acid	H_2CO_3	Magnesium hydroxide	$Mg(OH)_2$
Nitric acid	HNO_3	Ammonia	NH_3
Phosphoric acid	H_3PO_4	Sodium bicarbonate	$NaHCO_3$

Table 13.1: Some common acids and bases and their chemical formulae.

Most acids share certain characteristics, and most bases also share similar characteristics. It is important to be able to have a definition for acids and bases so that they can be correctly identified in reactions.

Defining acids and bases ESBQW

One of the first things that was noted about acids is that they have a sour taste. Bases were noted to have a soapy feel and a bitter taste. However you cannot go around tasting and feeling unknown substances since they may be harmful. Also when chemists started to write down chemical reactions more practical definitions were needed.

A number of definitions for acids and bases have developed over the years. One of the earliest was the **Arrhenius** definition. Arrhenius (1887) noticed that water dissociates (splits up) into hydronium (H_3O^+) and hydroxide (OH^-) ions according to the following equation:

TIP
For more information on *dissociation*, refer to Grade 10 (chapter 18: reactions in aqueous solution).

$2H_2O\ (l) \rightarrow H_3O^+(aq) + OH^-(aq)$

Arrhenius described an **acid** as a compound that increases the concentration of H_3O^+ ions in solution and a **base** as a compound that increases the concentration of OH^- ions in solution.

Look at the following examples showing the dissociation of hydrochloric acid and sodium hydroxide (a base) respectively:

1. $HCl\ (aq) + H_2O(l) \rightarrow H_3O^+(aq) + Cl^-(aq)$

   Hydrochloric acid in water increases the concentration of H_3O^+ ions and is therefore an *acid*.

2. $NaOH\ (s) \xrightarrow{H_2O} Na^+(aq) + OH^-(aq)$

   Sodium hydroxide in water increases the concentration of OH^- ions and is therefore a *base*.

   Note that we write $\xrightarrow{H_2O}$ to indicate that water is needed for the dissociation.

However, this definition could only be used for acids and bases *in water*. Since there are many reactions which do not occur in water it was important to come up with a much broader definition for acids and bases.

In 1923, Lowry and Bronsted took the work of Arrhenius further to develop a broader definition for acids and bases. The **Bronsted-Lowry model** defines acids and bases in terms of their ability to donate or accept protons.

DEFINITION: *Acids*

A Bronsted-Lowry **acid** is a substance that gives away protons (hydrogen cations H^+), and is therefore called a **proton donor**.

DEFINITION: *Bases*

A Bronsted-Lowry **base** is a substance that takes up protons (hydrogen cations H^+), and is therefore called a **proton acceptor**.

Below are some examples:

1. $HCl\ (aq) + NH_3(aq) \rightarrow NH_4^+(aq) + Cl^-(aq)$

   We highlight the chlorine and the nitrogen so that we can follow what happens to these two elements as they react. We do not highlight the hydrogen atoms as we are interested in how these change. This colour coding is simply to help you identify the parts of the reaction and does not represent any specific property of these elements.

   $HCl\ (aq) + NH_3(aq) \rightarrow NH_4^+(aq) + Cl^-(aq)$

In order to decide which substance is a proton donor and which is a proton acceptor, we need to look at what happens to each reactant. The reaction can be broken down as follows:

$HCl\ (aq) \rightarrow Cl^{-}(aq)$ and

$NH_3(aq) \rightarrow NH_4^{+}(aq)$

From these reactions, it is clear that HCl is a *proton donor* and is therefore an **acid**, and that NH_3 is a *proton acceptor* and is therefore a **base**.

2. $CH_3COOH\ (aq) + H_2O\ (l) \rightarrow H_3O^{+}(aq) + CH_3COO^{-}(aq)$

   Again we highlight the parts of the reactants that we want to follow in this reaction:

   $CH_3COOH\ (aq) + H_2O\ (l) \rightarrow H_3O^{+}(aq) + CH_3COO^{-}(aq)$

   The reaction can be broken down as follows:

   $CH_3COOH\ (aq) \rightarrow CH_3COO^{-}(aq)$ and

   $H_2O\ (l) \rightarrow H_3O^{+}(aq)$

   In this reaction, CH_3COOH (acetic acid or vinegar) is a proton donor and is therefore the **acid**. In this case, water acts as a **base** because it accepts a proton to form H_3O^{+}.

3. $NH_3(aq) + H_2O\ (l) \rightarrow NH_4^{+}(aq) + OH^{-}(aq)$

   Again we highlight the parts of the reactants that we want to follow in this reaction:

   $NH_3(aq) + H_2O\ (l) \rightarrow NH_4^{+}(aq) + OH^{-}(aq)$

   The reaction can be broken down as follows:

   $H_2O\ (l) \rightarrow OH^{-}(aq)$ and

   $NH_3(aq) \rightarrow NH_4^{+}(aq)$

   Water donates a proton and is therefore an **acid** in this reaction. Ammonia accepts the proton and is therefore the **base**.

Notice in these examples how we looked at the common elements to break the reaction into two parts. So in the first example we followed what happened to chlorine to see if it was part of the acid or the base. And we also followed nitrogen to see if it was part of the acid or the base. You should also notice how in the reaction for the acid there is one less hydrogen on the right hand side and in the reaction for the base there is an extra hydrogen on the right hand side.

Amphoteric substances

In examples 2 and 3 above we notice an interesting thing about water. In example 2 we find that water acts as a base (it accepts a proton). In example 3 however we see that water acts as an acid (it donates a proton)!

Depending on what water is reacting with it can either react as a base or as an acid. Water is said to be **amphoteric**. Water is not unique in this respect, several other substances are also amphoteric.

DEFINITION: *Amphoteric*

An amphoteric substance is one that can react as either an acid or base.

When we look just at Bronsted-Lowry acids and bases we can also talk about amphiprotic substances which are a special type of amphoteric substances.

DEFINITION: *Amphiprotic*

An amphiprotic substance is one that can react as either a proton donor (Bronsted-Lowry acid) or as a proton acceptor (Bronsted-Lowry base). Examples of amphiprotic substances include water, hydrogen carbonate ion (HCO_3^-) and hydrogen sulfate ion (HSO_4^-).

Note: You may also see the term **ampholyte** used to mean a substance that can act as both an acid and a base. This term is no longer in general use in chemistry.

Polyprotic acids [NOT IN CAPS]

A polyprotic (many protons) acid is an acid that has more than one proton that it can donate. For example sulfuric acid can donate one proton to form the hydrogen sulfate ion:

$$H_2SO_4(aq) + OH^-(aq) \rightarrow HSO_4^-(aq) + H_2O\ (l)$$

Or it can donate two protons to form the sulfate ion:

$$H_2SO_4(aq) + 2OH^-(aq) \rightarrow SO_4^{2-}(aq) + 2H_2O\ (l)$$

In this chapter we will mostly consider monoprotic acids (acids with only one proton to donate). If you do see a polyprotic acid in a reaction then write the resulting reaction equation with the acid donating all its protons.

Some examples of polyprotic acids are:

H_2SO_4, H_2SO_3, H_2CO_3 and H_3PO_4.

See simulation: 245H at www.everythingscience.co.za

Exercise 13 – 1: Acids and bases

1. Identify the Bronsted-Lowry acid and the Bronsted-Lowry base in the following reactions:

   a) $HNO_3(aq) + NH_3(aq) \rightarrow NO_3^-\ (aq) + NH_4^+\ (aq)$

   b) $HBr\ (aq) + KOH\ (aq) \rightarrow KBr\ (aq) + H_2O\ (l)$

2. a) Write a reaction equation to show HCO_3^- acting as an acid.

   b) Write a reaction equation to show HCO_3^- acting as an base.

   c) Compounds such as HCO_3^- are . . .

TIP
Up to now you have looked at reactions as starting with the reactants and going to the products. For acids and bases we also need to consider what happens if we swop the reactants and the products around. This will help you understand conjugate acid-base pairs.

TIP
The word conjugate means coupled or connected.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 245J 1b. 245K 2. 245M

www.everythingscience.co.za m.everythingscience.co.za

Conjugate acid-base pairs ESBQX

Look at the reaction between hydrochloric acid and ammonia to form ammonium and chloride ions (again we have highlighted the different parts of the equation):

$HCl\ (aq) + NH_3(aq) \rightarrow NH_4^+(aq) + Cl^-(aq)$

We look at what happens to each of the reactants in the reaction:

$HCl\ (aq) \rightarrow Cl^-(aq)$ and

$NH_3(aq) \rightarrow NH_4^+(aq)$

We see that HCl acts as the acid and NH_3 acts as the base.

But what if we actually had the following reaction:

$NH_4^+(aq) + Cl^-(aq) \rightarrow HCl\ (aq) + NH_3(aq)$

This is the **same** reaction as the first one, but the products are now the reactants.

Now if we look at the what happens to each of the reactants we see the following:

$NH_4^+(aq) \rightarrow NH_3(aq)$ and

$Cl^-(aq) \rightarrow HCl\ (aq)$

We see that NH_4^+ acts as the acid and Cl^- acts as the base.

When HCl (the acid) loses a proton it forms Cl^- (the base). And that when Cl^- (the base) gains a proton it forms HCl (the acid). We call these two species a **conjugate acid-base pair**. Similarly NH_3 and NH_4^+ form a conjugate acid-base pair.

We can represent this as:

conjugate pair

$$\underset{\text{acid 1}}{HCl} + \underset{\text{base 2}}{NH_3} \longrightarrow \underset{\text{acid 2}}{NH_4^+} + \underset{\text{base 1}}{Cl^-}$$

conjugate pair

Activity: Conjugate acid-base pairs

Using the common acids and bases in Table 13.1, pick an acid and a base from the list. Write a chemical equation for the reaction of these two compounds.

Now identify the conjugate acid-base pairs in your chosen reaction. Compare your results to those of your classmates.

TIP
In chemistry the word salt does not mean the white substance that you sprinkle on your food (this white substance is a salt, but not the only salt). A salt (to chemists) is a product of an acid-base reaction and is made up of the cation from the base and the anion from the acid.

See video: 245N at www.everythingscience.co.za

Exercise 13 – 2: Acids and bases

1. In each of the following reactions, label the conjugate acid-base pairs.
   a) $H_2SO_4(aq) + H_2O\ (l) \rightarrow H_3O^+(aq) + HSO_4^-(aq)$
   b) $NH_4^+(aq) + F^-(aq) \rightarrow HF(aq) + NH_3(aq)$
   c) $H_2O\ (l) + CH_3COO^-(aq) \rightarrow CH_3COOH\ (aq) + OH^-(aq)$
   d) $H_2SO_4(aq) + Cl^-(aq) \rightarrow HCl\ (aq) + HSO_4^-(aq)$
2. Given the following reaction:
$$H_2O\ (l) + NH_3(aq) \rightarrow NH_4^+(aq) + OH^-(aq)$$
   a) Write down which reactant is the base and which is the acid.
   b) Label the conjugate acid-base pairs.
   c) In your own words explain what is meant by the term conjugate acid-base pair.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 245P 1b. 245Q 1c. 245R 1d. 245S 2. 245T

www.everythingscience.co.za m.everythingscience.co.za

13.2 Acid-base reactions ESBQY

The reaction between an acid and a base is known as a **neutralisation** reaction. Often when an acid and base react a salt and water will be formed. We will look at a few examples of acid-base reactions.

1. Hydrochloric acid reacts with sodium hydroxide to form sodium chloride (a salt) and water. Sodium chloride is made up of Na^+ cations from the base (NaOH) and Cl^- anions from the acid (HCl).
$$HCl\ (aq) + NaOH\ (aq) \rightarrow H_2O\ (l) + NaCl\ (aq)$$

FACT
Bee stings are acidic and have a pH between 5 and 5,5. They can be soothed by using substances such as bicarbonate of soda and milk of magnesia. Both bases help to neutralise the acidic bee sting and relieve some of the itchiness!

2. Hydrogen bromide reacts with potassium hydroxide to form potassium bromide (a salt) and water. Potassium bromide is made up of K^+ cations from the base (KOH) and Br^- anions from the acid (HBr).

$$HBr\ (aq) + KOH\ (aq) \rightarrow H_2O\ (l) + KBr\ (aq)$$

3. Hydrochloric acid reacts with ammonia to form ammonium chloride (a salt). Ammonium chloride is made up of NH_4^+ cations from the base (NH_3) and Cl^- anions from the acid (HCl).

$$HCl\ (aq) + NH_3(aq) \rightarrow NH_4Cl\ (aq)$$

You should notice that in the first two examples, the base contained OH^- ions, and therefore the products were a *salt* and *water*. NaCl (table salt) and KBr are both salts. In the third example, NH_3 also acts as a base, despite not having OH^- ions. A salt is still formed as the only product, but no water is produced.

It is important to realise how useful these neutralisation reactions are. Below are some examples:

- **Domestic uses**

  Calcium oxide (CaO) is a base (all metal oxides are bases) that is put on soil that is too acidic. Powdered limestone ($CaCO_3$) can also be used but its action is much slower and less effective. These substances can also be used on a larger scale in farming and in rivers.

  Limestone (white stone or calcium carbonate) is used in pit latrines (or long drops). The limestone is a base that helps to neutralise the acidic waste.

- **Biological uses**

  Acids in the stomach (e.g. hydrochloric acid) play an important role in helping to digest food. However, when a person has a stomach ulcer, or when there is too much acid in the stomach, these acids can cause a lot of pain. **Antacids** are taken to neutralise the acids so that they don't burn as much. Antacids are bases which neutralise the acid. Examples of antacids are aluminium hydroxide, magnesium hydroxide ("milk of magnesia") and sodium bicarbonate ("bicarbonate of soda"). Antacids can also be used to relieve heartburn.

- **Industrial uses**

  Basic calcium hydroxide (limewater) can be used to absorb harmful acidic SO_2 gas that is released from power stations and from the burning of fossil fuels.

General experiment: Acid-base reactions

Aim:

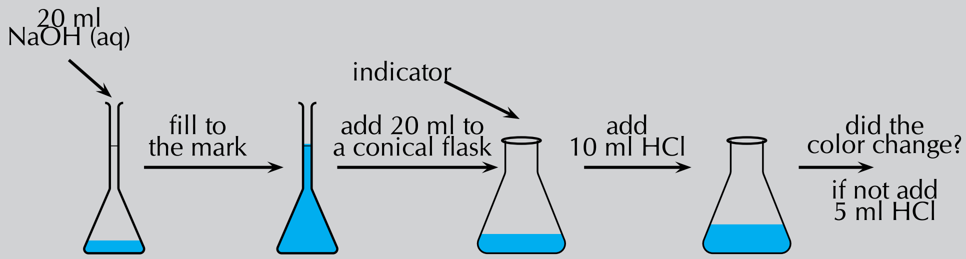
To investigate acid-base reactions.

Apparatus and materials:

- Volumetric flask
- conical flasks

- sodium hydroxide solution
- hydrochloric acid solution
- pipette
- indicator

Method:



1. Use the pipette to add 20 ml of the sodium hydroxide solution to a volumetric flask. Fill up to the mark with water and shake well.
2. Measure 20 ml of the sodium hydroxide solution into a conical flask. Add a few drops of indicator.
3. Slowly add 10 ml of hydrochloric acid. If there is a colour change stop. If not add another 5 ml. Continue adding 5 ml increments until you notice a colour change.

Observations:

The solution changes colour after a set amount of hydrochloric acid is added.

In the above experiment you used an indicator to see when the acid had neutralised the base. Indicators are chemical compounds that change colour depending on whether they are in an acid or in a base.

Informal experiment: Indicators

Aim:

To determine which plants and foods can act as indicators.

Apparatus and materials:

- Possible indicators: Red cabbage, beetroot, berries (e.g. mulberries), curry powder, red grapes, onions, tea (rooibos or regular), baking powder, vanilla essence
- acids (e.g. vinegar, hydrochloric acid), bases (e.g. ammonia (in many household cleaners)) to test
- Beakers

Method:

1. Take a small amount of your first possible indicator (do not use the onions, vanilla essence and baking powder). Boil the substance up until the water has changed colour.
2. Filter the resulting solution into a beaker being careful not to get any plant matter into the beaker. (You can also pour the water through a colander or sieve.)
3. Pour half the resulting coloured solution into a second beaker.
4. Place one beaker onto an A4 sheet of paper labelled "acids". Place the other beaker onto a sheet of paper labelled "bases".
5. Repeat with all your other possible indicators (except the onions, vanilla essence and baking powder).
6. Into all the beakers on the acid sheet carefully pour 5 ml of acid. Record your observations.
7. Into all the beakers on the base sheet carefully pour 5 ml of base. Record your observations.

   If you have more than one acid or base then you will need to repeat the above steps to get fresh indicator samples for your second acid or base. Or you can use less of the resulting coloured solution for each acid and base you want to test.
8. Observe the smell of the onions and vanilla essence. Place a small piece of onion into a beaker. This is for testing with the acid. Pour 5 ml of acid. Wave your hand over the top of the beaker to blow the air towards your nose. What do you notice about the smell of the onions? Repeat with the vanilla essence.
9. Place a small piece of onion into a beaker. This is for testing with the base. Pour 5 ml of base. Wave your hand over the top of the beaker to blow the air towards your nose. What do you notice about the smell of the onions? Repeat with the vanilla essence.
10. Finally place a teaspoon of baking powder into a beaker. Carefully pour 5 ml of acid into the beaker. Record your observations. Repeat using the base.

Observations:

Substance	Colour	Results with acid	Results with base
Red cabbage			
Beetroot			
Berries			
Curry powder			
Tea			
Red grapes			
Onions			
Vanilla essence			
Baking powder			

You should note that some of the substances change colour in the presence of either an acid or a base. The baking powder fizzes when it is in the acid solution, but no reaction is noted when it is in the base solution. Vanilla essence and onions should lose their characteristic smell when in the base.

We will now look at three specific types of acid-base reactions. In each of these types of acid-base reaction the acid remains the same but the kind of base changes. We will look at what kind of products are produced when acids react with each of these bases and what the general reaction looks like.

FACT
Vanilla and onions are known as olfactory (smell) indicators. Olfactory indicators lose their characteristic smell when mixed with acids or bases.

Acid and metal hydroxide ESBQZ

When an acid reacts with a metal hydroxide a *salt and water* are formed. We have already briefly explained this. Some examples are:

- $\text{HCl (aq)} + \text{NaOH (aq)} \rightarrow \text{H}_2\text{O (l)} + \text{NaCl (aq)}$
- $\text{2HBr (aq)} + \text{Mg(OH)}_2\text{(aq)} \rightarrow \text{2H}_2\text{O (l)} + \text{MgBr}_2\text{(aq)}$
- $\text{3HCl (aq)} + \text{Al(OH)}_3\text{(aq)} \rightarrow \text{3H}_2\text{O (l)} + \text{AlCl}_3\text{(aq)}$

We can write a general equation for this type of reaction:

$$n\text{H}^+\text{(aq)} + \text{M(OH)}_n\text{(aq)} \rightarrow n\text{H}_2\text{O (l)} + \text{M}^{n+}\text{(aq)}$$

Where n is the group number of the metal and M is the metal.

Exercise 13 – 3:

1. Write a balanced equation for the reaction between HNO_3 and KOH.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 245V

www.everythingscience.co.za **m.everythingscience.co.za**

Acid and metal oxide ESBR2

When an acid reacts with a metal oxide a *salt and water* are also formed. Some examples are:

- $\text{2HCl (aq)} + \text{Na}_2\text{O (aq)} \rightarrow \text{H}_2\text{O (l)} + \text{2NaCl}$
- $\text{2HBr (aq)} + \text{MgO} \rightarrow \text{H}_2\text{O (l)} + \text{MgBr}_2\text{(aq)}$
- $\text{6HCl (aq)} + \text{Al}_2\text{O}_3\text{(aq)} \rightarrow \text{3H}_2\text{O (l)} + \text{2AlCl}_3\text{(aq)}$

We can write a general equation for the reaction of a metal oxide with an acid:

$$2y\text{H}^{+}(\text{aq}) + \text{M}_x\text{O}_y(\text{aq}) \rightarrow y\text{H}_2\text{O}\ (\text{l}) + x\text{M}^{n+}(\text{aq})$$

Where n is the group number of the metal. The x and y represent the ratio in which the metal combines with the oxide and depends on the valency of the metal.

Exercise 13 – 4:

1. Write a balanced equation for the reaction between HBr and K_2O.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 245W

www.everythingscience.co.za m.everythingscience.co.za

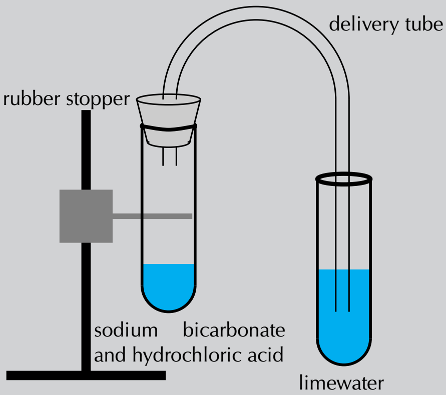
Acid and a metal carbonate ESBR3

General experiment: The reaction of acids with carbonates

Apparatus and materials:

- Small amounts of baking powder (sodium bicarbonate)
- hydrochloric acid (dilute) and vinegar
- retort stand
- two test tubes
- one rubber stopper for the test tube
- a delivery tube
- lime water (calcium hydroxide in water)

The experiment should be set up as shown below.



Method:

1. Carefully stick the delivery tube through the rubber stopper.
2. Pour limewater into one of the test tubes.
3. Carefully pour a small amount of hydrochloric acid into the other test tube.
4. Add a small amount of sodium carbonate to the acid and seal the test tube with the rubber stopper. Place the other end of the delivery tube into the test tube containing the lime water.
5. Observe what happens to the colour of the limewater.
6. Repeat the above steps, this time using vinegar.

Observations:

The clear lime water turns milky meaning that carbon dioxide has been produced. You may not see this for the hydrochloric acid as the reaction may happen to fast.

When an acid reacts with a metal carbonate a *salt, carbon dioxide and water* are formed. Look at the following examples:

- Nitric acid reacts with sodium carbonate to form sodium nitrate, carbon dioxide and water.

$$2HNO_3(aq) + Na_2CO_3(aq) \rightarrow 2NaNO_3(aq) + CO_2(g) + H_2O\ (l)$$

- Sulfuric acid reacts with calcium carbonate to form calcium sulfate, carbon dioxide and water.

$$H_2SO_4(aq) + CaCO_3(aq) \rightarrow CaSO_4(s) + CO_2(g) + H_2O\ (l)$$

- Hydrochloric acid reacts with calcium carbonate to form calcium chloride, carbon dioxide and water.

$$2HCl\ (aq) + CaCO_3(s) \rightarrow CaCl_2(aq) + CO_2(g) + H_2O\ (l)$$

Exercise 13 – 5:

1. Write a balanced equation for the reaction between HCl and K_2CO_3.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 245X

www.everythingscience.co.za m.everythingscience.co.za

Using what we have learnt about acids and bases we can now look at preparing some salts.

General experiment: Making salts

Aim:

To make some salts using acid-base reactions.

Materials:

- hydrochloric acid (1 $mol \cdot dm^{-3}$), sulfuric acid (dilute), sodium hydroxide, copper(II) oxide, calcium carbonate
- beakers, mass meter, funnels, filter paper, bunsen burner, measuring cylinders

Method:

WARNING!

Wear gloves and safety glasses when working with sulfuric acid. Work in a well ventilated room.

Part 1

1. Measure out 20 ml of hydrochloric acid into a beaker.
2. Measure out 20 ml of sodium hydroxide and carefully add this to the beaker containing hydrochloric acid.
3. Gently heat the resulting solution until all the water has evaporated. You should have a white powder left.

Part 2

1. Carefully add 25 ml of sulfuric acid to a clean beaker.
2. Add about a small amount (about 0,5 g) of copper(II) oxide to the beaker containing sulfuric acid. Stir the solution.
3. Once all the copper(II) oxide has dissolved, add another small amount of copper(II) oxide. Repeat until no more solid dissolves and there is a small amount of undissolved solid.
4. Filter this solution and discard the filter paper.
5. Gently heat the resulting liquid. You should get a small amount of solid.

Part 3

1. Measure out 20 ml of hydrochloric acid into a new beaker.
2. Add about a small amount (about 0,5 g) of calcium carbonate to the beaker containing hydrochloric acid. Stir the solution.
3. Once all the calcium carbonate has dissolved, add another small amount of calcium carbonate. Repeat until no more solid dissolves and there is a small amount of undissolved solid.
4. Filter this solution and discard the filter paper.
5. Gently heat the resulting liquid. You should get a small amount of solid.

Observations:

In the first reaction (hydrochloric acid with sodium hydroxide) the resulting solution was clear. When this solution was heated a small amount of white powder was noted. This powder is sodium chloride.

In the second reaction (sulfuric acid with copper(II) oxide) the resulting solution was blue in colour. When this solution was heated a small amount of white powder was noted. This powder is copper sulfate.

In the third reaction (hydrochloric acid with calcium carbonate) the resulting solution was clear. When this solution was heated a small amount of white powder was noted. This powder is calcium sulfate.

Try write reaction equations for the three reactions above.

Conclusion:

We used acid-base reactions to produce different salts.

TIP
You can remember this by using OiLRiG: Oxidation is Loss Reduction is Gain.

Exercise 13 – 6: Acids and bases

For each of the following reactants state what type of acid-base reaction the pair of reactants undergoes and write the balanced reaction equation.

1. HNO_3 and $Ca(OH)_2$
2. HCl and BeO
3. HI and K_2CO_3
4. H_3PO_4 and KOH
5. HCl and $MgCO_3$
6. HNO_3 and Al_2O_3

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 245Y 2. 245Z 3. 2462 4. 2463 5. 2464 6. 2465

www.everythingscience.co.za m.everythingscience.co.za

13.3 Redox reactions ESBR4

If you have seen a piece of rusty metal then you have seen the end result of a redox reaction (iron and oxygen forming iron oxide). Redox reactions are also used in electrochemistry and in biological reactions.

When some reactions occur, an exchange of electrons takes place. It is this exchange of electrons that leads to the change in charge that we noted in grade 10 (chapter 18, reactions in aqueous solution). When an atom gains electrons it becomes more negative and when it loses electrons it becomes more positive.

Oxidation is the *loss* of electrons from an atom, while **reduction** is the *gain* of electrons by an atom. In a reaction these two processes occur together so that one element or compound gains electrons while the other element or compound loses electrons. This is why we call this a redox reaction. It is a short way of saying reduction-oxidation reaction!

DEFINITION: *Oxidation*

Oxidation is the *loss* of electrons by a molecule, atom or ion.

DEFINITION: *Reduction*

Reduction is the *gain* of electrons by a molecule, atom or ion.

Before we look at redox reactions we need to first learn how to tell if a reaction is a redox reaction. In grade 10 you learnt that a redox reaction involves a change in the charge on an atom. Now we will look at why this change in charge occurs.

Oxidation numbers ESBR5

By giving elements an oxidation number, it is possible to keep track of whether that element is losing or gaining electrons during a chemical reaction. The loss of electrons in one part of the reaction must be balanced by a gain of electrons in another part of the reaction.

TIP
You will notice that some elements always have the same oxidation number while other elements can change oxidation numbers depending on the compound they are in.

DEFINITION: *Oxidation number*

Oxidation number is the charge an atom would have if it was in a compound composed of ions.

There are a number of rules that you need to know about oxidation numbers, and these are listed below.

1. A molecule consisting of only one element always has an oxidation number of zero, since it is neutral.

   For example the oxidation number of hydrogen in H_2 is 0. The oxidation number of bromine in Br_2 is also 0.

2. Monatomic ions (ions with only one element or type of atom) have an oxidation number that is equal to the charge on the ion.

   For example, the chloride ion Cl^- has an oxidation number of -1, and the magnesium ion Mg^{2+} has an oxidation number of $+2$.

3. In a molecule or compound, the sum of the oxidation numbers for each element in the molecule or compound will be zero.

   For example the sum of the oxidation numbers for the elements in water will be 0.

4. In a polyatomic ion the sum of the oxidation numbers is equal to the charge.

   For example the sum of the oxidation numbers for the elements in the sulfate ion (SO_4^{2-}) will be -2.

5. An oxygen atom usually has an oxidation number of -2. One exception is in peroxides (e.g. hydrogen peroxide) when oxygen has an oxidation number of -1.

   For example oxygen in water will have an oxidation number of -2 while in hydrogen peroxide (H_2O_2) it will have an oxidation number of -1.

6. The oxidation number of hydrogen is often $+1$. One exception is in the metal hydrides where the oxidation number is -1.

   For example the oxidation number of the hydrogen atom in water is $+1$, while the oxidation number of hydrogen in lithium hydride (LiH) is -1.

7. The oxidation number of fluorine is -1.

Worked example 1: Oxidation numbers

QUESTION

Give the oxidation number of sulfur in a sulfate (SO_4^{2-}) ion

SOLUTION

Step 1: Determine the oxidation number for each atom

Oxygen will have an oxidation number of -2. (Rule 5, this is not a peroxide.) The oxidation number of sulfur at this stage is uncertain since sulfur does not have a set oxidation number.

Step 2: Determine the oxidation number of sulfur by using the fact that the oxidation numbers of the atoms must add up to the charge on the compound

In the polyatomic SO_4^{2-} ion, the sum of the oxidation numbers must be -2 (rule 4).

Let the oxidation number of sulfur be x. We know that oxygen has an oxidation number of -2 and since there are four oxygen atoms in the sulfate ion, then the sum of the oxidation numbers of these four oxygen atoms is -8.

Putting this together gives:

$$\begin{aligned} x + (-8) &= -2 \\ x &= -2 + 8 \\ &= +6 \end{aligned}$$

So the oxidation number of sulfur is $+6$.

Step 3: Write down the final answer

In the sulfate ion, the oxidation number of sulfur is $+6$.

Worked example 2: Oxidation numbers

QUESTION

Give the oxidation number of both elements in ammonia (NH_3).

SOLUTION

Step 1: Determine the oxidation number for each atom

Hydrogen will have an oxidation number of $+1$ (rule 6, ammonia is not a metal hydride). At this stage we do not know the oxidation number for nitrogen.

Step 2: Determine the oxidation number of nitrogen by using the fact that the oxidation numbers of the atoms must add up to the charge on the compound

In the compound NH_3, the sum of the oxidation numbers must be 0 (rule 3).

Let the oxidation number of nitrogen be x. We know that hydrogen has an oxidation number of $+1$ and since there are three hydrogen atoms in the ammonia molecule, then the sum of the oxidation numbers of these three hydrogen atoms is $+3$.

Putting this together gives:

$$\begin{aligned} x + (+3) &= 0 \\ &= -3 \end{aligned}$$

So the oxidation number of nitrogen is -3.

Step 3: Write the final answer

Hydrogen has an oxidation number of $+1$ and nitrogen has an oxidation number of -3.

Worked example 3: Oxidation numbers

QUESTION

Give the oxidation numbers for all the atoms in sodium chloride (NaCl).

SOLUTION

Step 1: Determine the oxidation number for each atom in the compound

This is an ionic compound composed of Na^+ and Cl^- ions. Using rule 2 the oxidation number for the sodium ion is $+1$ and for the chlorine ion it is -1.

This then gives us a sum of 0 for the compound.

Step 2: Write the final answer

The oxidation numbers for sodium is $+1$ and for chlorine it is -1.

Exercise 13 – 7: Oxidation numbers

1. Give the oxidation numbers for each element in the following chemical compounds:

   a) MgF_2 b) $CaCl_2$ c) CH_4 d) $MgSO_4$

2. Compare the oxidation numbers of:

TIP

The word species is used in chemistry to indicate either a compound, a molecule, an ion, an atom or an element.

a) nitrogen in: NO_2 and NO

b) carbon in: CO_2 and CO

c) chromium in: $Cr_2O_7^{2-}$ and CrO_4^-

d) oxygen in: H_2O and H_2O_2

e) hydrogen in: NaH and H_2O

3. Give the oxidation numbers for each of the elements in all the compounds. State if there is any difference between the oxidation number of the element in the reactant and the element in the product.

    a) $\text{C (s)} + \text{O}_2(\text{g}) \rightarrow \text{CO}_2(\text{g})$

    b) $\text{N}_2(\text{g}) + 3\text{H}_2(\text{g}) \rightarrow 2\text{NH}_3(\text{g})$

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 2466 1b. 2467 1c. 2468 1d. 2469 2a. 246B 2b. 246C
2c. 246D 2d. 246F 2e. 246G 3a. 246H 3b. 246J

www.everythingscience.co.za m.everythingscience.co.za

Redox reactions ESBR6

Now that we know how to determine the oxidation number of a compound, we will go on to look at how to use this knowledge in reactions.

Oxidation and reduction

By looking at how the oxidation number of an element changes during a reaction, we can easily see whether that element is being **oxidised** (lost electrons) or **reduced** (gained electrons).

If the oxidation number of a species becomes more positive, the species has been **oxidised** and if the oxidation number of a species becomes more negative, the species has been **reduced**.

We will use the reaction between magnesium and chlorine as an example.

The chemical equation for this reaction is:

$$\text{Mg (aq)} + \text{Cl}_2(\text{aq}) \rightarrow \text{MgCl}_2(\text{aq})$$

As a *reactant*, magnesium has an oxidation number of zero, but as part of the *product* magnesium chloride, the element has an oxidation number of $+2$. Magnesium has *lost* two electrons and has therefore been *oxidised* (note how the oxidation number becomes more positive). This can be written as a **half-reaction**. The half-reaction for this change is:

$$\text{Mg} \rightarrow \text{Mg}^{2+} + 2e^-$$

As a *reactant*, chlorine has an oxidation number of zero, but as part of the *product* magnesium chloride, the element has an oxidation number of -1. Each chlorine atom has *gained* an electron and the element has therefore been *reduced* (note how the oxidation number becomes more negative). The half-reaction for this change is:

$$\mathrm{Cl_2} + 2e^- \rightarrow 2\mathrm{Cl}^-$$

DEFINITION: *Half-reaction*

A half reaction is either the oxidation or reduction reaction part of a redox reaction.

In the two half-reactions for a redox reaction the number of electrons donated is exactly the same as the number of electrons accepted. We will use this to help us balance redox reactions.

Two further terms that we use in redox reactions and that you may see are reducing agents and oxidising agents.

An element that is **oxidised** is called a **reducing agent**, while an element that is **reduced** is called an **oxidising agent**.

You can remember this by thinking of the fact that when a compound is oxidised, it causes another compound to be reduced (the electrons have to go somewhere and they go to the compound being reduced).

Redox reactions

DEFINITION: *Redox reaction*

A redox reaction is one involving oxidation and reduction, where there is always a change in the oxidation numbers of the elements involved. Redox reactions involve the transfer of electrons from one compound to another.

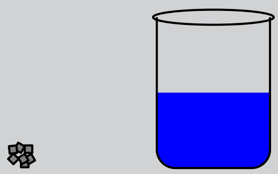
Informal experiment: Redox reaction - displacement reaction

Aim:

To investigate the redox reaction between copper sulfate and zinc.

Materials:

- A few granules of zinc
- 15 ml copper (II) sulfate solution (blue colour)
- glass beaker



Method:

Add the zinc granules to the copper sulfate solution and observe what happens. What happens to the zinc granules? What happens to the colour of the solution?

Results:

- Zinc becomes covered in a layer that looks like copper.
- The blue copper sulfate solution becomes clearer.

Cu^{2+} ions from the $CuSO_4$ solution are **reduced** to form copper metal. This is what you saw on the zinc crystals. The reduction of the copper ions (in other words, their removal from the copper sulfate solution), also explains the change in colour of the solution (copper ions in solution are blue). The equation for this reaction is:

$$\mathrm{Cu}^{2+}(\mathrm{aq}) + 2e^{-} \rightarrow \mathrm{Cu}\ (\mathrm{s})$$

Zinc is **oxidised** to form Zn^{2+} ions which are clear in the solution. The equation for this reaction is:

$$\mathrm{Zn}\ (\mathrm{s}) \rightarrow \mathrm{Zn}^{2+}(\mathrm{aq}) + 2e^{-}$$

The overall reaction is:

$$\mathrm{Cu}^{2+}(\mathrm{aq}) + \mathrm{Zn}\ (\mathrm{s}) \rightarrow \mathrm{Cu}\ (\mathrm{s}) + \mathrm{Zn}^{2+}(\mathrm{aq})$$

Conclusion:

A redox reaction has taken place. Cu^{2+} ions are reduced and the zinc is oxidised. This is a displacement reaction as the zinc replace the copper ions to form zinc sulfate.

Informal experiment: Redox reaction - synthesis reaction

Aim:

To investigate the redox reaction that occurs when magnesium is burnt in air.

Materials:

A strip of magnesium; bunsen burner; tongs; glass beaker.

Method:

WARNING!

Do not look directly at the flame.

1. Light the bunsen burner and use a pair of tongs to hold the magnesium ribbon in the flame.
2. Hold the lit piece of magnesium over a beaker. What do you observe?

Results:

The magnesium burns with a bright white flame. When the magnesium is held over a beaker, a fine powder is observed in the beaker. This is magnesium oxide.

The overall reaction is:

$$2\text{Mg (s)} + \text{O}_2\text{(g)} \rightarrow 2\text{MgO (s)}$$

Conclusion:

A redox reaction has taken place. Magnesium is oxidised and the oxygen is reduced. This is a synthesis reaction as we have made magnesium oxide from magnesium and oxygen.

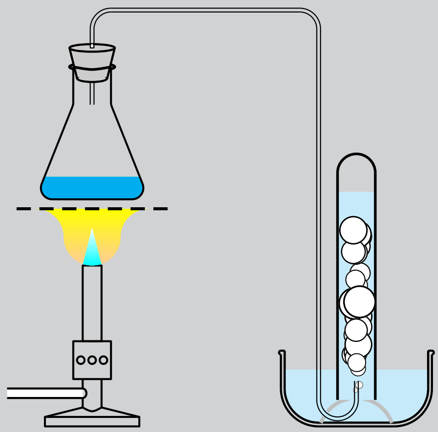
Informal experiment: Redox reaction - decomposition reaction

Aim:

To investigate the decomposition of hydrogen peroxide.

Materials:

Dilute hydrogen peroxide (about 3%); manganese dioxide; test tubes; a water bowl; stopper and delivery tube, Bunsen burner



WARNING!

Hydrogen peroxide can cause chemical burns. Work carefully with it.

Method:

1. Put a small amount (about 5 ml) of hydrogen peroxide in a test tube.
2. Set up the apparatus as shown above.
3. Very carefully add a small amount (about 0,5 g) of manganese dioxide to the test tube containing hydrogen peroxide.

Results:

You should observe a gas bubbling up into the second test tube. This reaction happens quite rapidly.

The overall reaction is:

$$2H_2O_2(aq) \rightarrow 2H_2O\ (l) + O_2(g)$$

Conclusion:

A redox reaction has taken place. H_2O_2 is both oxidised and reduced in this decomposition reaction.

See video: 246K at www.everythingscience.co.za

Using what you have learnt about oxidation numbers and redox reactions we can balance redox reactions in the same way that you have learnt to balance other reactions. The following worked examples will show you how.

Worked example 4: Balancing redox reactions

QUESTION

Balance the following redox reaction:

$$Fe^{2+}(aq) + Cl_2(aq) \rightarrow Fe^{3+}(aq) + Cl^-(aq)$$

SOLUTION

Step 1: Write a reaction for each compound

$$Fe^{2+} \rightarrow Fe^{3+}$$

$$Cl_2 \rightarrow Cl^-$$

Step 2: Balance the atoms on either side of the arrow

We check that the atoms on both sides of the arrow are balanced:

$$Fe^{2+} \rightarrow Fe^{3+}$$

$$Cl_2 \rightarrow 2Cl^-$$

Step 3: Add electrons to balance the charges

We now add electrons to each reaction so that the charges balance.

We add the electrons to the side with the greater positive charge.

$$Fe^{2+} \rightarrow Fe^{3+} + e^-$$

$$Cl_2 + 2e^- \rightarrow 2Cl^-$$

Step 4: Balance the number of electrons

We now make sure that the number of electrons in both reactions is the same.

The reaction for iron has one electron, while the reaction for chlorine has two electrons. So we must multiply the reaction for iron by 2 to ensure that the charges balance.

$$2Fe^{2+} \rightarrow 2Fe^{3+} + 2e^-$$

$$Cl_2 + 2e^- \rightarrow 2Cl^-$$

We now have the two half-reactions for this redox reaction.

The reaction for iron is the oxidation half-reaction as iron became more positive (lost electrons). The reaction for chlorine is the reduction half-reaction as chlorine has become more negative (gained electrons).

Step 5: Combine the two half-reactions

$$\begin{array}{lrcl} & 2\text{Fe}^{2+} & \rightarrow & 2\text{Fe}^{3+} + 2e^- \\ + & \text{Cl}_2 + 2e^- & \rightarrow & 2\text{Cl}^- \\ \hline & 2\text{Fe}^{2+} + \text{Cl}_2 + 2e^- & \rightarrow & 2\text{Fe}^{3+} + 2\text{Cl}^- + 2e^- \end{array}$$

Step 6: Write the final answer

Cancelling out the electrons gives:

$$2\text{Fe}^{2+}(\text{aq}) + \text{Cl}_2(\text{aq}) \rightarrow 2\text{Fe}^{3+}(\text{aq}) + 2\text{Cl}^-(\text{aq})$$

Note that we leave the co-efficients in front of the iron ions since the charge on the left hand side has to be the same as the charge on the right hand side in a redox reaction.

In the example above we did not need to know if the reaction was taking place in an acidic or basic medium (solution). However if there is hydrogen or oxygen in the reactants and not in the products (or if there is hydrogen or oxygen in the products but not in the reactants) then we need to know what medium the reaction is taking place in. This will help us to balance the redox reaction.

If a redox reaction takes place in an **acidic** medium then we can add water molecules to either side of the reaction equation to balance the number of oxygen atoms. We can also add hydrogen ions to balance the number of hydrogen atoms. We do this because we are writing the net ionic equation (showing only the ions involved and often only the ions containing the elements that change oxidation number) for redox reactions and not the net reaction equation (showing all the compounds that are involved in the reaction). If a Bronsted acid is dissolved in water then there will be free hydrogen ions.

If a redox reaction takes place in an **basic** medium then we can add water molecules to either side of the reaction equation to balance the number of oxygen atoms. We can also add hydroxide ions (OH^-) to balance the number of hydrogen atoms. We do this because we are writing the net ionic equation (showing only the ions involved and often only the ions containing the elements that change oxidation number) for redox reactions and not the net reaction equation (showing all the compounds that are involved in the reaction). If a Bronsted base is dissolved in water then there will be free hydroxide ions.

Worked example 5: Balancing redox reactions

QUESTION

Balance the following redox reaction:

$$Cr_2O_7^{2-}(aq) + H_2S\ (aq) \rightarrow Cr^{3+}(aq) + S\ (s)$$

The reaction takes place in an acidic medium.

SOLUTION

Step 1: Write a reaction for each compound

$$Cr_2O_7^{2-} \rightarrow Cr^{3+}$$
$$H_2S \rightarrow S$$

Step 2: Balance the atoms on either side of the arrow

We check that the atoms on both sides of the arrow are balanced.

In the first reaction we have 2 chromium atoms and 7 oxygen atoms on the left hand side. On the right hand side we have 1 chromium atom and no oxygen atoms. Since we are in an acidic medium we can add water to the right hand side to balance the number of oxygen atoms. We also multiply the chromium by 2 on the right hand side to make the number of chromium atoms balance.

$$Cr_2O_7^{2-} \rightarrow 2Cr^{3+} + 7H_2O$$

Now we have hydrogen atoms on the right hand side, but not on the left hand side so we must add 14 hydrogen ions to the left hand side (we can do this because the reaction is in an acidic medium):

$$Cr_2O_7^{2-} + 14H^+ \rightarrow 2Cr^{3+} + 7H_2O$$

We do not use water to balance the hydrogens as this will make the number of oxygen atoms unbalanced.

For the second part of the reaction we need to add 2 hydrogen ions to the right hand side to balance the number of hydrogens:

$$H_2S \rightarrow S + 2H^+$$

Step 3: Add electrons to balance the charges

We now add electrons to each reaction so that the charges balance.

We add the electrons to the side with the greater positive charge.

$$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$$

$$H_2S \rightarrow S + 2H^+ + 2e^-$$

Step 4: Balance the number of electrons

We now make sure that the number of electrons in both reactions is the same.

The reaction for chromium has 6 electrons, while the reaction for sulfur has 2 electrons. So we must multiply the reaction for sulfur by 3 to ensure that the charges balances.

$$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$$

$$3H_2S \rightarrow 3S + 6H^+ + 6e^-$$

We now have the two half-reactions for this redox reaction.

The reaction involving sulfur is the oxidation half-reaction as sulfur became more positive (lost electrons). The reaction for chromium is the reduction half-reaction as chromium has become more negative (gained electrons).

Step 5: Combine the two half-reactions

We combine the two half-reactions:

$$\begin{array}{lrcl} & Cr_2O_7^{2-} + 14H^+ + 6e^- & \rightarrow & 2Cr^{3+} + 7H_2O \\ + & 3H_2S & \rightarrow & 3S + 6H^+ + 6e^- \\ \hline & Cr_2O_7^{2-} + 14H^+ + 6e^- + 3H_2S & \rightarrow & 2Cr^{3+} + 7H_2O + 3S + 6H^+ + 6e^- \end{array}$$

Step 6: Write the final answer

Crossing off the electrons and hydrogen ions gives:

$$Cr_2O_7^{2-}(aq) + 3H_2S\ (aq) + 8H^+(aq) \rightarrow 2Cr^{3+}(aq) + 3S\ (s) + 7H_2O\ (l)$$

In Grade 12, you will go on to look at electrochemical reactions, and the role that electron transfer plays in this type of reaction.

Exercise 13 – 8: Redox reactions

1. Consider the following chemical equations:

$$Fe\ (s) \rightarrow Fe^{2+}(aq) + 2e^-$$

$$4H^+(aq) + O_2(g) + 4e^- \rightarrow 2H_2O\ (l)$$

Which one of the following statements is correct?

a) Fe is oxidised and H^+ is reduced
b) Fe is reduced and O_2 is oxidised
c) Fe is oxidised and O_2 is reduced
d) Fe is reduced and H^+ is oxidised

(DoE Grade 11 Paper 2, 2007)

2. Which one of the following reactions is a redox reaction?

a) $HCl\ (aq) + NaOH\ (aq) \rightarrow NaCl\ (aq) + H_2O\ (l)$
b) $AgNO_3(aq) + NaI\ (aq) \rightarrow AgI\ (s) + NaNO_3(aq)$
c) $2FeCl_3(aq) + 2H_2O\ (l) + SO_2(aq) \rightarrow H_2SO_4(aq) + 2HCl\ (aq) + 2FeCl_2(aq)$
d) $BaCl_2(aq) + MgSO_4(aq) \rightarrow MgCl_2(aq) + BaSO_4(s)$

3. Balance the following redox reactions:

a) $Zn\ (s) + Ag^+(aq) \rightarrow Zn^{2+}(aq) + Ag\ (s)$
b) $Cu^{2+}(aq) + Cl^-(aq) \rightarrow Cu\ (s) + Cl_2(g)$
c) $Pb^{2+}(aq) + Br^-(aq) \rightarrow Pb\ (s) + Br_2(aq)$
d) $HCl\ (aq) + MnO_2(s) \rightarrow Cl_2(g) + Mn^{2+}(aq)$
This reaction takes place in an acidic medium.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 246M 2. 246N 3a. 246P 3b. 246Q 3c. 246R 3d. 246S

www.everythingscience.co.za m.everythingscience.co.za

13.4 Chapter summary ESBR7

See presentation: 246T at www.everythingscience.co.za

- There are many different **types of reactions** that can take place. These include acid-base and redox reactions.
- The **Arrhenius** definition of acids and bases defines an acid as a substance that increases the concentration of hydronium ions (H_3O^+) in a solution. A base is a substance that increases the concentration of hydroxide ions (OH^-) in a solution. However this description only applies to substances that are in water.
- The **Bronsted-Lowry** model defines acids and bases in terms of their ability to donate or accept protons.
- A Bronsted-Lowry **acid** is a substance that gives away protons (hydrogen cations H^+), and is therefore called a **proton donor**.

- A Bronsted-Lowry **base** is a substance that takes up protons (hydrogen cations H^+), and is therefore called a **proton acceptor**.
- An **amphoteric** substance is one that can react as either an acid or base. Water (H_2O) is an example of an amphoteric substance.
- An **amphiprotic** is one that can react as either a proton donor (Bronsted-Lowry acid) or as a proton acceptor (Bronsted-Lowry base). HCO_3^- and HSO_4^- are examples of amphiprotic substances.
- A **conjugate acid-base pair** refers to two compounds in a reaction (one reactant and one product) that transform or change into the other through the loss or gain of a proton.
- The reaction between an acid and a base is a **neutralisation** reaction.
- Acids and bases are used in domestic uses (for example calcium carbonate on acidic soil) in biology (for example in antacids for stomach ulcers) and in industry (for example in absorbing harmful SO_2 gas),.
- Indicators are chemical compounds that change colour depending on whether they are in an acid or in a base.
- When an acid reacts with a metal hydroxide a **salt** and water are formed. The salt is made up of a cation from the base and an anion from the acid. An example of a salt is potassium chloride (KCl), which is the product of the reaction between potassium hydroxide (KOH) and hydrochloric acid (HCl).
- When an acid reacts with a metal oxide a salt and water are formed. An example is the reaction between magnesium oxide (MgO) and hydrochloric acid (HCl).
- When an acid reacts with a metal carbonate a salt, water and carbon dioxide are formed. An example is the reaction between calcium carbonate ($CaCO_3$) and hydrochloric acid (HCl).
- **Oxidation** is the loss of electrons by a molecule, atom or ion.
- **Reduction** is the gain of electrons by a molecule, atom or ion.
- A **redox reaction** is one involving oxidation and reduction, where there is always a change in the oxidation numbers of the elements involved. Redox reactions involve the transfer of electrons from one compound to another.
- An **oxidation number** is the charge an atom would have if it was in a compound composed of ions.
- If the oxidation number of a species becomes more positive, the species has been oxidised and if the oxidation number of a species becomes more negative, the species has been reduced.
- A half reaction is either the oxidation or reduction reaction part of a redox reaction. In the two half-reactions for a redox reaction the number of electrons donated is exactly the same as the number of electrons accepted.
- An element that is **oxidised** is called a **reducing agent**, while an element that is **reduced** is called an **oxidising agent**.

Exercise 13 – 9:

1. Give one word/term for each of the following descriptions:
   a) A chemical reaction during which electrons are transferred.
   b) A substance that takes up protons and is said to be a proton acceptor.
   c) The loss of electrons by a molecule, atom or ion.
   d) A substance that can act as either an acid or as a base.
2. Given the following reaction:

   $$HNO_3(aq) + NH_3(aq) \rightarrow NH_4^+(aq) + NO_3^-(aq)$$

   Which of the following statements is true?
   a) HNO_3 accepts protons and is a Bronsted-Lowry base
   b) HNO_3 accepts protons and is a Bronsted-Lowry acid
   c) NH_3 donates protons and is a Bronsted-Lowry acid
   d) HNO_3 donates protons and is a Bronsted-Lowry acid
3. When chlorine water (Cl_2 dissolved in water) is added to a solution of potassium bromide, bromine is produced. Which one of the following statements concerning this reaction is correct?
   a) Br is oxidised
   b) Cl_2 is oxidised
   c) Br^- is the oxidising agent
   d) Cl^- is the oxidising agent

   (IEB Paper 2, 2005)
4. Given the following reaction:

   $$H_2SO_3(aq) + 2KOH\ (aq) \rightarrow K_2SO_3(aq) + 2H_2O\ (l)$$

   Which substance acts as the acid and which substance acts as the base?
5. Use balanced chemical equations to explain why HSO_4^- is amphoteric.
6. Milk of magnesia is an example of an antacid and is only slightly soluble in water. Milk of magnesia has the chemical formula $Mg(OH)_2$ and is taken as a powder dissolved in water. Write a balanced chemical equation to show how the antacid reacts with hydrochloric acid (the main acid found in the stomach).
7. In an experiment sodium carbonate was used to neutralise a solution of hydrochloric acid. Write a balanced chemical equation for this reaction.
8. Write a balanced chemical equation for phosphoric acid (H_3PO_4) reacting with calcium oxide (CaO).
9. Label the acid-base conjugate pairs in the following equation:

   $$HCO_3^-(aq) + H_2O \rightarrow CO_3^{2-}(aq) + H_3O^+(aq)$$

10. Look at the following reaction:

$$2H_2O_2(l) \rightarrow 2H_2O\ (l) + O_2(g)$$

a) What is the oxidation number of the oxygen atom in each of the following compounds?

i. H_2O_2

ii. H_2O

iii. O_2

b) Does the hydrogen peroxide (H_2O_2) act as an oxidising agent or a reducing agent or both, in the above reaction? Give a reason for your answer.

11. Challenge question: Zinc reacts with aqueous copper sulfate ($CuSO_4(aq)$) to form zinc sulfate ($ZnSO_4(aq)$) and copper ions. What is the oxidation number for each element in the reaction?

12. Balance the following redox reactions:

a) $Pb\ (s) + Ag^{+}(aq) \rightarrow Pb^{2+}(aq) + Ag\ (s)$

b) $Zn^{2+}(aq) + I^{-}(aq) \rightarrow Zn\ (s) + I_2(g)$

c) $Fe^{3+}\ (aq) + NO_2(aq) \rightarrow Fe\ (s) + NO_3^{-}(aq)$
This reaction takes place in an acidic medium.

13. A nickel-cadmium battery is used in various portable devices such as calculators. This battery uses a redox reaction to work. The equation for the reaction is:

$$Cd\ (s) + NiO(OH)\ (s) \rightarrow Cd(OH)_2(s) + Ni(OH)_2(s)$$

This reaction takes place in a basic medium. Balance the equation.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 246V	1b. 246W	1c. 246X	1d. 246Y	2. 246Z	3. 2472
4. 2473	5. 2474	6. 2475	7. 2476	8. 2477	9. 2478
10. 2479	11. 247F	12a. 247G	12b. 247H	12c. 247J	13. 247K

www.everythingscience.co.za m.everythingscience.co.za

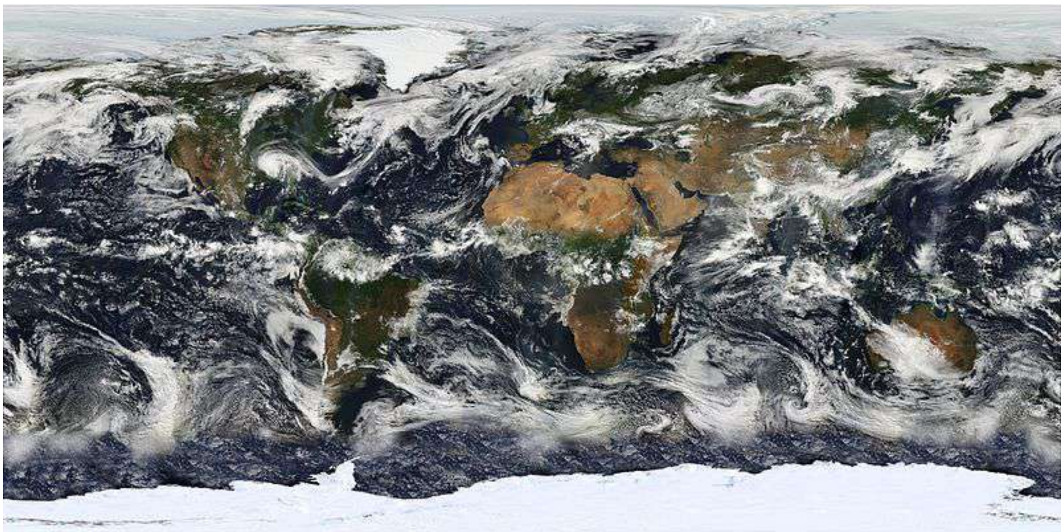
CHAPTER 14

The lithosphere

14.1 Introduction ESBR8

In Grade 10 we studied the elements. We learnt about the different elements and the compounds that could be formed from those elements. But where do all these elements come from? Where did mankind find them and how has he used them?

This chapter will explore the part of the Earth known as the lithosphere. We will find out what the lithosphere is and how the elements are distributed within it.



14.2 The lithosphere ESBR9

If we were to cut the Earth in half we would see that our planet is made up of a number of layers, namely the **core** at the centre (separated into the inner and outer core), the **mantle**, the **upper mantle**, the **crust** and the **atmosphere** (Figure 14.1). The core is made up mostly of iron. The mantle, which lies between the core and the crust, consists of molten rock, called **magma** which moves continuously because of convection currents. The crust is the thin, hard outer layer that "floats" on the magma of the mantle. It is the upper part of the mantle and the crust that make up the **lithosphere** (lith means types of stone and sphere refers to the round shape of the Earth).

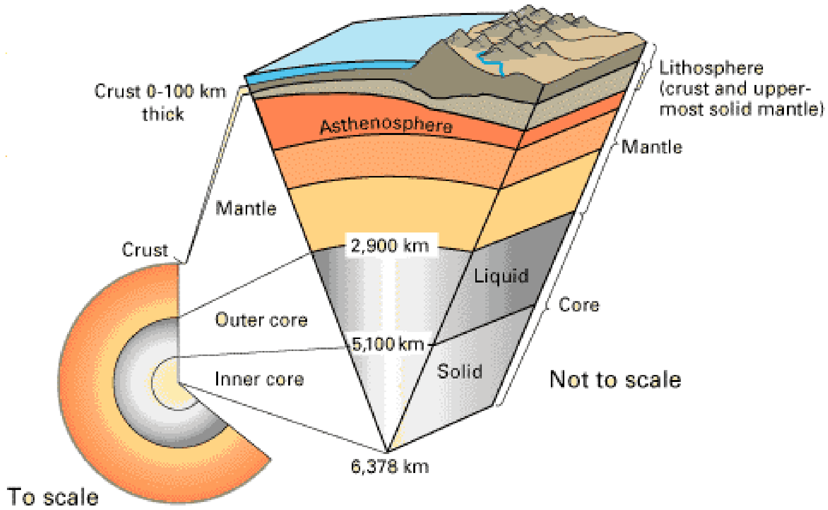


Figure 14.1: A cross-section through the Earth to show its different layers.

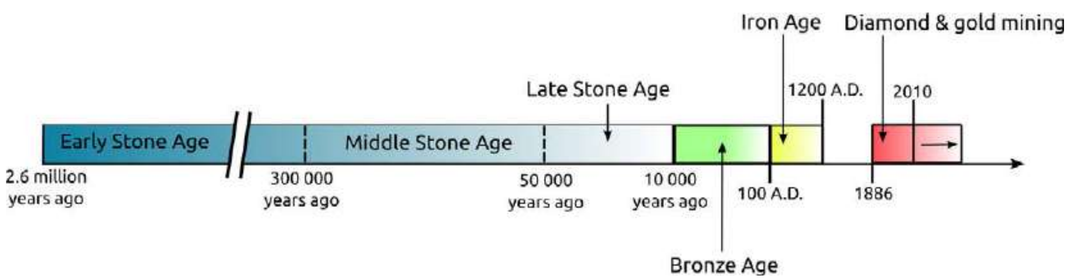
DEFINITION: *Lithosphere*

The lithosphere is the solid outermost shell of our planet. The lithosphere includes the crust and the upper part of the mantle, and is made up of material from both the continents and the oceans on the Earth's surface.

The lithosphere is vital to humans as this is the part of the Earth that we live on and can easily access. This is the part of the Earth that supports us, provides for us and gives us a wealth of materials to use. We are now going to explore how man slowly became aware of the wealth of minerals at his feet and how he has learnt to use these minerals for his benefit.

History of mankind ESBRB

The ancient history of humankind can be divided into several periods: the Stone Age or Palaeolithic period (from approximately 2,6 million years to 10 000 years ago); the Bronze Age or Mesolithic period (from approximately 10 000 B.C. to 100 A.D.); and the Iron Age or Neolithic period (approximately 100 A.D. to 1200 A.D.) In many places in the world these periods overlapped in time and it is important to understand that the above dates are very general approximations.



The Stone Age can be further divided into three periods: the Early Stone Age (2,6 million – 300 000 years ago); the Middle Stone Age (approximately 300 000 – 50 000 years ago); and the Late Stone Age or (approximately 50 000 – 10 000 years ago).

FACT
The largest Iron Age site in South Africa is found at Mapungubwe, in the Limpopo Province. Mapungubwe (inhabited from about 1000 – 1300 A.D.) is thought to be the ancient capital of an African king. The site was rediscovered in 1932 and has proved to be a treasure trove of archaeological evidence for an advanced society. The remains of royal burial sites, houses, tools, art and farming implements have been found. Gold-plated jewellery, sculptures and artifacts (thought to have belonged to members of the royal family) have also been discovered at the site.

Age	Time	Information	Major material	Images
Early stone age	2,6 million - 300 000 years ago	Basic stone tools were used. Mankind mainly used the rocks and stones he found lying around him. Evidence for this has been found in the Sterkfontein caves in the Cradle of Humankind.	Stone	
Middle stone age	300 000 - 50 000 years ago	Mankind learnt to use fire to treat stones before they were made into tools. Stone tools became more refined during this period. Evidence for this has been found at Swartkrans (Cradle of Humankind), Montagu Cave (Klein Karoo), Klasies River Mouth (Tsitsikamma), Stilbaai and Blombos Cave (Southern Cape).	Stone treated with fire	
Late stone age	50 000 - 10 000 years ago	Mankind started turning to other materials that were easily available. He started working with different materials and seeing how these new materials could be combined with stone. Evidence for this has been found in the Melkhoutboom Cave, in the Suurberg Mountains (Eastern Cape).	Stone with other natural materials	
Bronze age	10 000 B.C. - 100 A.D.	The Bronze Age saw the first use of smelted copper and bronze in the manufacture of tools and weapons. Evidence for this has been found in North Africa.	Copper and tin	
Iron age	100 A.D. - 1200 A.D.	Primitive mining for materials occurred during this period. Furnaces were used to heat metal, in order to smelt it into weapons and tools. Evidence for this has been found at Melville Koppies and at Mapungubwe (Limpopo).	Iron and other metals	

Table 14.1: The different ages of civilisation.

After the Iron Age ended, mankind started to explore many more ways to get at the precious metals that they were using. Mining for the minerals started to become more common and man began to look for new sources of the metals and minerals he needed.

Many different cultures in Africa used gold, diamonds and other precious metals for different things. A large number of these artefact's were looted from these people and taken to Europe by the colonists and explorers that came from Europe. Efforts have now begun to return these stolen artifacts to the people they came from, but many museums still hold onto their collections.

When colonists and explorers first began exploring and settling in Africa they started looking at ways to use the natural resources they found.

Although diamonds had long been known to the indigenous people of South Africa, colonists found them on a farm outside Kimberley (Northern Cape) in the 1860's. The original hill on which the diamonds were found was mined extensively by the hands of thousands of hopeful diamond prospectors who rushed to the area. Today the site of that hill is now the Big Hole in Kimberley – one of the largest holes in the world that was dug by hand. In 1888, several small diamond mining companies merged to form De Beers, the company that still dominates the global diamond mining and retail industry today.

Around the same time (1880), the colonists discovered gold reefs just south of present-day Johannesburg. The rush of gold-prospectors to the area resulted in the birth of the city of Johannesburg. A massive labour force was required to dig deep into the bedrock to extract the rock containing the gold. Indeed, huge numbers of mine workers are still employed by gold mines in South Africa today, but (as with diamond mining too), due to technological advancements over the last 150 years, much of their work is now aided by industrial mining equipment.

Many farmers and indigenous people were forced to work in the new diamond and gold mines. These people were taken by force from their homes and put to work on the mines. Many of them died from a result of the poor working conditions and from mining disasters that occurred.

FACT
The deepest mine in the world is currently the TauTona Mine (or Western Deep No. 3 Shaft) outside Carletonville, Gauteng. Owned by AngloGold Ashanti, it is 3,9 km deep and has about 800 km of tunnels. The rock face at that depth reaches temperatures of 60°C and air-conditioners have to be used to cool the air in the mine from 55°C to 28°C.

Group discussion: Minerals and the rights to own them

Many indigenous people share the same central belief that the land and all it produces are for all the people to use equally. The land and what it produces is often vital to the survival of these people.

When the colonists came to Southern Africa, they largely ignored the indigenous people and exploited them for the knowledge they held and the work they could provide. A great number of atrocities were committed against the indigenous people.

Now the indigenous people are rising up and asking that what is rightfully theirs should be returned to them. Organisations (such as the UN working group on indigenous people) have formed to address these issues.

Shortly after De Beers formed they claimed the right to all the diamonds in the area and anyone found with diamonds could be killed for taking the diamonds illegally. Other large mining companies have tried to claim the rights to the minerals that they produce.

In groups or as a class discuss whether or not a few select people should hold the rights to the land and the minerals in it. Who owns the minerals? Should big corporates hold the rights? Or should it belong to all the people?

Elements and minerals ESBRC

Now that we know what mankind used to make tools and shelter from, we will take a look at why he chose the materials he did. In the early ages (early and middle stone age) mankind used whatever was to hand and easy to get to. Later on (from the late stone age to the present day) he started wondering how these different materials could be improved. So what exactly is in the lithosphere and how did mankind use this knowledge?

The crust is made up of about 80 elements, which occur in over 2000 different compounds and minerals. However, most of the mass of the material in the crust is made up of only 8 of these elements. These are oxygen (O), silicon (Si), aluminium (Al), iron (Fe), calcium (Ca), sodium (Na), potassium (K) and magnesium (Mg). These elements are seldom found in their pure form, but are usually part of other more complex **minerals**. A mineral is a compound that is formed through geological processes, which give it a particular structure. A mineral could be a pure element, but more often minerals are made up of many different elements combined. *Quartz* is just one example. It is a mineral that is made up of silicon and oxygen. Some more examples are shown in Table 14.2.

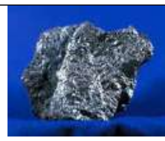
Element	**Most common mineral**	**Chemistry**	
Gold	Calaverite or pure element	Au (pure element) or $AuTe_2$ (Calaverite, a gold mineral)	
Iron	Hematite	Fe_2O_3 (iron oxide)	
Copper	Pure element or chalcocite	Cu (pure element) or Cu_2S (copper sulfide)	
Carbon	Diamond, graphite, coal	C (pure element)	
Platinum	Pure element, combined with other elements	Pt (pure element)	
Zinc	Sphalerite	ZnS	
Manganese	Manganese dioxide	MnO_2	
Chromium (chrome)	Chromite	$FeCr_2O_4$	

Table 14.2: Table showing examples of minerals and their chemistry.

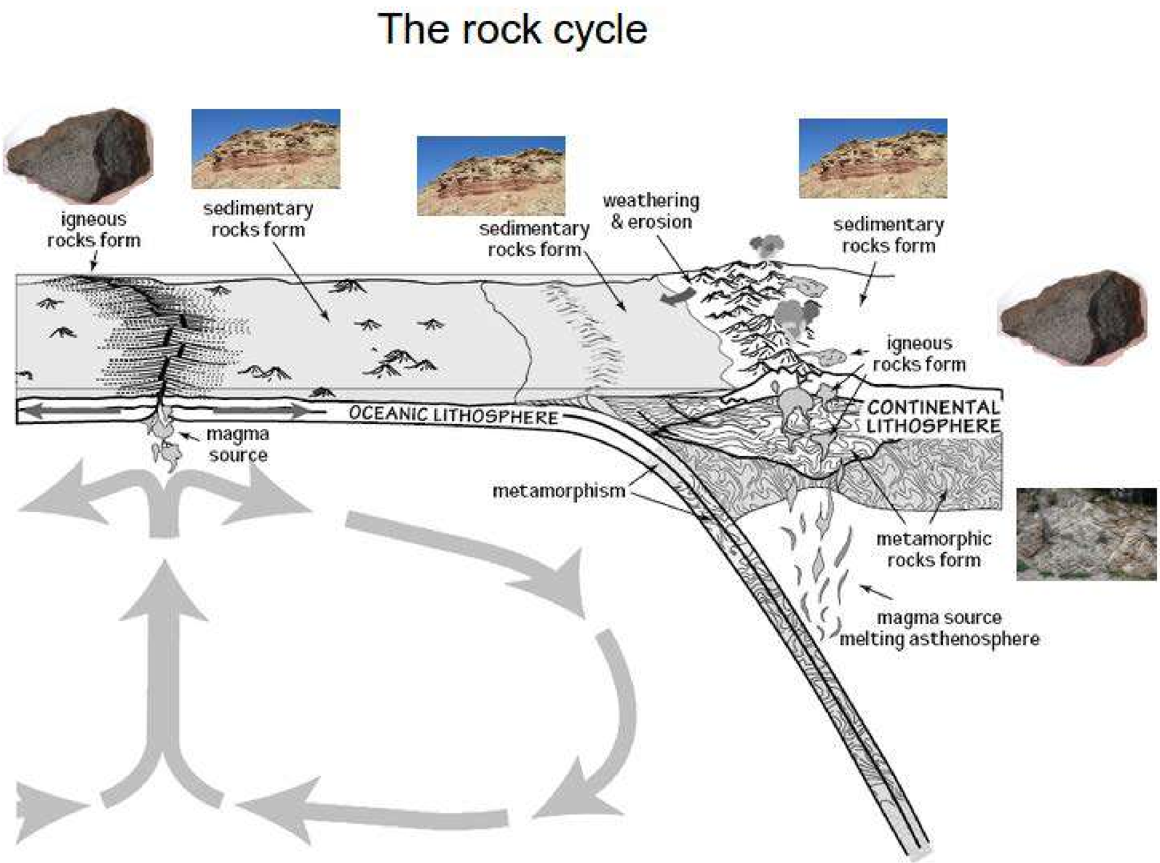
DEFINITION: *Mineral*

Minerals are natural compounds formed through geological processes. The term "mineral" includes both the material's chemical composition and its structure. Minerals range in composition from pure elements to complex compounds.

In this chapter we will mainly look at metal minerals (gold, copper, iron). There are also non-metal minerals (sand, stone) and fuel minerals (coal, oil).

A **rock** is a combination of one or more minerals. *Granite* for example, is a rock that is made up of minerals such as SiO_2, Al_2O_3, CaO, K_2O, Na_2O and others. There are three different types of rocks: **igneous**, **sedimentary** and **metamorphic**. Igneous rocks (e.g. granite, basalt) are formed when magma is brought to the Earth's surface as lava, and then solidifies. Sedimentary rocks (e.g. sandstone, limestone) form when rock fragments, organic matter or other sediment particles are deposited and then compacted over time until they solidify. Metamorphic rock is formed when any other rock types are subjected to intense heat and pressure over a period of time. Examples include slate and marble.

The figure below shows how these different types of rock are formed in the lithosphere.



Many of the elements that are of interest to us (e.g. gold, iron, copper), are unevenly distributed in the lithosphere. In places where these elements are abundant, it is profitable to extract them (e.g. through mining) for economic purposes. If their concentration is very low, then the cost of extraction becomes more than the money that would be made if they were sold. Rocks that contain valuable minerals are called **ores**.

FACT

A **gemstone** (also sometimes called a **gem**, **semi-precious stone** or **precious stone**), is a highly attractive and valuable piece of mineral which, when cut and polished, is used in jewellery and other adornments. Examples of gemstones are amethyst, diamond, cat's eye and sapphire.

As humans, we are particularly interested in the ores that contain metal elements, and also in those minerals that can be used to produce energy.

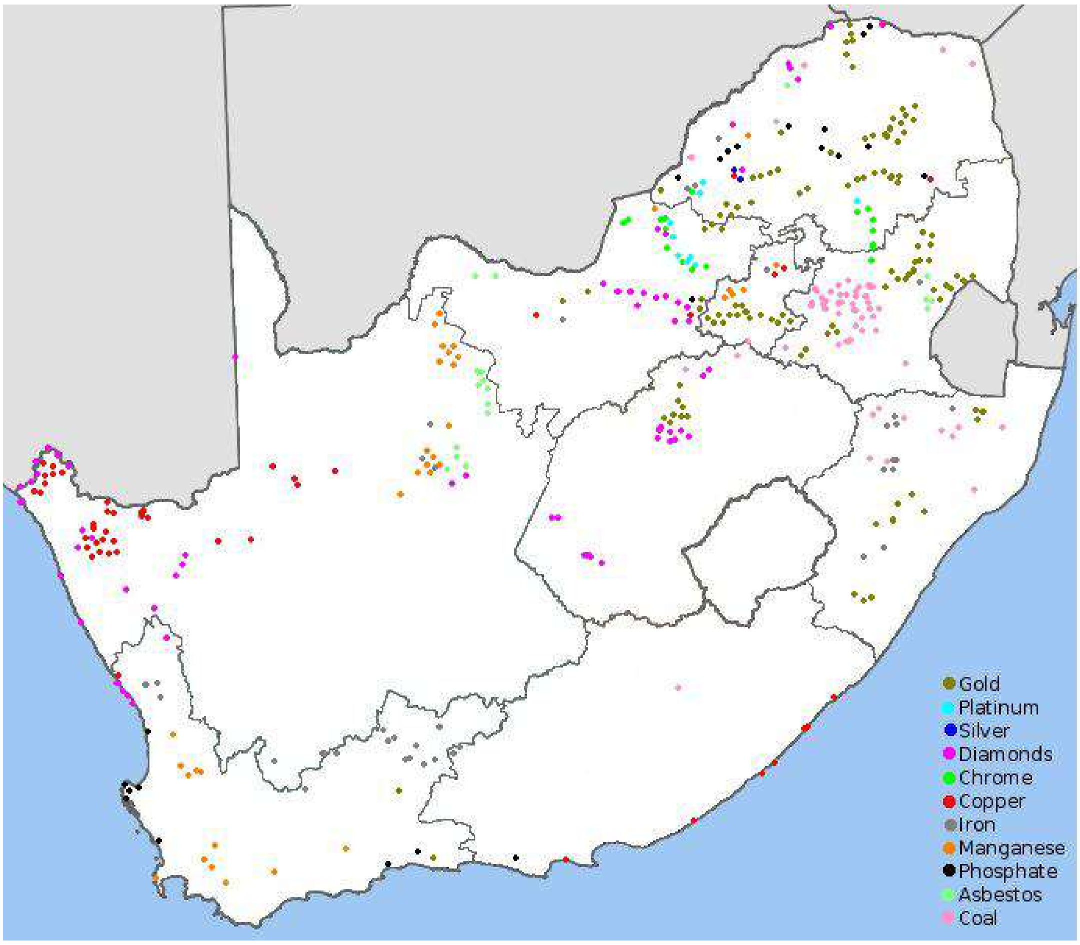


Figure 14.2: Location of minerals in South Africa.

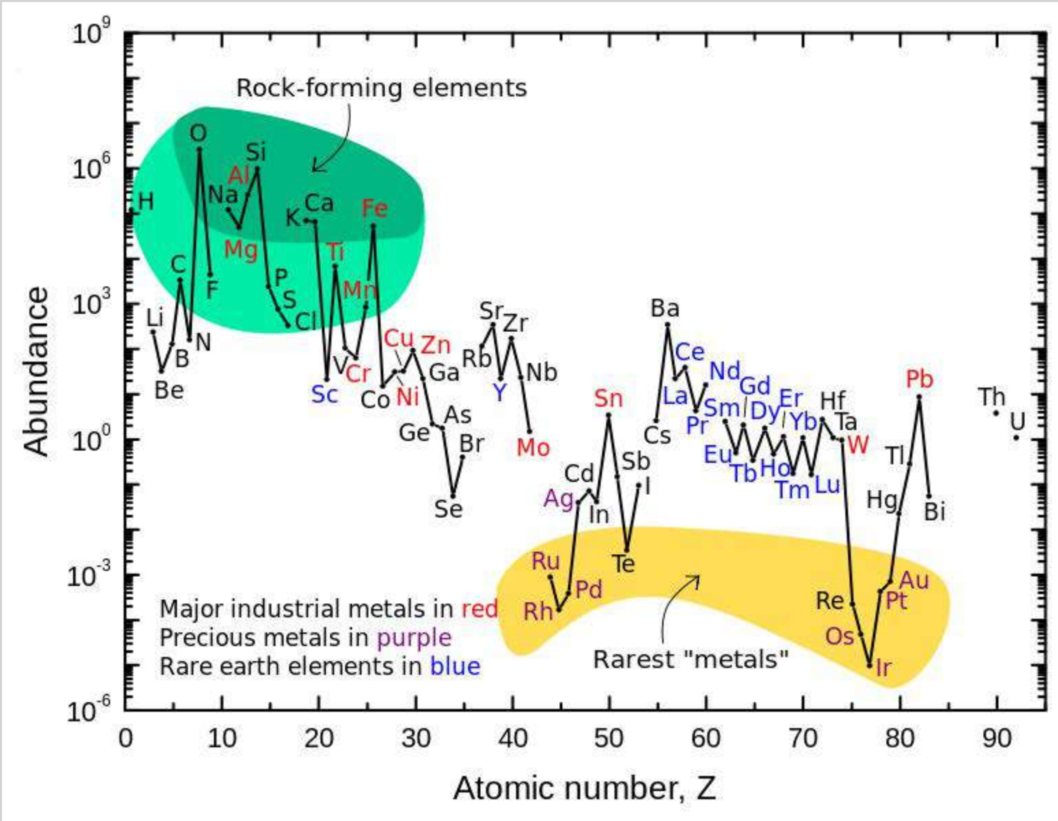
DEFINITION: *Ore*

An ore is a volume of rock that contains minerals which make it valuable for mining.

See video: 247M at www.everythingscience.co.za

Activity:

The following diagram shows the abundance of all the elements.



Look at the diagram and answer the following questions:

1. Name three of the rarest metals.
2. Name four rock-forming elements.
3. What element is the rarest?
4. What element is the most abundant?
5. Find the following elements on the image: gold, copper, iron, manganese, platinum, zinc, chromium, phosphorus, oxygen and carbon. Which of these is the most abundant? Which is the least abundant?
6. Classify the following elements as rock-forming, rarest metals and other: gold, copper, iron, manganese, platinum, zinc, chromium, phosphorus, oxygen and carbon.

14.3 Mining and mineral processing ESBRD

Now that we know where the minerals that mankind uses can be found, we can look at how he accesses the minerals. We have seen that often the minerals are not found just lying around waiting to be picked up, but rather are embedded in rocks and combined with other elements.

As man learnt more and made new discoveries, the techniques used to extract the ores

FACT
Indigenous people often have unique ways of finding mineral deposits, such as observing the behaviour of animals or noting the abundance of particular plants.

FACT
Coal miners used to take a canary with them down the mines. If the canary died, they knew that there was a high level of gas building up and that it was not safe to stay down in the mine.

improved and the amount of mineral that could be extracted increased greatly. Large tunnels started to be cut into the Earth to access minerals buried deep underground. New processing methods meant new kinds of metals could be mined.

The main stages in mining are: exploration, extraction, refining, manufacturing and marketing. We will take a brief look at exploration, extraction and refining.

Exploration ESBRF

Ancient humans did not worry about whether they could make money from the ores they mined. All they cared about was accessing metals and minerals that would help them eat, keep warm and beat the neighbouring tribes.

In modern times money matters and so the first step in mining is to find a suitably sized deposit of ore. For example, diamonds were found in Kimberley and gold ore was found in Johannesburg. These deposits were large enough to make mining for the gold and diamonds profitable.

As the number and size of known deposits is shrinking, geologists (who study the lithosphere) are developing new ways of finding suitable deposits. Geologists will often spend years learning how to find ores and then spend years exploring for the ores.

Obtaining the ore ESBRG

Once a suitably sized deposit of a mineral has been found, mining can begin. In ancient times mining and mineral processing were very primitive and the main method used to extract ores was digging by hand.

Mining is largely divided into surface and underground mining. Minerals are often found in river beds, beach sands and other sandy areas. These are known as alluvial deposits. These minerals can be fairly easily removed by surface mining techniques. Other minerals occur in long streaks known as veins or in pipes, and underground mining techniques are used to access these minerals.

Surface mining includes open pit mines, quarrying, strip mining and landfill mining. Coal and copper are often mined in this way.

Underground mining mainly consists of digging tunnels and shafts into the Earth's crust. Gold is often mined in this way. Underground mining is more risky than surface mining as tunnels can collapse and dangerous gases build up underground.

We will look more at underground and surface mining shortly.

Extracting the minerals ESBRH

Once the ore has been mined it is usually crushed into smaller pieces that can be processed more easily. The mineral then needs to be removed from the ore.

Ancient peoples soon learnt that fire could be used to refine and purify the metals. This technique is known as smelting. This was the earliest method used to extract the

mineral from the ore.

Smelting involves heating the ore to a very high temperature, often in a blast furnace. A reducing agent is usually added and the mineral is removed using chemical reactions.

In modern times several new techniques are used to extract the mineral from the ore, in addition to smelting. The most common methods are flotation, leaching and use of redox reactions.

Leaching involves mixing the ore with a carefully chosen liquid that dissolves either the mineral or the unwanted minerals. The liquid is often an acid.

Flotation involves the use of air bubbles to separate the valuable minerals from the unwanted rock. The valuable mineral becomes attached to the air bubbles and rises to the top of the mixture from where they can be removed.

The following table lists several metals and the typical methods used to mine and extract them.

Metal	Mining techniques	Extraction techniques
Gold	Underground mining (shaft mining) and surface mining.	Gold cyanidation is used. The ore is chemically treated to extract the gold.
Iron	Surface mining (open pit mining).	Smelting and chemical reduction.
Phosphate	Surface mining (open pit mining).	Treatment with acid.
Coal	Surface mining (open pit mining). Underground mining is now starting to become more common.	Coal is extracted in almost pure form.
Diamonds	Surface mining (alluvial deposits) and underground mining (pipe mining).	Diamonds are extracted from rocks and in almost pure form.
Copper	Surface mining (open pit mining).	Leaching is used to extract the copper using an acid.
Platinum	Underground mining (shaft mining).	Chemical methods and as a byproduct of copper mining.
Zinc	Underground mining (shaft mining).	Smelting and leaching.
Chromium (Chrome)	Surface mining (open pit mining) and underground mining (shaft mining).	Smelting, redox reactions.
Asbestos	Surface mining (open pit mining).	Extracted in fairly pure form.
Manganese	Surface mining (open pit mining) and underground mining (shaft mining).	Smelting and chemical processes.

Table 14.3: Table showing the mining and mineral extraction techniques for several minerals.

We will now discuss gold mining in more detail to gain a better understanding of the mining process.

Gold mining ESBRJ

Gold had long been known in Africa by the indigenous people, but in the late 1800's colonists found gold reefs and started exploiting the resource. Since then it has played a very important role (and often a controversial one too) in South Africa's *history* and *economy*. Its discovery brought many foreigners into South Africa, lured by the promises of wealth. They set up small mining villages which later grew into larger settlements, towns and cities. One of the first of these settlements was the beginning of present-day Johannesburg, also known as Egoli or Place of Gold.

Most of South Africa's gold is concentrated in the Golden Arc which stretches from Johannesburg to Welkom. Geologists believe that, millions of years ago, this area was a massive inland lake. Rivers feeding into this lake brought sand, silt, pebbles and fine particles of gold, depositing them over a long period of time. Eventually these deposits accumulated and became compacted to form gold-bearing sedimentary rock or **gold reefs**. It is because of this complex but unique set of circumstances that South Africa's gold deposits are so concentrated in that area. In other countries like Zimbabwe, gold occurs in smaller pockets which are scattered over a much greater area.

Mining the gold ESBRK

A number of different techniques can be used to mine gold and other minerals. The three most common methods in South Africa are **panning**, **open pit** (not typically used for gold) and **shaft** mining.

1. **Panning**

   Panning for gold is a manual technique that is used to sort gold from other sediments. Wide, shallow pans are filled with sand and gravel (often from river beds) that may contain gold. Water is added and the pans are shaken so that the gold is sorted from the rock and other materials. Because gold is much more dense, it settles on the bottom of the pan. **Pilgrim's Rest** in Mpumalanga was the first site for gold panning in South Africa.

2. **Surface mining (open pit mining)**

   This type of mining takes place from the surface of the Earth. In open pit mining, the topsoil is removed first and placed on the side of a trench. Then the surface layers of rock are removed to expose the deeper, valuable mineral layers. The valuable rocks are then blasted into smaller rocks using explosives. The rocks are loaded onto huge trucks and taken away for further crushing and processing. Any mineral that is found close to the surface (even up to 1000 m) can be mined using surface mining techniques. If minerals are found deeper, as is the case with most of the gold in South Africa, underground mining is used.

3. **Underground mining (shaft mining)**

   South Africa's thin but extensive gold reefs slope at an angle underneath the ground, and this means that some deposits are very deep and often difficult to reach. Shaft mining is needed to reach the gold ore. After the initial drilling, blasting and equipping of a mine shaft, tunnels are built leading outwards from the main shaft so that the gold reef can be reached. Shaft mining is a dangerous operation, and roof supports are needed so that the rock does not collapse. In addition the intense heat and high pressure below the surface make shaft mining very complex, dangerous and expensive. A diagram illustrating open cast and shaft mining is shown in Figure 14.3.

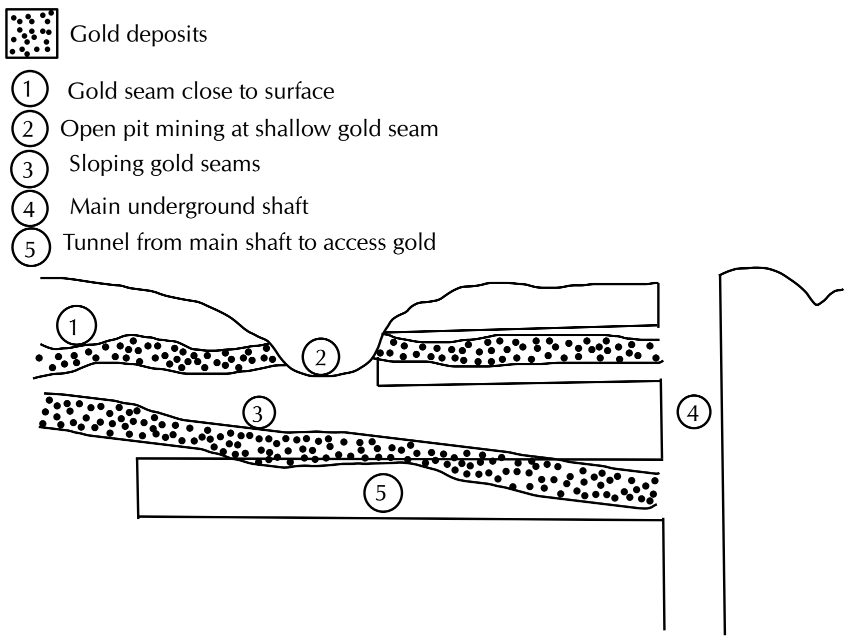


Figure 14.3: Diagram showing open pit and shaft mining.

Processing the gold ore

ESBRM

For every ton of ore that is mined, only a very small amount (about 5 g) of gold is extracted. A number of different methods can be used to separate gold from its ore, but one of the more common methods is called **gold cyanidation**.

In the process of gold cyanidation, the ore is crushed and then cyanide (CN^-) solution is added so that the gold particles are chemically separated from the ore. In this stage of the process, gold is oxidised. Zinc dust is then added to the cyanide solution. The zinc then takes the place of the gold, so that the gold is precipitated out of the solution. This process is shown in Figure 14.4.

STEP 1 - The ore is crushed until it is like a fine powder

↓

STEP 2 - A sodium cyanide (NaCN) solution is mixed with the finely ground rock

$$4Au + 8NaCN + O_2 + 2H_2O \rightarrow 4NaAu(CN)_2 + 4NaOH$$

Gold is **oxidised**.

↓

STEP 3 - The gold-bearing solution can then be separated from the remaining solid ore through filtration.

↓

STEP 4 - Zinc is added. Zinc replaces the gold in the gold-cyanide solution. The gold is precipitated from the solution. This is the **reduction** part of the reaction.

Figure 14.4: Flow diagram showing how gold is processed.

Characteristics and uses of gold

ESBRN

Gold has a number of uses because of its varied and unique characteristics. Below is a list of some of these characteristics that have made gold such a valuable metal:

- *Shiny*

  Gold's beautiful appearance has made it one of the favourite metals for use in jewellery.

- *Durable*

  Gold does not tarnish or corrode easily, and therefore does not deteriorate in quality. It is sometimes used in dentistry to make the crowns for teeth.

- *Malleable and ductile*

  Gold can be bent and twisted into shape, as well as being flattened into very thin sheets. For this reason gold is used to make fine wires and thin, flat sheets.

- *Good conductor*

  Gold is a good conductor of electricity and is therefore used in transistors, computer circuits and telephone exchanges.

- *Heat ray reflector*

  Because gold reflects heat very effectively, it is used in space suits and in vehicles. It is also used in the protective outer coating of artificial satellites. One of the more unusual applications of gold is its use in firefighting, where a thin layer of gold is placed in the masks of the firefighters to protect them from the heat.

Exercise 14 – 1: Gold mining

1. In Mapungubwe (in the Limpopo Province) there is evidence of gold mining in South Africa as early as 1200. Today, South Africa is a world leader in the technology of gold mining. The following flow diagram illustrates some of the most important steps in the recovery of gold from its ore.

$$\boxed{\text{Gold bearing ore}} \xrightarrow{\textbf{A}} \boxed{NaAu(CN)_2} \xrightarrow{\textbf{B}} \boxed{\text{Gold precipitate}} \xrightarrow{\textbf{C}} \boxed{\text{Pure gold}}$$

   a) Name the process indicated by A.

   b) During process A, gold is removed from the ore. Is gold oxidised or reduced during this process?

   c) Use oxidation numbers to explain your answer to the question above.

   d) Name the chemical substance that is used in process B.

   e) During smelting (illustrated by C in the diagram), gold is sent to a large oven called a furnace. Why do you think this process is needed, and explain what happens to the gold during this process.

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 247N

www.everythingscience.co.za m.everythingscience.co.za

Mining and the environment ESBRP

Environmental impacts of gold mining

However, despite the incredible value of gold and its usefulness in a variety of applications, all mining is undertaken at a cost to the environment. The following are just a few of the environmental impacts of gold mining:

- *Resource consumption*

  Gold mining consumes large amounts of electricity and water. Electricity is often generated using non-renewable resources.

- *Poisoned water*

  If the mining process is not monitored properly, acid and other chemicals from gold processing can leach into nearby water systems such as rivers. This causes damage to animals and plants, as well as to humans who may rely on that water for drinking.

- *Changing the landscape*

  This applies particularly to surface mines (open pit mines), where large amounts of soil and rock must be displaced in order to access the mineral reserves. The shape of the landscape can be changed when large amounts of rocks are dug up from the Earth and stacked on the surface. These are called mine dumps. Open pit mines also create very large holes (pits) in the ground that change the shape of the land.

- *Air pollution*

  Dust from open pit mines, as well as harmful gases such as sulphur dioxide and nitrogen dioxide, could be released from mining processes and contribute to air pollution.

- *Threaten natural areas*

  Mining activities often encroach on protected areas and threaten biodiversity in their operation areas.

Group discussion: Mine rehabilitation

There is a growing emphasis on the need to rehabilitate old mine sites that are no longer in use. If it is too difficult to restore the site to what it was before, then a new

type of land use might be decided for that area. Any mine rehabilitation programme should aim to achieve the following:

- ensure that the site is safe and stable
- remove pollutants that are contaminating the site
- restore the biodiversity that was there before mining started
- restore waterways to what they were before mining

There are different ways to achieve these goals. For example plants can be used to remove metals from polluted soils and water, and can also be used to stabilise the soil so that other vegetation can grow. Land contouring can help to restore drainage in the area.

In groups of 3-4, discuss the following questions:

1. What are the main benefits of mine rehabilitation?
2. What are some of the difficulties that may be experienced in trying to rehabilitate a mine site?
3. Suggest some creative ideas that could be used to encourage mining companies to rehabilitate old sites.
4. One rehabilitation project that has received a lot of publicity is the rehabilitation of dunes that were mined for titanium by Richards Bay Minerals (RBM). As a group, carry out your own research to try to find out more about this project.
    - What actions did RBM take to rehabilitate the dunes?
    - Was the project successful?
    - What were some of the challenges faced?

Group discussion: Mining and Mapungubwe

Read the following information about Mapungubwe.

Because of the history preserved at the site, Mapungubwe was declared a National Heritage Site in the 1980's and a World Heritage Site in 2003. The area surrounding it was declared a National Park in 1995. Unfortunately, the site is currently under huge threat from an Australian mining company who have been granted rights to construct an opencast and underground coal mine less than 6 km from the border of the National Park. There is much concern that mining operations will have a negative impact on the ecosystems, flora and fauna around Mapungubwe, and that it will hinder preservation of the site. Several environmental agencies have taken the matter to court in order to prohibit the planned mine and protect the environment around Mapungubwe.

As a class discuss, whether or not the proposed mining should go ahead. Divide into two teams. Assign one team to be in favour of the mining and one team against it. Each team should find facts relevant to their argument and then present it to the other team. At the end of the discussion, draw conclusions about whether or not the mining should take place.

Other minerals ESBRQ

We have largely focused on the process involved in mining gold. However gold is not the only mineral of interest to humankind. Many other minerals are useful such as copper, iron and platinum.

Activity: Other mining activities

Choose one of the minerals given in the list below and find information on how that mineral is mined. Use the information given in this chapter about gold mining and try to find similar information for the mineral you have chosen. Write up your findings in a report. If possible, split the class into groups and assign a different mineral to each group.

- Iron
- Phosphate
- Coal
- Diamond
- Copper
- Platinum
- Zinc
- Chrome
- Asbestos
- Manganese

14.4 Energy resources ESBRR

At present we rely mainly on oil and coal resources to provide our energy needs. However these resources are non-renewable energy sources and have many negative effects on the environment. Alternative sources of energy are constantly being investigated to assess if they will be able to provide for our growing energy needs. If we ignore these alternatives, one day we will run out of oil and coal reserves.

One of the most debated topics around energy resources is climate change. Scientists and environmentalists disagree as to the causes and effects of climate change and whether or not it actually exists.

Activity: Climate change

Climate change is a significant and lasting change in the distribution of weather patterns over periods of time ranging from decades to millennia. It is seen as changes in extreme weather events (freak storms, tornadoes, tsunamis, etc.) and changes in the average weather conditions for cities and regions.

Scientists collect data about climate change from ice core drilling, floral and faunal records, glacial and periglacial processes, isotopic analysis and from many other methods. Each of these methods provides different indicators of how the Earth's climate has changed over time.

Many factors influence the Earth's climate such as the amount of solar radiation received, changes in the Earth's orbit, changes on the Earth's surface, such as continental drift and size of polar ice caps, etc. All these factors combined make it difficult to pinpoint exactly what causes a particular change in climate. Also the effects of these events can take centuries to fully develop.

Human activities such as mining, burning of fossil fuels and agriculture also have an effect on the climate. Scientists are largely in agreement that the climate is changing irreversibly and that humans are a big cause of this change.

What is not agreed upon is how big an effect humans have and how much of a problem it is. At present global temperatures are found to be increasing and polar ice caps are found to be decreasing in size. Whether humans are simply making this worse or are actually causing the change is not yet understood.

Despite the debate, humans do need to be aware of their impact on the environment and will need to come up with ways to reduce it.

1. Discuss the above information.
2. Discuss ways in which human activities impact on the enviroment.
3. Discuss ways in which this impact can be reduced.
4. Conduct research on climate change and the debates about it. Critically analyse the information and draw your own conclusions. Discuss these ideas and conclusions with your classmates.

14.5 Summary ESBRS

See presentation: 247P at www.everythingscience.co.za

- The Earth's lithosphere is a rich source of many minerals.
- The lithosphere consists of the upper crust and mantle of the Earth.
- The minerals can be extracted using mining techniques.
- People have long been exploiting the Earth's mineral resources.
- Mining and mineral use has a significant environmental impact.
- Other resources in the lithosphere are fuels.

Exercise 14 – 2:

1. Give one word/term for each of the following descriptions:

    a) The part of the Earth that includes the crust and in which all minerals are found.

    b) The process in which minerals are extracted from the ores.

    c) An age in which humans discovered the use of fire to improve the properties of stone.

2. Read the following extract and answer the questions that follow.

    More profits, more poisons

    Since the last three decades gold miners have made use of cyanidation to recover gold from the ore. Over 99% of gold from ore can be extracted in this way. It allows miners to obtain gold flakes – too small for the eye to see. Gold can also be extracted from the waste of old operations which sometimes left as much as a third of the gold behind.

    The left-over cyanide can be re-used, but is more often stored in a pond behind a dam or even dumped directly into a local river. A teaspoonful of two-percent solution of cyanide can kill a human adult.

    Mining companies insist that cyanide breaks down when exposed to sunlight and oxygen which render it harmless. They also point to scientific studies that show that cyanide swallowed by fish will not 'bio-accumulate', which means it does not pose a risk to anyone who eats the fish. In practise, cyanide solution that seeps into the ground will not break down because of the absence of sunlight. If the cyanide solution is very acidic, it could turn into cyanide gas which is toxic to fish. On the other hand, if the solution is alkaline, the cyanide does not break down.

    There are no reported cases of human death from cyanide spills. If you don't see corpses, everything is okay.

    a) What is cyanidation?

    b) What type of chemical reaction takes place during this process: precipitation; acid-base; redox?

    c) Is the solution after cyanidation acidic, basic or neutral?

    d) How is the solid gold recovered from the solution?

    e) Refer to cyanidation and discuss the meaning of the heading of the extract: More profits, more poisons.

    DBE exemplar paper - Nov 2007

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 247Q   1b. 247R   1c. 247S   2. 247T

www.everythingscience.co.za   m.everythingscience.co.za

CHAPTER 15

Quantities used in the book

Physical Quantities		
Quantity	**Unit name**	**Unit symbol**
Acceleration (a)	metres per second squared	$m{\cdot}s^{-1}$
Charge (q)	coulomb	C
Concentration (C)	moles per cubic decimetre	$mol{\cdot}dm^{-3}$
Current (I)	amperes	A
Distance (d)	metre	m
Electrical energy (E)	joules	J
Electric field (E)	newtons per coulomb	$N{\cdot}C^{-1}$
Force (F)	newton	N
Induced emf ($\mathcal{E}$)	volt	V
Magnetic field (B)	tesla	T
Magnetic flux (ϕ)	weber	Wb
Mass (m)	kilogram	kg
Moles (n)	moles	mol
Pressure (p)	pascals	Pa
Power (P)	watts	W
Resistance (R)	ohms	Ω
Temperature (T)	kelvin	K
Tension (T)	newton	N
Time (t)	seconds	s
Voltage (V)	volts	V
Volume (V)	meters cubed	m^3
Weight (N)	newton	N

Table 15.1:

Units used in **the book**

1 Vectors in two dimensions

Exercise 1 – 2:

1. $\vec{R}_x = 1$ N
2. $\vec{R}_x$ is 1,3 N and $\vec{R}_y = -1$ N
3. $\vec{R}_x = 2$ N and $\vec{R}_y = 3$ N
4. $\vec{R}_x = -2{,}5$ N and $\vec{R}_y = 2{,}5$ N
5. $F_x = 2{,}8$ N and $F_y = -5$ N

Exercise 1 – 5: Algebraic addition of vectors

1. 28,6 N at $53{,}53°$
2. 25,4 N at $159{,}21°$
3. 2,69 N at $328{,}67°$
4. $186{,}4°$

Exercise 1 – 6:

1.
   - $F_x = 3{,}54$ N, $F_y = 3{,}54$ N
   - $F_x = 6{,}81$ N, $F_y = 13{,}37$ N
   - $F_x = -6{,}80$ N, $F_y = 9{,}02$ N
   - $F_x = -52{,}83$ N, $F_y = -113{,}29$ N
2.
   - $F_x = 9{,}22 \times 10^4$ N, $F_y = 5{,}99 \times 10^4$ N
   - $F_x = 13{,}2$ GN, $F_y = 7{,}04$ GN
   - $F_x = -2{,}54$ kN, $F_y = 11{,}01$ kN
   - $F_x = 9{,}14 \times 10^6$ N, $F_y = -8{,}52 \times 10^6$ N

Exercise 1 – 7:

3. $\vec{R}_x = 1$ N, $\vec{R}_y = 0$ N.
4. $\vec{R}_x = 1{,}6$ N, $\vec{R}_y = 0{,}2$ N
5. $\vec{R}_x = 1{,}4$ N, $\vec{R}_y = 1{,}7$ N
12. 3,25 N at $45°$
13.
   a) $F_x = 96{,}29$ N and $F_y = 41{,}87$ N
   b) $F_x = 13{,}95$ N and $F_y = 23{,}12$ N
   c) $F_x = 9{,}02$ N and $F_y = -6{,}8$ N
   d) $F_x = -62{,}97$ N and $F_y = -135{,}04$ N
   e) $F_x = 3{,}89$ N and $F_y = 14{,}49$ N
   f) $F_x = 14{,}43$ N and $F_y = 3{,}71$ N
   g) $F_x = -6{,}22$ N and $F_y = 9{,}43$ N
   h) $F_x = -136{,}72$ N and $F_y = 99{,}34$ N

14. have equal magnitudes but opposite directions.

15. 15 N due north and 7 N due west

16. $0°$

17. 4 N

18. 902,98 N

19. a) B

    b) 781 N

2 Newton's laws

Exercise 2 – 1:

1. Kinetic friction = 0 N and static friction = 20 N to the left.

2. a) 0 N for both static and kinetic frictional forces.

   b) Kinetic friction: 0 N and static friction: 5 N

   c) 8 N

   d) 4 N

   e) Kinetic friction: 4 N and static friction: 0 N

3. a)

$$F_s \leq \mu_s N$$

Exercise 2 – 2:

1. $\vec{F}_{gy} = 172{,}30$ N in the negative y-direction, $\vec{F}_{gx} = 422{,}20$ N in the negative x-direction.

2. $42{,}61°$

Exercise 2 – 3: Forces and motion

1. c) $\vec{F}_R = 55$ N to the right

2. c) $\vec{F}_f = 346{,}41$ N

Exercise 2 – 5:

2. 1700 N

3. 12 N

4. $0{,}1\ \text{m·s}^{-2}$

5. 17,5 N

6. 20 kg

7. $0{,}15\ \text{m·s}^{-2}$

8. 8,33 kg
9. 0,25 $m{\cdot}s^{-2}$ and 1 $m{\cdot}s^{-2}$
10. a) 40 N
    b) 20 $m{\cdot}s^{-2}$
11. a) 5 $m{\cdot}s^{-2}$
    b) 100 $m{\cdot}s^{-1}$
    c) 7 s
    d) 562,5 m
12. a) 100 N
    b) −25 N
    c) 173,2 N
13. 8,5 N
14. 4,2 $m{\cdot}s^{-2}$
16. 2500 N
17. 6 N
18. a) 10,2 $m{\cdot}s^{-2}$
19. a) −10 $m{\cdot}s^{-2}$
    b) 10 000 N
20. 277,76 N, 113,35 N and 45°
21. 25,53°
22. 25,53°
23. a) −1340 N
    b) 562,5 N
24. a) −441,14 N
    b) 20,97 N

Exercise 2 – 6:

1. the same
2. A or B

Exercise 2 – 7:

1. $1{,}92 \times 10^{18}$ N
2. $8{,}80 \times 10^{17}$ N
3. Earth: $5{,}65 \times 10^{6}$ m, Jupiter: $1{,}00 \times 10^{8}$ m
4. Jupiter: 24,8 $m{\cdot}s^{-2}$ Moon: 1,7 $m{\cdot}s^{-2}$
5. a) 7×10^{-7} N
   b) $1{,}35 \times 10^{-6}$ N
   c) 0,52
6. a) $4{,}7 \times 10^{12}$ N
   b) $9{,}4 \times 10^{16}$ N

Exercise 2 – 8:

1. a) 323,75 N
   b) 326,04 N
   c) 980,88 N
2. Object 2

Exercise 2 – 9:

1. 5 F
2. decrease
3. $r - R = 2R - R = R$
4. $\frac{1}{9}$
5. 2 M
7. $6{,}67 \times 10^{-12}$ N
8. 0,13 m
9. 0,5 kg
10. a) Newton's law of universal gravitation
    b) It is the same
    c) increases
    d) $2{,}37 \times 10^{7}$ N

Exercise 2 – 10: Forces and Newton's Laws

2. a) non-contact
   b) contact
   c) contact
   d) non-contact
4. acceleration
5. 2 N
6. 250
7. The box moves to the left.
8. 200
9. $F cos 60° - ma$ in the direction of A
10. his inertia
12. The resultant of the forces is zero.
13. F
14. F is the reaction to the force that the rocket exerts on the gases which escape out through the tail nozzle.
15. move to the left
16. b) 19,6, 49
17. increase only
18. graph (b)
19. 40 N
21. $\frac{5}{4}a$
22. There is zero resultant force.
23. The load has a weight equal in magnitude to F.
24. It accelerates to the left, moving along the smooth horizontal surface.
25. A zero resultant force acts on the passenger.
26. b) 696 000 N
    c) 2784 N
27. b) i. 5 m·s^{-2}
       ii. 5000 N
    c) 2500 N
29. a) 11 760 N
    b) $-2{,}25$ m·s^{-2}
    c) 9060 N
30. a) 1,33 m·s^{-2}
    b) 26,7 m·s^{-1}
    c) 1,25 m·s^{-2}
    d) 51,6 m·s^{-1}
31. b) $W_{\perp} = 16{,}86$ N and $W_{||} = 24{,}01$ N
32. c) 1171,28 kg
33. b) 0,33 m·s^{-2}
    c) $F_f = 300$ N
    e) 50 m
34. a) $-6{,}67$ m·s^{-2}
    c) 3,0 s
    d) 2 s

35. $\sin\theta = \frac{N}{w}$

36. Force F_3 is the resultant of forces F_1 and F_2.

37. c) $1{,}63$ N

38. b) i. $5497{,}2$ N

ii. $204{,}17$ kg

39. a) 2352 N

b) 1568 N

c) $2715{,}9$ N

d) $1810{,}6$ N

Exercise 2 – 11: Gravitation

1. $9F$

2. weight of the object

3. a) $778{,}2$ N

b) $3112{,}8$ N

4. $8{,}2 \times 10^{-2}$ m

5. a) 448 N

b) 192 N

6. 5488 N

7. $1{,}6 \times 10^{6}$ N

9. $4{,}1 \times 10^{22}$ N

11. a) Agree

b) Disagree

c) Disagree

12. a) 3000 N

b) $76{,}53$ kg

13. $1{,}8$ N

14. b) $1{,}63$

c) on the moon

3 Atomic combinations

Exercise 3 – 5: Atomic bonding and Lewis diagrams

3. a) Nitrogen: 5, hydrogen: 1
Carbon: 4, hydrogen: 1

c) ammonia and methane

4. a) 6

b) 1

c) 2 single bonds

d) Y: oxygen and X: hydrogen.

Exercise 3 – 6: Molecular shape

1. linear

2. linear

3. trigonal pyramidal

4. octahedral

5. linear

6. tetrahedral

7. bent or angular

8. trigonal planar

Exercise 3 – 7:

1. $1{,}0$, $0{,}4$, $3{,}0$, $0{,}6$ and $1{,}0$

Exercise 3 – 8: Electronegativity

1. a) $2{,}1$
   b) $3{,}0$
   c) Hydrogen will have a slightly positive charge and chlorine will have a slightly negative charge.
   d) Polar covalent bond
   e) polar

Exercise 3 – 9:

1. a) Bond length
   b) Covalent bond
   c) Electronegativity
2. Dative covalent (co-ordinate covalent) bond
4. $\text{Ne} + \text{Ne} \rightarrow \text{Ne}_2$
6. a) 1
   b) 5
   c) X could be hydrogen and Y could be nitrogen.
7. a) linear
   b) linear
   c) trigonal planar
   d) linear
   e) tetrahedral
   f) tetrahedral
   g) linear
   h) octahedral
9. a) non-polar
   b) non-polar
   c) non-polar
   d) polar

4 Intermolecular forces

Exercise 4 – 1:

1. dipole-dipole
2. induced dipole
3. ion-dipole
4. induced dipole

Exercise 4 – 4:

1. a) Intermolecular force
   b) Polar molecule
   c) Specific heat
2. The melting point of NH_3 will be higher than for Cl_2
3. a) HI and NH_3 only
4. a) Water

5 Geometrical optics

Exercise 5 – 1: Rays and Reflection

3. a) E
   b) C
   c) D
   d) B
   e) A
6. B
7. $15°$
8. $45°$
9. $65°$
10. $25°$

Exercise 5 – 2: Refractive index

1. $2{,}29 \times 10^{8}$ m·s^{-1}
2. crown glass

Exercise 5 – 4: Snell's Law

2. b) $25{,}88°$
3. $18{,}08°$
4. a) decreases
   b) remains the same
   c) Towards the normal.
5. a) increases
   b) remains the same
   c) Away from the normal
7. $1{,}327$
8. $30{,}34°$
9. $78{,}07°$
10. A: acetone, B: fuzed quartz, C: diamond, D: cubic zirconia, E: 80% sugar solution
11. c) water

Exercise 5 – 5: Total internal reflection and fibre optics

3. No
4. no
5. $47,33°$
6. The light will be trapped in the diamond.
7. a glass to water interface
8. $41,8°$
9. $62,46°$
10. $2,42°$
11. $37,86°$

Exercise 5 – 6: End of chapter exercises

1. a) normal
   b) refraction
   c) reflection
2. a) False
   b) True
   c) True
4. $35°$
5. b) $27,78°$
   c) increases
   d) stays the same
6. sapphire

6 2D and 3D wavefronts

Exercise 6 – 1:

2. light waves

Exercise 6 – 2:

2. blue, green, yellow, red
4. a) $3,4 \times 10^{-6}$ m
   b) $2,4 \times 10^{-6}$ m

7 Ideal gases

Exercise 7 – 2: Boyle's law

1. 100 kPa
2. $112,5$ kPa
3. $8,33$ cm^3
4. c) Yes
5. a) 217 Pa

Exercise 7 – 3: Charles' law

2. $4,67\text{ dm}^3$
3. $56,67\text{ K}$

Exercise 7 – 4: Pressure-temperature relation

1. c) linear relationship
2. $7,24$ atm
3. $8,14$ atm
5. $465,6\text{ K}$

Exercise 7 – 5: The general gas equation

1. $0,87$ atm
2. $0,6$ atm
3. $1,26$ atm

Exercise 7 – 6: The ideal gas equation

1. $0,008\text{ m}^3$
2. $106,54$ kPa
3. $0,02$ mol
4. b) No
   c) $193,3$ kPa
5. a) $73,1\text{ g}$

Exercise 7 – 7:

1. a) Ideal gas
   b) Charles' law
   c) Temperature
2. Temperature must be kept constant.
3. The sample of helium gas will be at the greatest pressure.
4. $V \propto n$, with p, T constant.
7. b) $311,775$ Pa
8. $106,39$ kPa

9. c) 322,0 kPa
   d) 246,31 g
10. 1363,0 kPa
11. 5,78 L
12. a) 201,9 K
    b) Decreased.
13. d) 64 kPa
    e) Increases.

8 Quantitative aspects of chemical change

Exercise 8 – 1:

1. 2 dm^3

Exercise 8 – 2: Gases and solutions

1. 7 dm^3
2. a) 4,56 $mol{\cdot}dm^{-3}$
   b) 60,95 g
3. 1,105 $mol{\cdot}dm^{-3}$
4. 0,24 $mol{\cdot}dm^{-3}$
5. 0,185 $mol{\cdot}dm^{-3}$

Exercise 8 – 3:

1. 2,74 kg

Exercise 8 – 4:

1. 65,69%

Exercise 8 – 5:

1. $C_2H_2O_4$

Exercise 8 – 6:

1. 69%

Exercise 8 – 7: Stoichiometry

1. 1,69 kg
2. 88,69%
3. C_6H_6
4. 93,5%
5. 48,6%

Exercise 8 – 8: Gases 2

2. 0,224 dm^3

Exercise 8 – 9:

1. a) Limiting reagent
   b) Empirical formula
   c) Theoretical yield
   d) Titration
2. 67,2 dm^3
3. 1,8 dm^3
4. 0,1 $mol{\cdot}dm^{-3}$
5. 58%
6. a) $2NaOH + H_2SO_4 \rightarrow Na_2SO_4 + 2H_2O$
   b) 0,03 mol
   c) the sodium hydroxide will be fully neutralised
7. 1,52 $mol{\cdot}dm^{-3}$
8. a) O_3: 0,0154 mol and NO: 0,0223 mol
   b) O_3
   c) 0,71 g
9. 96,9%
10. a) 88,461 g
    b) 66,051 dm^3
    c) 77,69 g
11. $C_2Cl_2F_4$
12. 48,6%
13. a) 1,33 dm^3
    b) Yes

9 Electrostatics

Exercise 9 – 1: Electrostatic forces

1. $1,35 \times 10^{-2}$ N
2. $1,27 \times 10^{-3}$ N
3. $0,26$ N
4. $0,56$ N
5. $0,2$
6. 9000 N
7. $4,6 \times 10^{-3}$ m
8. $8,77$ N $to\ the\ right.$
9. $4,2 \times 10^{-8}$ C
10. $3,42 \times 10^{-5}$ N acting at an angle of $83,8°$ to the negative x-axis
11. $1,52 \times 10^{-6}$ N acting at an angle of $82,80°$ to the positive x-axis
12. $1,05 \times 10^{-4}$ N acting at an angle of $34,76°$ to the positive x-axis

Exercise 9 – 2: Electric fields

1. $0,15$ N·C^{-1}
2. $-5,8 \times 10^{-8}$ N·C^{-1}

Exercise 9 – 3:

1. $8,44 \times 10^{-7}$ N·C^{-1}
2. a) $1,35 \times 10^{-3}$ N·C^{-1}
   b) -2 nC
   c) $1,00 \times 10^{-5}$ N·C^{-1}
3. $3,3 \times 10^{-4}$ m
5. $\frac{1}{3}F$
6. a
7. b) $1,5 \times 10^{-5}$ N·C^{-1}
8. $2E$

10 Electromagnetism

Exercise 10 – 1: Magnetic Fields

3. a) Out of the page
   b) into the page
4. A - D: counterclockwise, E - H: clockwise

Exercise 10 – 2: Faraday's Law

5. 2,45 V
6. a) $-0{,}14$ V
   b) 0,22 m
7. a) 28,8 Wb
   b) 9,22 s

Exercise 10 – 3:

3. a) 0,1225 Wb
   b) The change in flux is 0,117 Wb and the induced emf will be zero because the cardboard is not a conductor.
4. 3,4 Wb and 0 Wb
5. 7,26 Wb
6. 9,34 V
7. a) 0,00249 V
   b) 0,014 m
8. 0,012 Wb
9. 2,65 V

11 Electric circuits

Exercise 11 – 1: Ohm's Law

1. b) straight-line
   c) 0,13
   d) Yes

Exercise 11 – 2: Ohm's Law

1. 4 Ω
2. 3 A
3. 15 V

Exercise 11 – 3: Series and parallel resistance

1. 20 kΩ
2. 90 Ω
3. 9,09 kΩ
4. 6 Ω
5. a) 1,2 Ω b) 0,48 Ω c) 5 Ω d) 10 Ω

Exercise 11 – 4: Ohm's Law in series and parallel circuits

1. 2 Ω
2. 1,8 A
3. 0,75 A
4. 12 A
5. 30 Ω
6. a) 26 A
   b) $R_1 = 20$ A, $R_2 = 4$ A, $R_3 = 2$ A

Exercise 11 – 5: Series and parallel networks

1. a) 3,33 Ω
   b) 10,67 Ω
   c) 2,652 Ω
2. a) 4,52 A
   b) 0,59 A
3. Total voltage: 12 V, $R_1 = 9{,}32$ V, $R_2 = 2{,}66$ V, $R_3 = 2{,}66$ V,
4. a) 2,5 A
   b) 3,75 V
   c) 1,25 A

Exercise 11 – 6:

1. $2{,}00 \times 10^{12}$ W
2. 0,99 W
3. a) 1,5 W
   b) 7,5 W
4. a) 6,48 W, 1,44 W, 2,88 W
   b) 1,88 W, 1,25 W, 0,16 W, 0,32 W
   c) 30,2 Ω, 0,89 W, 1,17 W, 5,83 W, 3,5 W
5. a) 2,64 A
   b) 0,34 A
   c) 1,7 W
6. 18,64 W, 10,72 W, 5,36 W

Exercise 11 – 7:

1. a) Voltage
   b) Electrical power
   c) Ohm's law
2. 0,5 A
3. 17 Ω
4. 1,05 Ω
5. 24 W
6. 0,5 A, 1,5 V, 5 V, 2,5 V
7. a) 0,09 A
   b) 2,25 V
   c) 13,5 V
8. a) 38,4 Ω
   b) 270 kJ
9. b) $-1{,}014 \times 10^{-16}$ J

12 Energy and chemical change

Exercise 12 – 1: Exothermic and endothermic reactions 1

1. a) taken in
   b) released
   c) released
   d) taken in

2. a) Exothermic
   b) Endothermic
   c) Exothermic
   d) Endothermic

Exercise 12 – 2: Endothermic and exothermic reactions

1. a) Exothermic
   b) Exothermic
   c) Endothermic
   d) Exothermic

2. a) Endothermic
   b) Exothermic
   c) Exothermic
   d) Endothermic
   e) Exothermic
   f) Exothermic

Exercise 12 – 3: Energy and reactions

1. Endothermic

2. a) -15 kJ
   b) 0 kJ
   c) 15 kJ
   d) 40 kJ

Exercise 12 – 4:

1. a) Activation energy
   b) Bond energy
   c) Exothermic reaction
   d) Endothermic reaction

2. Energy is released when the bonds in H_2O form.

3. The energy of the product is less than the energy of the reactants.

4. The second graph (b) is correct.

6. a) Negative
   c) Photosynthesis
   d) Endothermic

13 Types of reactions

Exercise 13 – 1: Acids and bases

2. c) Amphoteric

Exercise 13 – 3:

1. $HNO_3(aq) + KOH\ (aq) \rightarrow KNO_3\ (aq) + H_2O\ (l)$

Exercise 13 – 4:

1. $2HBr\ (aq) + K_2O\ (aq) \rightarrow 2KBr\ (aq) + H_2O\ (l)$

Exercise 13 – 5:

1. $2HCl\ (aq) + K_2CO_3(aq) \rightarrow 2KCl\ (aq) + H_2O\ (l) + CO_2(g)$

Exercise 13 – 7: Oxidation numbers

1. a) Magnesium: $+2$, fluorine: -1
   b) Calcium: $+2$, chlorine: -1
   c) Hydrogen: $+1$, carbon: -4
   d) Magnesium: $+2$, oxygen: -2, sulfur: $+6$

Exercise 13 – 8: Redox reactions

1. Fe is oxidised and O_2 is reduced
2. $$FeCl_3(aq) + 2H_2O\ (l) + SO_2(aq) \rightarrow H_2SO_4(aq) + 2HCl\ (aq) + 2FeCl_2(aq)$$

Exercise 13 – 9:

1. a) Redox reaction
   b) Bronsted-Lowry base
   c) Oxidation
   d) Amphoteric substance
2. HNO_3 donates protons and is a Bronsted-Lowry acid
3. Br^- is the oxidising agent (chlorine is reduced in this reaction)
4. H_2SO_3 is the Bronsted-Lowry acid and KOH is the Bronsted-Lowry base.
6. $2HCl\ (aq) + Mg(OH)_2(aq) \rightarrow MgCl_2(aq) + 2H_2O\ (l)$
7. $2HCl\ (aq) + Na_2CO_3(aq) \rightarrow 2NaCl\ (aq) + H_2O\ (l) + CO_2(g)$
8. $2H_3PO_4(aq) + 3CaO\ (s) \rightarrow Ca_3(PO_4)_2\ (aq) + 3H_2O\ (l)$
10. a) i. -1
       ii. -2
       iii. 0
13. $Cd\ (s) + 2NiO(OH)\ (s) + 2H_2O\ (l) \rightarrow Cd(OH)_2(s) + 2Ni(OH)_2(s) + 2OH^-(aq)$

14 The lithosphere

Exercise 14 – 1: Gold mining

1. b) Gold is oxidised.
   d) Zinc

Exercise 14 – 2:

1. a) Lithosphere
   b) Minerals processing or extraction
   c) Middle Stone Age
2. b) Redox reaction
   c) acidic

List of Definitions

Image Attribution

VERSION 1 CAPS

GRADE 11
PHYSICAL SCIENCES

WRITTEN BY VOLUNTEERS

THIS TEXTBOOK IS AVAILABLE ON YOUR MOBILE

This book is available on web, mobi and mxit.
Read, practice intelligently or see solutions at m.everythingscience.co.za

ISBN 978-1-920423-83-4

SIYAVULA
TECHNOLOGY-POWERED LEARNING